Reconfigurable Flight Control Methods and Related Issues
A Survey

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Chapter 1

Current research topics

"Control reconfiguration" has a broad meaning. This kind of technique is applied mainly in three situations:

1. the establishment of the system operating regime
2. performance improvement during operation
3. as part of fault accommodation.

Control reconfiguration is a critical technology. All methods presume a certain initial knowledge of the system. In time, this may change gradually due to environmental effects or rapidly on account of faults. Hence, various impairments may significantly degrade the performance of the aircraft as well as the decoupling of the aircraft longitudinal and lateral axes. A reconfigurable controller possesses the capability to redistribute and coordinate the control effort during a system failure among the aircraft's remaining effective control surfaces, such that satisfactory flight performance is retained if possible. As the system changes the necessity for updating the model and the control strategy increases. The flight control system is designed not only to cope with failures but to be robust with respect to variations in aerodynamic stability derivatives, high frequency unmodelled dynamics, time delays, sensor noise and atmospheric turbulence. If the aircraft dynamics is altered due to uncontrollable changes such as failed sensors, actuators or damaged structural elements the reconfigurable control should ensure a response to pilot's command inputs still close to the nominal performance if possible.

Reconfigurable control tries to avoid excessive hardware replication. Of course, the set of redundant components allows the flight control system to continuously perform its functions in spite of the failures some of its components might have. The trade off, when is about replication, is between the weight, size, complexity and power consumption of such redundant sets of instruments. Instead of relying on hardware redundancy a reconfigurable control system has to exploit inherent control redundancies in order to restructure in real time and preserve stability and tracking. Increased survivability and reduced support requirements involve redundant aerodynamic control surfaces and excessive power during manoeuvres.

As a validation scheme the adaptive control reconfiguration concepts use the full non-linear simulation of the aircraft, the main performance goal being to land the aircraft safely whenever possible. Owing to the time constraints, in many failure scenarios the automated control law redesign process has to make use of the algorithms as numerically efficient as possible. When an impairment occurs the most urgent task is to stabilise the system as to avoid catastrophic situations. Once the stabilisation has been carried out, there is more time for the control reconfiguration mechanism to manipulate the control in order to obtain better system performance.

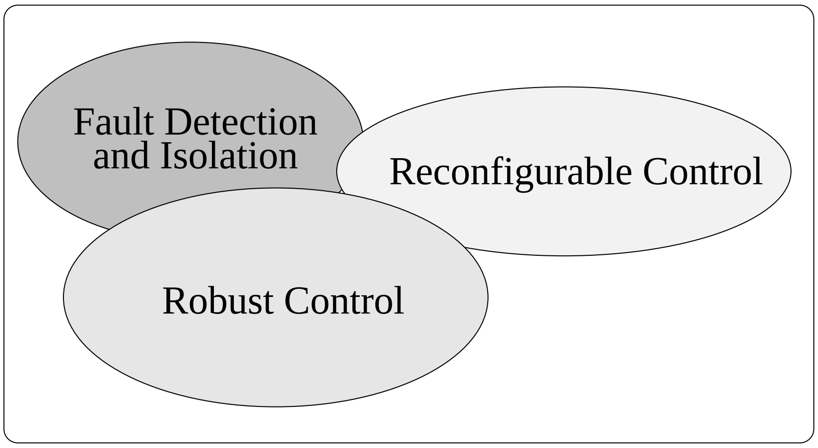

Figure 1.1: The Venn diagram of current research topics

Three main research topics rise at the moment the interest of the control community. All of them closely inter-related and contributing towards an integrated way of tackling control issues. The Figure 1.1 shows in a Venn diagram, originally depicted in [Pat96] the three topics. Actually in his survey [Pat96] the author discuses from the perspective of a researcher, involved for a long period of time in fault tolerant control, the contributions made by various authors especially in the areas situated at the intersection in between these main topics.

Chapter 2

Requirements for control reconfiguration

Considerable effort has to be put into all stages of the system design in order to enable it to cope with the various requirements of reconfigurable control.

Before carrying the effective design process, an integrated and systematic understanding of the system structure, reliability and types of redundancy in sensors and actuators is required.

2.1 System requirements

There are authors focusing on these various aspects of reconfigurable system design, among them relevant studies were generated by [Bla95, Pat96].

According to [Bla95] several steps must be taken prior to designing a reconfigurable control system:

1. Essential properties of the system must be designed with respect to previous and likely fault conditions.

2. All potential failures and their effects have to be systematically determined, if possible. Techniques such as failure mode effect analysis (FMEA) can be used and their results eventually used in a supervisor type of algorithm.

3. System reliability analysis should provide information on various component failure rates. Additionally a reliability distribution chart must be produced.

Practical control problems often involve more actuators and sensors than are needed for designing a control system in an economic way. As a result an appropriate set of actuators and sensors must be selected from the available candidates and then partitioned and paired for decentralised control. In general establishing a control structure refers to both actuator/sensor selection and the process of partitioning and pairing.

The choice of partitioning/pairing problem in the case of decentralised control was studied by [SP96] and many practical tools such as relative gain array and other interaction measures have been developed. The RGA is used as a measure of the sensitivity of a control structure to diagonal input uncertainty.

The complementary problem of actuator/sensor selection was addressed in [LBMP95]. Its authors try to answer the question "What makes a control structure more desirable than others?". Screening tools that select the desirable candidate from a set of control structures were developed relying on measuring the potential performance of the candidates.

Most of the research in the field was carried out in the stochastic setting and in general the criteria developed were based on the nominal performance like in [HMW80]. More advanced and systematic developments for this issue were provided by [CT78] relying on the "Condition Number Criterion" which is valid for some types of model redundancy.

Other advances were made by [LM91] suggesting a criterion based on the structured singular value, a criterion that can deal with structured type of uncertainty. It is possible to see now, after having browsed through several techniques, that most of the criteria rely on a particular uncertainty description or a specific design approach.

On another hand during operation we encounter similar problems in the sense that plant inputs and/or outputs can be lost or appear at any moment as an effect of diverse hardware failures. This will lead to a dynamic change of degrees of freedom degrees available to the controller.

With respect to the two distinctive notions: controlled and manipulated variables we are able to provide a system classification, used by [KDG96], in square (rare), "fat" (more manipulated than control variables, a type common in the real world) and "slim" plants (the reverse case to the "fat" one). The case of a "fat" plant is desired due to the extra degrees of freedom that can be used either together with additional objectives or in the case of reconfiguration.

In the case of reconfigurable control the regulator has to be designed in such a way that even if signals are lost it will stay awake trying to make the best of the sub-plant in control in terms of stability and performance. Unfortunately a well conditioned plant might contain a number of poorly conditioned sub-plants.

We consider that another problem raised by the existence of such diverse sub-plants is that the controller has to provide them with a similar behaviour in closed loop, which might cause completely different weights to be used in the process of translating the control requirements in method specific parameters and might require sophisticated automatic tuning procedures.

2.2 Types and degrees of redundancy

It is impossible to ensure control reconfiguration in the case of actuator or sensor failure without redundancy in the initial system subjected to control. As an inherent part of the design process for control reconfiguration the nature and location of redundancy available has to be analysed. If the level of safety for the closed loop does not match the level required then additional redundancy has to be provided at this stage in either direct or indirect form. Aerospace applications usually take advantage of both types of redundancies so as to cut the costs of flight control systems without affecting their safety.

The redundancy rate therefore has a direct connection with the system reliability and the complexity of the FDI system in use. The type and level of redundancy provided determines the way in which control reconfiguration is provided. Hence the failed sensors or actuators in systems with redundancy will imply sometimes dramatic changes in the controller structure.

2.2.1 Direct redundancy

By definition direct redundancy means that a number of independent hardware channels are used. Generally a voting system is used in order to decide in between the good and impaired channels. A discussion of such voting systems is provided in [Wes77].

As long as several hardware devices automatically provide redundant signals the analysis being carried out in the design process will detect them and use them to keep the number of physical devices low. Often, different channels are supplied with different types of sensors, actuators or even software that have the same input-output features. Actually what is referred to by parallel redundancy is equivalent to direct redundancy with dissimilar pieces of hardware and/or software.

2.2.2 Analytical redundancy

In most control systems that rely on dynamic models there is an amount of built in analytical redundancy. For example [LP93] in a flight control system the roll rate p can be estimated/computed analytically by assuming measurements of yaw (Ψ) and roll (Φ) angle derivatives obtained via a filter having the Laplace transform $\frac{1}{1+0.1s}$.

Other researchers [Wil76] are applying this method backing up available measurements using the mathematical model. Kalman filtering can be considered as another means of providing redundancy but, the accuracy of this methodology suffers from the system nonlinearty and estimation errors.

Chapter 3

Control reconfiguration methods

Reconfigurable control can be tackled by various methods described in a succinct manner by the tree chart depicted in Figure 3.1. As we are able to see from this figure the precise nature of reconfigurable control depends upon whether the approach is passive or active.

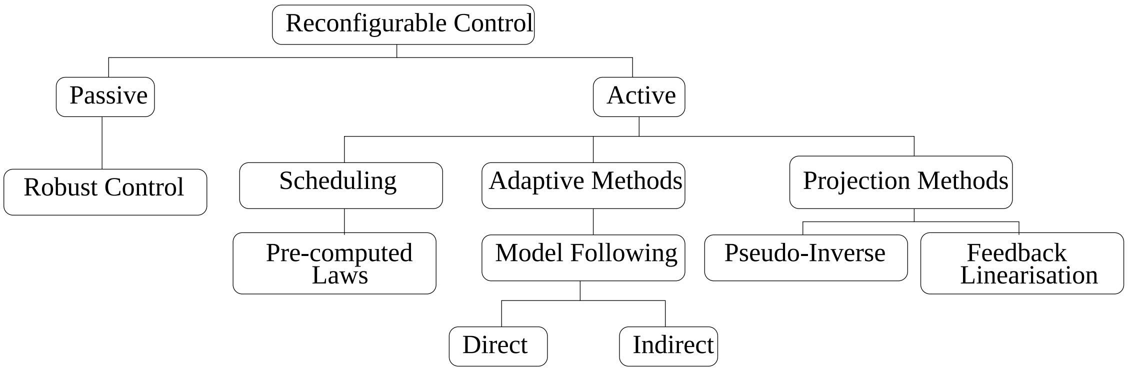

Figure 3.1: The tree diagram of reconfigurable control methods

3.1 Passive methods

The passive approaches use the robust control theory and techniques [KDG96, GL95] to ensure robust stability of the closed loop in face of certain faults that can be regarded as various types of model uncertainties: additive, multiplicative or more general coprime factors which are well suited for aerospace applications [PGSP97].

The impaired system will operate in a satisfactory manner with the same controller designed for the nominal case, providing that it is situated within the stability radius ensured by the robust controller. The performance of the closed loop system is going to be affected to an extent depending on the robust performance provided by the same controller.

Of course with one single controller we have small such regions and therefore the requirement for active approaches such as scheduling [Hyd95]. Therefore it is considered that passive approaches are suitable in restricted cases when a fault has an incipient effect on the controlled system.

3.2 Active methods

Among various active approaches to reconfigurable control suggested in the literature we see a possible organisation in terms of following methods: model following, model reference adaptive control, multi-model adaptive identification and reconfigurable control, pseudo-inverse reconfiguration, control distribution and polynomial networks super-controller strategy. The reader will be able in the next few sections to get more insight into these methods, which will be required later to evaluate the newly suggested strategy, and see how the drawbacks of these methods are addressed.

3.2.1 Pre-computed laws

The concept of scheduling originated in the flight control area as an effective solution to deal with aerodynamic coefficient changes with respect to airspeed, angle of attack (AOA) or altitude. In such applications measured data from the environment triggers the right control action. This principle can be used within the reconfigurable flight control framework as well.

The idea extended in the field of reconfigurable flight control is altogether different. Pre-computed control laws are stored and activated in accordance with the information from an FDI system that monitors the plant and provides state estimates used as basis of a controller scheduling scheme.

In the case of complex systems for which fast parameter changes might be encountered due to the system envelope, failures or external disturbances, we measure the intelligence of a controller for such a system via its speed and accuracy in responding to such changes [Bla95]. From the set of controllers corresponding to such environments encountered during operation we have to determine the best one to be used.

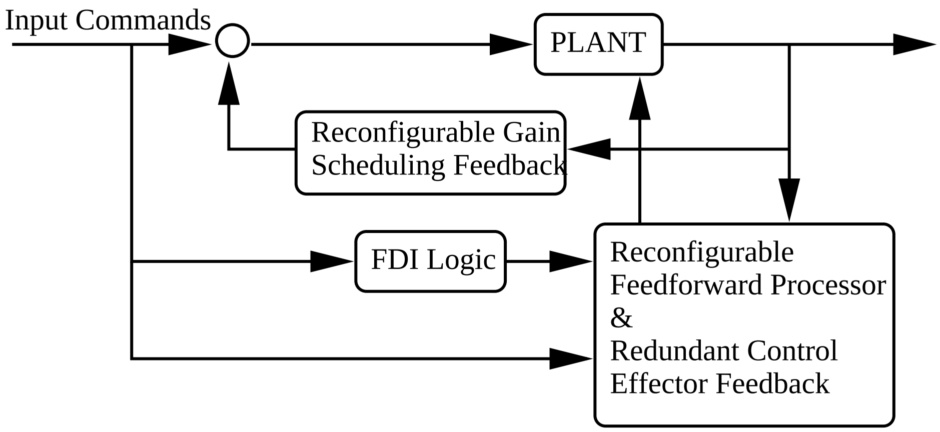

Figure 3.2: Restructurable flight control based on pre-computed laws

One of the schemes suggested by [MHBC89], quoted in [Pat96] is based on the following principles emphasised in the block diagram of Figure 3.2. The role of the processor shown in the figure is to cancel out the effect of the command error integrators as "surfaces are taken out" from the system (i.e. the plant becomes "slim"). The FDI logic is designed to predict the plant states and to trigger the scheduled feedback. This feedback is in fact adjusted by a feedforward compensator (i.e. in open loop fashion) which means that no information about the closed loop is available resulting in a great drawback during failures, when the FDI system may induce instability and poor performance as a result of malfunctioning. This represent a major concern in most of the FDI based schemes.

Two main streams can be distinguished in the literature: the first one is the direct switching (i.e) an output based choice switching towards the next controller in a predetermined structure) and the second one called indirect switching is a strategy aiming to have multiple models used in determining when and to which controller we should switch.

Researchers like [MGHM88], the initiator of the indirect switching, and [Mor93, NB97] concentrate their attention on proving the stability of such schemes.

Another approach that falls into the category of pre-computed control laws was developed by [MS91]. This method involves multiple model adaptive estimation and control as shown in the structure of Figure 3.3.

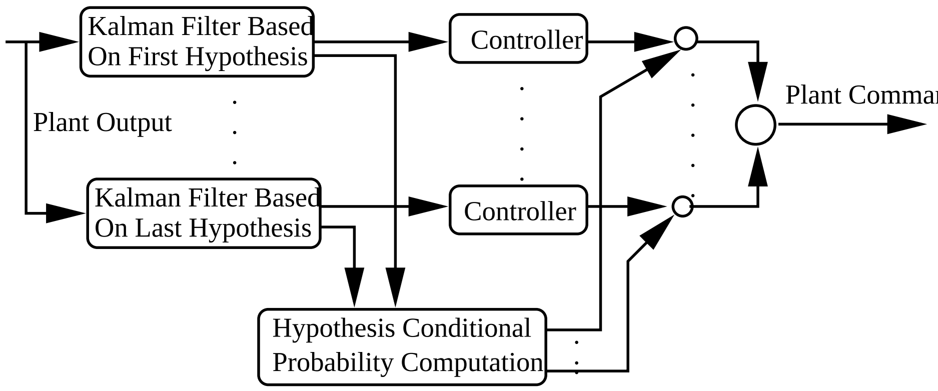

Figure 3.3: Restructurable flight control based on pre-computed laws and a probabilistic structure

For an adequate model of the aircraft and parameters being allowed to take discrete values, a Kalman filter is designed for each parameter value resulting in a bank of separate filters. Based on the residuals of these filters the conditional probability of each discrete parameter value is evaluated at each time step. The output value of each controller from the bank is weighted by its corresponding probability.

The nonlinear system is described by the following equations:

$$\begin{aligned} x(k+1) &= f(x(k), u(k), q) + \mu(k) \\ y(k) &= h(x(k), u(k), q) + \nu(k) \end{aligned}$$

where the meaning of the variable is: $x(k)$ the state, $u(k)$ the input, $\mu(k)$ the input noise, $y(k)$ the output, $\nu(k)$ the output noise and q the parameter vector.

The feedback law can be described as:

$$u(k) \quad = \quad g(\hat{x}(k), u(k), \hat{q})$$

Considering N discrete values for the parameter vector q (q_i, $i = 1 \ldots N$) we are able to write the state and output estimates using the corresponding nonlinear Kalman estimator:

$$\begin{aligned} \hat{x}_i(k+1) &= f(\hat{x}_i(k), u(k), q_i) + K_i(y(k) - \hat{y}_i(k)) \\ \hat{y}_i(k) &= h(\hat{x}_i(k), u(k), q_i) \end{aligned}$$

Then the probability for each of the assumed parameters q_i is calculated as a function of $y(k)$ and $\hat{y}_i(k)$. Hence $\hat{q}_i$ is determined as the conditional mean over all q_i assumed. Alternatively $\hat{q}_i$ can be the q_i with the highest probability.

Examples of this method in the literature, when several controllers were designed for various conditions such as: healthy aircraft, failed pitch rate sensor, failed tail-plane and failed combination of other control surfaces equivalent to a "pseudo structural damage", can be found in [MS91].

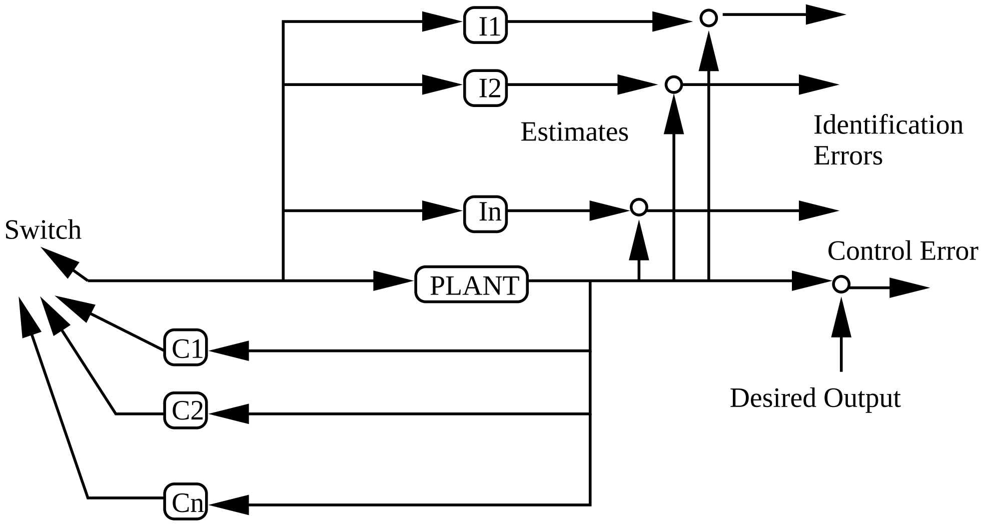

Figure 3.4: Restructurable flight control based on n identification models and controllers

The third approach of interest to this presentation is due to [NB97]. In their work Narendra et al. [NB97] suggests the architecture of the reconfigurable/adaptive control system presented in Figure 3.4.

In this approach the objective is to minimise the control error $e_i = \hat{y}_i - y$ between the real and desired output. All the errors between the output of the identification model I_i and the plant output are computed. Corresponding to each identification model I_i there is a controller C_i whose parameter q_i is chosen such that C_i achieves the control objective for I_i.

The switching rule that selects the appropriate $\hat{y}_i$ tries to yield the best performance for a given objective while stabilising the system. The choice for the switching rule was to determine the performance cost indexes of the form:

$$J_i(k) = \alpha e_i^2(k) + \beta \sum_{j=0}^{k} \lambda^{-(k-j)} e_i(j)$$

Using the parameters α and β of the above performance index we can describe various choices of instantaneous and long term accuracy measures. The exponential weighting determines the memory of the cost function in rapid switching environments and ensures its boundedness when e_i is bounded.

The index $J_i(k)$ is read at each time step. An elapsed time $T_m > 0$ is allowed after every switch and only then the controller corresponding to the model having minimum $J_i(k)$ is chosen for control. The dwell time T_m is required in order to prevent arbitrarily fast switching.

The complexity of the problem in the multiple model case increases when a wider range of failure conditions is addressed. So, the additional computing resources can constitute a serious disadvantage. Examples of simplifications made to improve the computational speed for a terminal missile guidance can be found in [Rau95]. In [FR94] a novel approach employs each model from the bank in simulation done off-line storing the results. Just one Kalman filter is runing on-line of which estimated results are compared with the stored ones enabling the decision of which model should be used.

3.2.2 Feedback linearisation

The authors of various schemes using feedback linearisation for reconfigurable flight control systems (RFCS) [MH84, SM87, OK91, Och93, OK95] claim that the methodology is well established in flight control despite the drawbacks it presents.

As this section aims for a concise presentation we will show first the principle of the method, followed by the theoretical approach, and a number of pertinent conclusions drawn after surveying the above mentioned papers. This should enable us to pinpoint the advantage of the approach suggested in the thesis.

The research on reconfigurable control using feedback linearisation was motivated by the fact that failures often cause severe nonlinearities due to large movements away from the nominal model used in linear design. Of course in such situations the linearised models cannot describe the dynamics of the faulty aircraft well. Such kind of reasons drove us towards an approach to RFCS that avoids these problems and provides a certain robustness.

The scheme to be addressed, presented in [OK91, Och93, OK95], assumes that failures are identified as parameter changes in the dynamic model. A recursive least squares algorithm is employed on-line to provide the estimated parameters, which are updated at each time step simultaneously with the ones belonging to the controller.

The basic structure of the control strategy suggested is given in Figure 3.5. The design procedure involves all the blocks defined in Figure 3.5 but the implementation (due to the system complexity) may present difficulties requiring a high performance on-board computer.

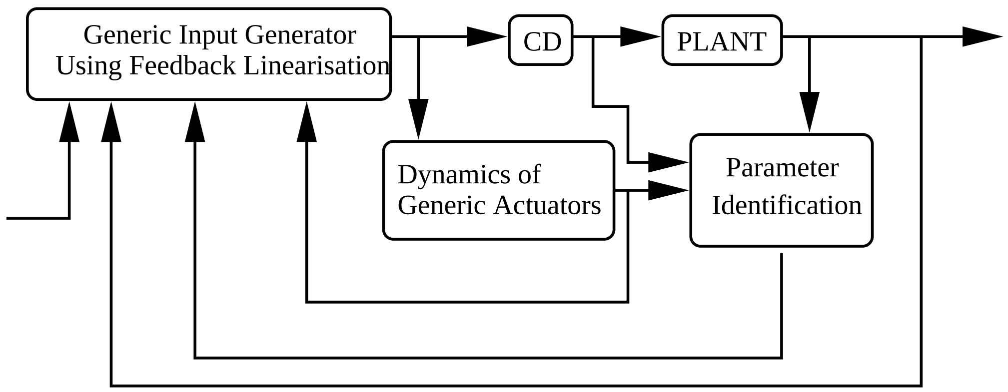

Figure 3.5: RFCS structure using feedback linearisation

The state and output equations are defined in their usual forms with the exception that $A(x) \in \mathbb{R}^n$, $B(x) \in \mathbb{R}^{n\times m}$ have the form of linear combinations of constant parameters and C is a constant matrix when linear feedback control is employed or $C(x)$ represents a vector of nonlinear functions when feedback nonlinear control is used:

$$\begin{aligned}\dot{x} &= A(x) + B(x)u \\ y &= C(x)\end{aligned}$$

where $u \in \mathbb{R}^m$, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ are control, state and output vector respectively.

The control objective is to make y track the desired reference output y^* which results in a control law derived by feedback linearisation. Sometimes actuator dynamics augment the plant model due to the possibility of degrading the guaranteed nominal performance:

$$\dot{u} = \Lambda(-u + u_c)$$

where Λ contains on the diagonal the inverse of each actuator time constant and u_c has the meaning of the actuator model input.

The next step is the introduction of the controller distributor (CD) denoted via matrix P and the generic inputs u_g:

$$u = Pu_g$$

where $P \in \mathbb{R}^{m\times q}$, $q < m$. Of course P can be chosen in any way as long its full rank properties are kept.

The RFCS suggested is an adaptive control system based on parameters that are identified using the generic inputs instead of the real ones in order to maintain the complexity of the algorithm low. This algorithm has the feature of accommodating failures that change the aircraft dynamics.

In [OK91] the following rules for choosing P are stated:

1. The generic inputs should corresponding to the actual control surfaces (i.e.longitudinal generic inputs should address control surfaces such as canards and elevators).

2. In principle the magnitude of the elements should be in proportion with the related surfaces deflection limits, scaling being applied at this level.

The class of systems for which the method is applicable is restricted to the one where the dynamical model is well known (i.e. aircraft). Hence, we do not expect to see this method being applied extensively to processes where dynamical systems are not so well defined (i.e. process industries). A generic problem is the one of robustness to modelling errors, measurement noises and sensor failure. The main issue that makes this method so vulnerable to the above problems is the inversion of the nonlinear plant involved in the control strategy.

3.2.3 Model following

The model following as a reconfigurable control technique was employed by many researchers such as [MO90, SKB82, Cha84, GB95, Bod97]. Aircraft models from F-8 to F-16 and F-18 were used in performing various simulation checks of the schemes suggested, simulations that were considered succesfull providing an improved system response both in the case when model parameters are varying of impairments can be found in the plant (aircraft).

In essence model following means to adjust a constant feedback gain, assuming that this gain is used in the nominal system, so that the reconfigured system approximates the nominal (ideal) one in some sense.

Adaptive reconfiguration via model following method ensures the redesign of the flight control law by a parameter estimation of the failed system and a re-optimisation. The resulting controller can be either a total new design or an augmented version of the initial controller depending on the strategy.

Many schemes employ the conventional controller in parallel with the adaptive one each of them being used in different situations. The nominal one is employed during the flight with the healthy aircraft or when the adaptive controller exhibits failures as opposite to the adaptive one which is introduced in the algorithm when parameters vary and/or adaptation is needed. Associated with the on-line redesign of the control law there are associated several problems.

The linear dependence between the error signal and the unknown parameters makes possible the use of a least square algorithm where a forgetting factor is introduced in the algorithm when parameters vary and adaptation is needed. The least square algorithm becomes unstable when there is insufficient excitation (ie. the case of a steady level flight). This method is discussed in detail in the identification theory. Such situations are tackled by the conventional controller that runs in parallel with the adaptive one or by a persistent excitation of the regressors vector z in order to guarantee convergence of the parameters to the nominal values. Unfortunately the second approach might induce discontinuities and transients in the response of the algorithm. A stabilised version of the least square on-line identification with forgetting factor was developed in [CPM95].

In the case of flight control systems the form of the linear differential equations that reflect the plant model are:

$$\begin{aligned}\dot{x} &= Ax + Bu + d\\ y &= Cx\end{aligned}$$

The usual meaning in state space formulations is given to A, B, C matrices or u, x, y vectors. An input disturbance term d is introduced on purpose to account for the trim values of the input necessary to maintain steady flight at the operating point – feature that if used in the automation of trim computation will free the pilot from this task.

A reference model with states available for measurement is defined as:

$$\dot{y}_m = A_m y_m + B_m r$$

where $y \in \mathbb{R}^m$ and $r \in \mathbb{R}^m$, A_m, $B_m \in \mathbb{R}^{m\times m}$ square matrices and A_m stable.

Simple solutions in the case of model reference control are obtained when the plant can be made to follow the reference model transfer matrix using an implementable (proper) compensator condition which mathematically boils down to $det(CB) \neq 0$ [1]. A more complex reference model is required if the above assumption is not satisfied.

A main requirement when designing compensators is to guarantee the internal stability of the closed loop. Because $m < n$ then $n - m$ modes must be made unobservable.

In [GB95] was shown that model reference control places m modes of the plant at desired locations making the others unobservable by placing them at locations of the plant transmission zeros. Therefore the requirement to have a system minimum phase (i.e. all transmission zeros in the RHP).

The feedback control law defined in the assumption of measurable states, valid in flight control case, is:

$$u = C_0 r + G_0 x + v$$

where C_0, $G_0 \in \mathbb{R}^{m\times m}$ and $v \in \mathbb{R}^m$ are free controller parameters to be determined by the algorithm.

Considering the closed loop dynamics:

$$\begin{aligned}\dot{x} &= Ax + +BG_0 x + BC_0 r + Bv + d\\ y &= Cx\end{aligned}$$

or directly in terms of y:

$$\dot{y} = (CA + CBG_0)x + CBC_0 r + CBv + Cd$$

The above equation will lead to the values of the controller parameter in the nominal case (i.e. when the plant model is considered known):

$$\begin{aligned}C_0^* &= (CB)^{-1} B_m\\ G_0^* &= (CB)^{-1}(A_m C - CA)\\ v^* &= -(CB)^{-1}(Cd)\end{aligned}$$

[1] The matrix CB is called the plant high frequency gain matrix, a critical parameter in adaptive algorithms [Bod97].

The implementation of the model following control strategy is given in Figure 3.6.

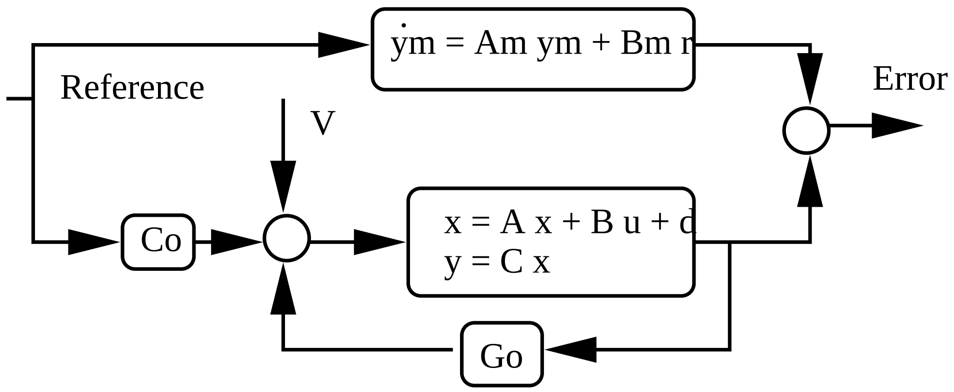

Figure 3.6: RFCS structure using model following

According with one of the most active researchers in this particular field [Bod97] the adaptive model following can be regarded as being direct (implicit) or indirect (explicit), following sections being aimed at clarifying the difference in between these two.

Direct model following

The defining feature of direct adaptive control is that the algorithm estimates the controller parameters C_0, G_0, v directly from data.

Two main approaches to this estimation problem are considered in literature [GA92, GB95]:

The output error ($e_0 = y - y_m$):

It is possible to write this error in the following form:

$$e_0 = (sI - A_m)^{-1} CB[(C_0 - C_o^*)r + (G_0 - G_0^*)x + (v - v^*)]$$

or

$$e_0 = (sI - A_m)^{-1} CB[\Phi w]$$

where $\Phi = [(C_0 - C_o^*), (G_0 - G_0^*), (v - v^*)]$ and $w = [r, x, 1]^T$.

The update law $\dot{\Phi} = -Ge_0 w^T$ leads to a stable adaptive system in Lyapunov sense, as proved in [Bod97] when $(sI - A_m)^{-1}$ is strictly positive real transfer matrix ($A_m - A_m^T$ is negative definite) and $(CB)^T G^{-1} > 0$ a positive definite matrix. Conditions for having this approach implementable require knowledge of CB and G.

The input error ($e_u = u_m - u$)

Using the following identity:

$$u^* = C_0^* B_m^{-1}(\dot{y} - A_m y) + G_0^* x + v^*$$

the input error is defined as:

$$e_u = C_0 B_m^{-1}(\dot{y} - A_m y) + G_0 x + v - u$$

or $e_u = \Phi z$ where Φ is defined before and $z = [B_m^{-1}(\dot{y} - A_m y), x, 1]^T$ is the regressors vector.

This approach avoid the existence of the transfer matrix between the parameter error and the error signal which eliminates conditions more strictly stated above making possible the use of least square algorithms in a straight forward manner. Least square algorithms are used due to their faster convergence comparative with gradient algorithms.

In general the direct adaptive control methods used for control reconfiguration were proved to perform well following tasks: the selection of proper trim values for inputs (a failure causing rapid changes in the command trim), the system decoupling of the inputs and outputs during the failure when the symmetry of the aircraft is lost and the tracking of the pilot commands which is ensured despite a major reduction in the control effectiveness.

The implicit model following can be implemented with lower gains [Pat96] which are affected by the error directly, in real time.

Several issues are still questionable such as: the performance of the algorithm when noise is present or when state variables were not filtered from noise. Another undressed issue is the constraints upon inputs (saturations) that at the moment are handled by the aircraft outer loop controller.

Indirect model following

The indirect adaptive control methods are based on a recursive identifier combined with a control algorithm which uses the estimated parameters of the plant or the corresponding model as if they were perfect estimates. Additionally, it uses the estimated uncertainty affecting these parameters or the model.

The error vector using plant estimates $\hat{A}$, $\hat{B}$, $\hat{d}$ is:

$$\begin{aligned} e_i &= \hat{A}x + \hat{B}u + \hat{d} - \dot{x} \\ &= (\hat{A} - A)x + (\hat{B} - B)u + (\hat{d} - d) \end{aligned}$$

Least square algorithms of various types are used to find $\hat{A}$, $\hat{B}$, $\hat{d}$ and filtering via filters having transfer functions like $\frac{1}{1+0.1s}$ to obtain the state derivative $\dot{x}$.

Once the estimates of $\hat{A}$, $\hat{B}$, $\hat{d}$ were obtained the controller parameter are determined via the following expressions:

$$\begin{aligned} C_0^* &= (C\hat{B})^{-1} B_m \\ G_0^* &= (C\hat{B})^{-1}(A_m C - CA) \\ v^* &= -(C\hat{B})^{-1}(C\hat{d}) \end{aligned}$$

This method is less susceptible to parameter variation but model states must be generated and high feedback gains are required for adequate performance. The estimate of $C\hat{B}$ has to have $det(C\hat{B}) \neq 0$ at all times.

In [DG96] a method based on pseudo-inverse calculations for $(CB)^{-1}$ is regarded as a possible solution that offers feasibility of the indirect approach at all times.

The number of parameters to be identified is bigger than in the direct case (the plant has more parameters than the model $n > m$). Moreover two optimisations are required to produce the solution which slows down significantly the computations.

Such a control scheme is attractive, especially as a replacement of the FDI combined with the multi-model approach, but has to be robust to uncertainties and well behaved in the transient regime. The indirect approach as part of adaptive control schemes is able to provide the control of multivariable plants in the presence of drastic changes in the dynamics such as failures.

The presence of failures rises various problems due to a strong coupling for the original plant, a flight envelope to be covered by the continuous use of the adaptive algorithm and a system that maybe highly unstable leaving little time for reconfiguration and limiting the actuators authority enlarging the sensor noise significantly.

Mixed methods, considered to lie under the umbrella of indirect adaptive reconfigurable control, are based on: neural networks, fuzzy logic or knowledge based systems. They are called mixed due to the identification algorithm that is carried in general in an unconventional manner.

Sometimes accommodation methods based on statistical approach are restrictive in terms of dynamic changes and failure patterns. Extensive on-line tuning being still needed after FDI. All these represent time consuming procedures.

A typical scheme to accommodate failures via intelligent control will involve a low level layer, running at a high sampling rate in order to accommodate hard type of faults such as: actuator failure, structure break or sensor breakdown, stores a set of predictable failure patterns. Then a timely action is produced to prevent the vehicle from being uncontrollable. Afterwards higher layer accommodates the soft fault, more difficult to detect and isolate because of the induced ambiguities in the configuration management system.

An example of such mixed methods is given in [CMP93] when via the on-line optimising networks the model is identified on-line using a constrained least square algorithm. Then the Hopfield network generates an optimal model following the open loop control law.

The work in the area of receding horizon control, predictive control applied to reconfigurable control is mainly due to [CMP93, PCM95, WB95].

The model based predictive control ($MBPC$) approach to reconfiguration is in fact an indirect reconfigurable control system that performs an on-line system identification, sometimes taking advantage about the particular form of the aircraft linear dynamics expressed in terms of stability derivatives, followed by on-line optimisation which is maximising the aircraft tracking performance before and after control surface failure, preventing instability.

This method was employed with reasonably success by [CMP93, WB95]. Actually the paper by [WB95] concentrates much more on the identification algorithm, a modified parameter sequential least square identification.

In the case of [PCM95] a combined LQR with LP control strategy is employed such that a suboptimal tracking control law that does not violate the constraints is synthesised. This is not actually a proper $MBPC$ formulation but is closest the algorithm to the ones the we are already used to. The LQR controller closes an inner loop, the input constraints being addressed in an outer on-line LP optimisation.

We would like to emphasise the latest contribution in this field provided by [PCS97] where a point-wise linearisation that replace the identification part from their previous approach is employed.

3.2.4 Pseudo-inverse method

As in the case of other intelligent control approaches to control reconfiguration the research on pseudo-inverse reconfigurable control was largely motivated by problems encountered in aircraft industry - flight control design. In accommodating on-line unanticipated failures the self repairing system [2] based on the pseudo-inverse method (PIM) is considered a key approach. Researchers like [Ost85, Rat85] have used PIM successfully in flight simulations.

The basic idea featured by PIM is the an on-line modification of the feedback gain produced such that properties of the nominal system are still retained despite of the impairment. Theoretically speaking the method relies on the computation of the pseudo-inverse of a constant matrix via least square approximation as described in [KDG96].

In control system design first issue to be addressed, before performance, is the one regarding stability. Unfortunately at this level is located one of the main drawbacks of PIM because only few stability guarantees were given in the literature [GA91] and even those are quite conservative. Most of the works talk about the instability encountered when applying PIM without any safeguard but do not provide enough insight when the method works or fails and under what circumstances.

Considering the standard form of the state space representation of the open loop plant:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{3.1}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$. This plant has warped around a state feedback $u = Kx$. Note the assumption of measurable states which is generally valid in flight control. The closed loop system is:

$$\begin{aligned} \dot{x} &= (A + BK)x + Bu \\ y &= Cx \end{aligned}$$

The system model containing impairments is expressed as:

$$\begin{aligned} \dot{x}_f &= A_f x_f + B_f u_f \\ y_f &= C_f x_f \end{aligned}$$

where A_f, B_f, C_f have the same dimensions as A, B, C.

The closed loop with the new feedback K_f, to be determined will be:

$$\begin{aligned} \dot{x}_f &= (A_f + B_f K_f)x_f + B_f u_f \\ y_f &= C_f x_f \end{aligned}$$

[2] Some authors refer by reconfigurable control laws designed *a priori* to accommodate certain anticipated failures as opposite to restructurable control that implies an automatic design that accommodates unanticipated impairments

For the first time, in [Ost85], the idea of finding K_f such that the closed loop matrix which defines the failed system dynamics is similar in least square sense with the one belonging to the healthy system:

$$K_f = B_f^{\dagger}(A - A_f + BK)$$

where $B_f^{\dagger}$ denotes the pseudo-inverse of B_f in the least square sense is mentioned.

The row rank of B_f defines the type of problem we are going to solve. If B_f has full row rank then K_f obtained via the pseudo-inverse calculation always satisfies the equation $A + BK = A_f + N_f K_f$ otherwise an approximate solution can be found. Having such an approximate solution means that we do not know how closed the behaviour of the reconfigured system as to the nominal one. Hence some guarantees are required.

There is a quick solutions both from the implementation and stability guarantees point of view. This employs the PIM method in the restructurable control sense by having the K_f computed and stored for many anticipated failures. The stability guarantees can be given by checking off line all the plant plus controller combinations to be identified by the FDI system.

Another solution is the reconfigurable control, solution which relies on a modified approach by [GA91]. It is based on the PIM method in conjunction with the theory of robust stability of systems with structured uncertainty. Effectively this method tries to recover the performance of the impaired system as much as possible retaining the closed loop stability.

In [ZK87, Yed88] stability bounds for linear systems with structured uncertainty are derived. Both methods use a state space perturbation of the form $\dot{x} = (A + E)x$ where $E = \sum_{i=1}^{q} k_i E_i$ with E_i constant matrices and $A \in \mathbb{R}^{n \times n}$ the stable state matrix of an LTI system. The idea presented in [GA91] is to take advantage of these bounds and derive a stable $A_f + B_f K_f$ closed loop state matrix based on them.

The SISO approach has an analytical solution for the problem:
$\min K_f J$ where $J = \| A + BK - A_f + B_f K_f \|_F$ The corresponding algorithm that determine the solution of modified PIM has several steps. A first attempt is made to compute K_f via the classic PIM followed, in the unstable closed loop case, by a more computationally involved algorithm that determines K_f via minimisation of J subject to the constrained stated in [ZK87] or [Yed88]. This in effect pick up from the class of stabilising controllers for the impaired plant the one that gives a closed loop behaviour closed to the nominal one.

For the MIMO case the procedure is almost similar. In the unstable case effectively K_f is perturbed (i.e. $\hat{K}_f = K_f + \Psi$) with a Ψ based on the bounds derived in [ZK87] or [Yed88].

As an important remark we have to mention that the constraints used in minimisation are sufficient but not necessary conditions to guarantee stability; the constraints tend to be more conservative as q increases [GA91].

In [Pat96] two other serious limitations of the PIM approach to control reconfiguration are mentioned:

1. PIM make no explicit use of the robustness properties of the FDI system relying to much in the supervision system used in determining the fault signature.

2. The state measurement in other plants than aircrafts is not always available.

The second alternative of employing model based predictive control (MPBC) as a reconfigurable control system will use this approach modified at the level of controller synthesis.

As a final conclusion we would like to point out the resemblance of the pseudo-inverse method with model following – see [GA92] – another technique extensively used in aircraft reconfigurable control. In the the first case a nominal design of the closed loop is approximated whenever in the second an ideal model tends to be followed.

Bibliography

[Ast96] K.J. Astrom. Tuning and Adaptation. In *Proceedings of the 13th Congress*, San Francisco, California, June 1996. IFAC.

[Bla95] M. Blanke. Aims and Means in the Evolution of Fault-tolerant Control. In *European Science Foundation Workshop COSY*, September 1995.

[Bod97] M. Bodson. Multivariable Adaptive Algorithms for Reconfigurable Control. *IEEE Transactions on Control Systems Technology*, 7(2):217–227, Mar. 1997.

[Cha84] P.R. Chandler. Self repairing flight control system reliability and maintanaibility program executive overview. In *Proceedings to National Aerospace Electronics Conference*, pages 586–590, Dayton, OH, 1984.

[CMP93] P. Chandler, M. Mears, and M. Patcher. On-Line Optimising Networks for Reconfigurable Control. In *Proceedings of the 32nd Conference on Decision and Control*, pages 2272–2277. IEEE, 1993.

[CPM95] P.R. Chandler, M. Patcher, and M. Mears. System Identification for Adaptive and Reconfigurable Control. *Journal of Guidance Control and Dynamics*, 18(3):516–524, May–Jun. 1995.

[CT78] C.B.Borosilow and T.M. Tong. The structure and dynamics of inferential control systems. *AIChE*, 24:492–500, 1978.

[DG96] N. Dhayagunde and Z. Gao. Novel Approach to Reconfigurable Control Systems Design. *Journal of Guidance Control and Dynamics*, 19(4):963–966, 1996.

[FR94] W.A. Fisher and H.E. Rauch. Augmentation of an Extended Kalman Filter with a Neural Network. In *Proceedings of IEEE International Conference on Neural Networks*. IEEE, June 1994.

[GA91] Z. Gao and P.J. Antsaklis. Stability of the pseudo-inverse method for reconfigurable control systems. *International Journal of Control*, 53(3):717–729, 1991.

[GA92] Z. Gao and P.J. Antsaklis. Reconfigurable control systems design via perfect model following. *International Journal of Control*, 56(4):783–798, 1992.

[GB95] J.E. Groszkiewicz and M. Bodson. Flight Control Reconfiguration Using Adaptive Methods. In *Proceedings of the 34th Conference on Decision and Control*, pages 1159–1164, New Orleans, LA, Dec. 1995. IEEE.

[GL95] M. Green and D.J.N. Limebeer. *Linear Robust Control*. Prentice Hall, 1995.

[HMW80] T.J. Harris, J.F. MacGregor, and J.D. Wright. Optimal sensor location with an application to a packed-bed tubular reactor. *AIChE*, 26:910–916, 1980.

[Hyd95] R.A. Hyde. *H_∞ Aerospace Control Design*. Springer, 1995.

[KDG96] K.Zhou, J.C. Doyle, and K. Glover. *Robust and Optimal Control*. Prentice Hall, 1996.

[LBMP95] J.H. Lee, R.D. Braatz, M. Morari, and A. Packard. Screening Tools for Robust Control Structure Selection. *Automatica*, 31(2):229–235, 1995.

[LM91] J.H. Lee and M. Morari. Robust measurement selection. *Automatica*, 27:519–527, 1991.

[LP93] M. Labarrere and R.J. Patton. *Concise Encyclopedia of Aeronautics and Space Systems*. Pergamon Press, 1993.

[Mac97] J.M. Maciejowski. Reconfigurable Control Using Constrained Optimisation. In *Proceedings of ECC'97*, Bruxelles, Belgium, 1997.

[MGHM88] R.H. Middleton, G.C. Goodwin, D.J. Hill, and D.Q. Mayne. Design issues in adaptive control. *IEEE Transactions on Automatic Control*, 33:50–58, 1988.

[MH84] G. Meyer and L.R. Hunt. Application of non-linear transformations to automatic flight control. *Automatica*, 20(1):103–107, 1984.

[MHBC89] D.D. Moerder, N. Halyo, J.R. Broussard, and A.K. Caglayan. Applications of pre-computed control laws in a reconfigurable aircraft flight control system. *Journal of Guidance Control and Dynamics*, 12(3):325–333, 1989.

[MO90] W.D. Morse and K.A. Ossman. Model following reconfigurable flight control system for the AFTI/F–16. *Journal of Guidance Control and Dynamics*, 13(6):969–976, 1990.

[Mor93] A.S. Morse. Supervisory control of families of linear set point controllers. In *Proceedings of the 32nd Conference on Decision and Control*, San Antonio, Texas, Dec 1993. IEEE.

[MS91] P.S. Maybeck and R.D. Stevens. Reconfigurable Flight Control Via Multiple Adaptive Control Methods. *IEEE Transactions on Aerospace and Electronic Systems*, 27(3):470–479, 1991.

[NB97] K.S. Narendra and J. Balakrishnan. Adaptive Control Using Multiple Models. *IEEE Transactions on Automatic Control*, 42(2):171–187, 1997.

[Och93] Y. Ochi. Application of Feedback Linearisation Method in a Digital Restructurable Flight Control System. *Journal of Guidance Control and Dynamics*, 16(1):111–117, Jan.–Feb. 1993.

[OK91] Y. Ochi and K. Kanai. Design of Restructurable Flight Control Systems Using Feedback Linearisation. *Journal of Guidance Control and Dynamics*, 14(5):903–911, Sept.–Oct. 1991.

[OK95] Y. Ochi and K. Kanai. Application of Restructurable Flight Control System to Large Transport Aircraft. *Journal of Guidance Control and Dynamics*, 18(2):365–370, Mar.–Apr. 1995.

[Ost85] A.J. Ostroff. Techniques for accommodating control effector failures on a mildly statically unstable aircraft. In *Proceedings of the American Control Conference*, pages 903–906, 1985.

[Pat96] R.J. Patton. Where Are We in Fault-Tolerant Control? In E. Garcia and P. Albertos, editors, *European Science Foundation Workshop COSY*, October 1996.

[PCM95] M. Patcher, P.R. Chandler, and M. Mears. Reconfigurable Tracking Control with Saturation. *Journal of Guidance Control and Dynamics*, 18(5):1016–1022, Sep.–Oct. 1995.

[PCS97] M. Patcher, P.R. Chandler, and L. Smith. Maneuvering Flight Control. In *Proceedings of the American Control Conference*, July 1997.

[PGSP97] G. Papageorgiou, K. Glover, A. Smerlas, and I. Postlethwaite. *Robust Flight Control*, volume J.F. Magni and S. Bennani and J. Terlow of *Lecture Notes in Control and Information Engineering*, chapter H_∞ Loop Shaping, pages 52–64. Springer, 1997.

[QB96] S.J. Qin and T.A. Badgwell. An overview of industrial model predictive control technology. In *Chemical Process Control – CPC V*, Tahoe City, California, 1996. CACHE.

[Rat85] K.S. Rattan. Evaluation of control mixer concept for reconfiguration for flight control system. In *Proceedings on the 1985 IEEE National Aerospace and Electronics Conference*, volume 3, pages 560–569, 1985.

[Rau95] H.E. Rauch. Autonomous Control Reconfiguration. *IEEE Control Systems Magazine*, 15(6):37–48, 1995.

[SKB82] K.M. Soebel, H. Kaufmann, and M. Balas. Implicit adaptive control for a class of MIMO systems. *IEEE Transactions on Aerospace and Electronic Systems*, 18(5):576–589, 1982.

[SM87] G.A. Smith and G. Meyer. Aircraft automatic flight control system with model inversion. *Journal of Guidance Control and Dynamics*, 10(3):101–107, 1987.

[SP96] S. Skogestad and J. Postelthwaite. *Multivariable Feedback Control Analysis and Design.* John Wiley and Sons, 1996.

[WB95] D.G. Ward and R.L. Barron. A Self-Designing Receding Horizon Optimal Flight Controller. In *Proceedings of the American Control Conference*, pages 3490–3494, June 1995.

[Wes77] T.F. Westermeier. Redundancy management of digital FBW systems. In *Joint Automatic Control Conference*, pages 272–282, 1977.

[Wil76] A.S. Willsky. A survey of design methods for fault detection in dynamic systems. *Automatica*, 12:601–611, 1976.

[Yed88] R.K. Yedavalli. Stability robustness measures under dependent uncertainty. In *Proceedings of the American Control Conference*, pages 820–823, 1988.

[ZK87] K. Zhou and P.P. Khargonekar. Stability robustness bounds for linear state-space models with structured uncertainty. *IEEE Transactions on Automatic Control*, 32:621–623, 1987.