A Combined $MBPC/\mathcal{H}_\infty$ Automatic Pilot for a Civil Aircraft

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Abstract

This paper motivates the combined use of $\mathcal{H}_\infty$ loop-shaping and $MBPC$ as a method of designing automatic pilots for civil aircraft. The $\mathcal{H}_\infty$ loop-shaping controller will provide stability augmentation and guidance. The $MBPC$ controller will act as a flight manager. The design procedure developed was tested by designing an autopilot for the Research Civil Aircraft Model (RCAM) used in the GARTEUR design challenge. The resulting controller was subjected to a standard evaluation procedure. Satisfactory results were achieved.

Keywords: Flight Control, Automatic Pilot, Flight Management, Guidance, Stability Augmentation, $\mathcal{H}_\infty$ loop-shaping, Model Based Predictive Control ($MBPC$).

1 Introduction

A modern automatic pilot consists of a stability augmentation system (SAS), a guidance system (GS) and a flight management system (FMS) [9, 5]. The SAS, very short term control, provides robust stabilisation, disturbance rejection, noise rejection and sets the bandwidth of the closed loop. The GS, short term control, ensures output decoupling and tracking. The FMS, long term control, optimises long term objectives such as passenger comfort and fuel consumption over an a priori known trajectory provided by air traffic control (ATC) or a terrain map.

Constrained model based predictive control ($MBPC$) [2, 4] has been used to design an autopilot with reasonable success [10]. Two main problems were encountered: the relatively long time required to solve the optimisation problem on a standard autopilot computer which restricted the bandwidth of the controller (a technological constraint) and the availability of an analytical expression for the controller which affected the assessment of robustness and stability properties (a theoretical limitation). Robust control design methods do not usually account for future reference signals or plant time variations, although there is no reason why measurement of the future reference signals cannot be included in the setup. Hence they have not been used for the FMS functions.

This paper motivates the combined use of $\mathcal{H}_\infty$ loop-shaping and $MBPC$ as a method of designing automatic pilots for civil aircraft. The $\mathcal{H}_\infty$ loop-shaping controller, inner loop controller, will provide the stability augmentation and guidance functions. The $MBPC$ controller, outer loop controller, will act as a flight manager and overall supervisor.

After presenting the two individual design methods the autopilot architecture is justified. The design method is illustrated by designing an autopilot for the RCAM used in the Group for Aeronautical Research and Technology in Europe (GARTEUR) design challenge [6]. To enable comparisons with other GARTEUR designs a standard evaluation over a *Frankfurt descent procedure* was performed [1].

2 The design methods

2.1 $\mathcal{H}_\infty$ loop-shaping

$\mathcal{H}_\infty$ loop-shaping developed by McFarlane and Glover [11] is a very intuitive method of designing robust controllers as the notions of classical loop-shaping readily carry through. The designer can specify closed loop requirements such as disturbance and noise rejection by simply shaping the loop gains [17]. An important feature of $\mathcal{H}_\infty$ loop-shaping is that it enables the designer to push for the best achievable closed loop performance subject to robustness. This is because the designer has direct control over the crossover frequency of each loop singular value [12].

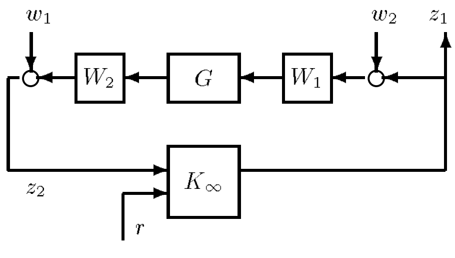

Figure 1: The $\mathcal{H}_\infty$ loop-shaping standard block diagram

When setting up a robust control problem a decision has to be made about the uncertainty structure to be used. This can be difficult as it requires good knowledge of the plant. Minimising γ

$$\left\| \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \to \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right\|_\infty = \gamma$$

provides robust stability to coprime factor uncertainty which requires no assumptions to be made about the stability of the perturbed plant. Coprime factor uncertainty

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is a more general type of uncertainty similar to the SISO gain and phase margins. Therefore, when there is little detailed knowledge about the uncertainty present in a plant, $\mathcal{H}_\infty$ loop-shaping is a good method for designing robust controllers.

Performance is specified by shaping the singular values of the plant G with weights W_1 and W_2, Figure 1. In [11] it is proved that shaping G corresponds to shaping the loop gains $G_s K_\infty$ and $K_\infty G_s$, where $G_s = W_2 G W_1$ is the augmented plant. Hence the controller K_∞ does not alter the desired loop shape G_s very much. All closed loop transfer functions are bounded in terms of G_s, W_1, W_2 and γ. A full tutorial on how to design using $\mathcal{H}_\infty$ loop-shaping can be found in [8, 12, 14].

The cost function minimised provides robust stability. A measure of the robustness of the desired loop shape is given by γ. Application of the method to real plants has shown that a value of $1/\gamma$ between 0.2 and 0.3 is satisfactory in the same way that a gain margin of $\pm 6dB$ and a phase margin of 45^o is in the SISO case. A "good" value of γ should guarantee that the required gain and phase margins are achieved. In [14] the phase and gain margins of a SISO system are related to γ.

A lot of research has gone into $\mathcal{H}_\infty$ control methods and, as a result, a great deal of powerful analysis tools are available. Such tools will aid certification issues.

2.2 *MBPC*

The defining feature of *MBPC* is the repeated optimisation of a performance objective over a finite horizon extending from a future time (N_1) up to a prediction horizon (N_2) [3, 2]. Figure 2 characterises the way prediction is used within the *MBPC* control strategy. Given a set-point, a reference $r(k+l)$ is produced and used within the optimisation of the cost function (1). Manipulating the control variable $u(k+l)$, over the control horizon (N_u), the algorithm drives the predicted output $\tilde{y}(k+l)$, over the prediction horizon, towards the reference.

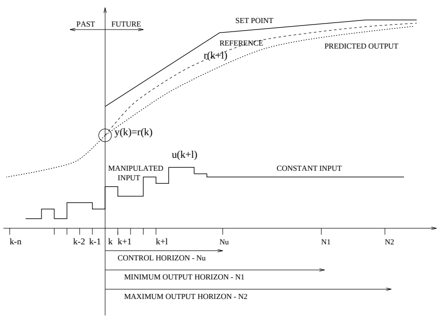

Figure 2: Prediction Strategy

The future control movement is determined by minimising the cost function:

$$J(k) = \sum_{l=N_1}^{N_2} \|(\tilde{y}(k+l) - r(k+l)\|^2_{Q(l)} + \\ + \sum_{l=0}^{N_u} \|\Delta u(k+l)\|^2_{R(l)} \tag{1}$$

subject to constraints on:

- the inputs levels: $u_l(l) \leq u(l) \leq u_u(l)$ where $k \leq l \leq k + N_u - 1$
- the input rates of change: $\Delta u_l(l) \leq \Delta u(l) \leq \Delta u_u(l)$ where $k \leq l \leq k + N_u - 1$
- the output (and state) levels: $y_l(l) \leq \tilde{y}(l) \leq y_u(l)$ where $k + N_1 \leq l \leq k + N_2$

$Q(l)$ and $R(l)$ are weights independent of time k and the norm $\|.\|^2_Q$ within the cost function is defined as $\|\alpha\|^2_Q = \alpha^T Q \alpha$. It is assumed that $\Delta u(k) = 0$ for $l \geq k + N_u$. As in [3, 7] the optimisation is carried out using a quadratic program (QP).

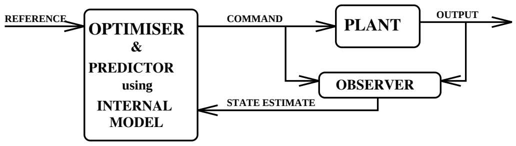

Figure 3: The structure of *MBPC* schemes

The general structure, Figure 3, of *MBPC* schemes is the following:

Optimiser contains the constrained cost function and works iteratively in real time. The main task of the optimiser is to compute the present and future manipulated variable moves such that the predicted outputs follow the reference in a desirable manner.

Predictor employing the internal model and the measurement or estimate of the current state provides the optimiser with future predicted values of states and outputs.

Internal model represents the plant. In this paper we use a state-space model for the plant. This is a natural choice for flight control, since the states can be chosen to be physical variables, and this correspondence can be exploited to some extent.

Observer provides state estimates, which can be used in the optimiser.

MBPC has some very appealing attributes: *Simplicity* — the basic idea of *MBPC* is fairly intuitive, and can be understood without advanced mathematics; *Richness* — the common elements of *MBPC* schemes, such as models, objective functions, prediction horizons, etc, can be tailored to specific problems; and *Practicality* — the usual combination of linear dynamics and inequality constraints allows realistic nonlinearities to be handled.

3 Combined $MBPC/\mathcal{H}_\infty$ autopilot

3.1 The architecture

Stability augmentation and guidance systems must be designed by a method that provides disturbance rejection at both input and output of the plant, noise rejection, robust stability, flexibility in specifying the bandwidth of the controller, output decoupling, tracking and a rich variety of analysis tools that will aid certification. For the reasons described in Section 2 $\mathcal{H}_\infty$ loop-shaping will be used to design the SAS and GS.

The flight management system must optimise long term objectives such as passenger comfort and fuel consumption over an a priori known trajectory provided by air traffic control. Moreover, current requirements in flight management are increasingly concerned with 4D navigation – the flight path is time parametrised and corrected with the along-track error [6]. The use of $MBPC$ as a method to design a FMS seems desirable as all the above goals can be expressed by the standard $MBPC$ problem formulation.

$MBPC$ can also remedy some of the drawbacks associated with fixed gain controllers:

1. Disturbances of large amplitude on the plant output might saturate the actuators. This could lead to poor output decoupling and potential loss of stabilisability in the case of an unstable open loop plant.
2. By making an a priori decision on the structure of the pre-filters it is possible to degrade the achievable performance. Pre-filters are usually chosen to give the fastest possible response without saturating the actuators during a typical pilot demand. This implies that small demands are achieved in the same time as large demands.
3. Respecting flight envelope constraints is not straightforward and again requires a priori decisions.

By monitoring the actuator states and plant outputs and imposing constraints on their behaviour [13, 15, 16] $MBPC$ can help deal with the above points. Therefore, the actuators are used to their limits while rejecting a disturbance without destabilising the closed loop. $MBPC$ will also adjust the reference to the inner closed loop ($\mathcal{H}_\infty$ controller and aircraft) so as to avoid violation of the flight envelope and ensure good time response characteristics. All the above motivate the following structure shown in Figure 4.

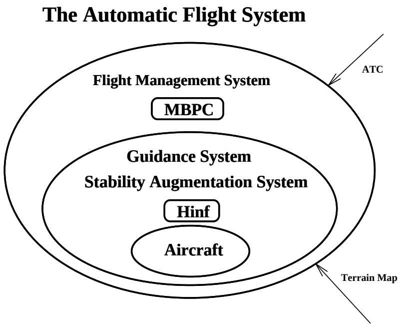

Figure 4: The autopilot architecture

The FMS must not destabilise or cancel the effect of the inner closed loop and must be robust to modelling uncertainty. $MBPC$ stability and robustness issues are addressed in [13]. When employing this novel architecture the $MBPC$ controller must be robust to uncertainty within the inner closed loop bandwidth. Hence the $MBPC$ controller can be less robust than the $\mathcal{H}_\infty$ controller as the uncertainty within the bandwidth is much smaller. It is our intention that the $MBPC$ controller provides an optimised reference to the inner closed loop without interfering with the stabilisation and guidance. Note that the $MBPC$ problem variable, $\Delta u(k+l)$, depends on the inner loop design and hence the inner loop indirectly affects the $MBPC$ step. Therefore the blended structure operates as a multi-rate system having a high sampling rate for the $\mathcal{H}_\infty$ controller and a low one, $1s$, for the $MBPC$ controller. It could be argued that if the autopilot goes unstable the $MBPC$ controller could be switched off.

In [13] both inner and outer loop controllers are assumed to operate at the same sampling rate. This may result in the $MBPC$ controller interfering with the effect of the inner loop controller. Our present research concentrates on investigating the stability and robustness properties of such single-rate and the proposed multi-rate controller.

3.2 Performance of the combined scheme

Figures 5 and 6 depict time responses of the RCAM. The simulations were run on the full non-linear model. The optimisation problem was solved with constraints placed upon the actuator deflections and rates.

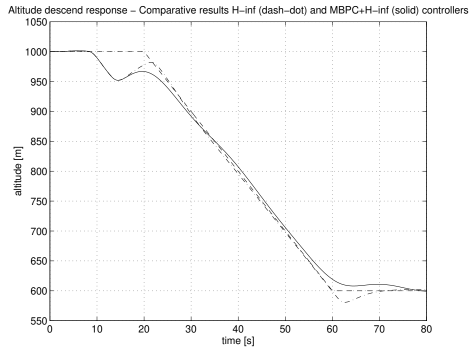

Figure 5: Altitude response subject to wind-shear

At $20s$, Figure 5, the RCAM goes into a descent at a rate of $10m/s$. At $10s$ there is a wind-shear of $10m/s$ for a $10s$ duration. The prediction horizon of the $MBPC$ is $N_2 = 10s$. The $\mathcal{H}_\infty$ controller by itself tries to recover from the disturbance as fast as possible and return to the original altitude of $1000m$. The blended structure though takes into account that the aircraft is going to start descending at $20s$ and does not try to reach the original set-point hence improving passenger comfort. In effect the $MBPC$ controller is modifying the reference to the inner closed loop. At the bottom of the descent there is no overshoot. Note that the $\mathcal{H}_\infty$ loop-shaping controller K_∞ is placed in the forward loop with no pre-filters.

The transients in the first $5s$, Figure 6, are because the non-linear model is not perfectly trimmed. Again the combined controller minimises the overshoot and follows the reference well within the design specs [6]. At $75s$ there is a head wind of $3m/s$ that lasts for $17s$. As the prediction horizon is $10s$ it can be deduced that the disturbance rejection capability of both controllers is the same. Hence the $MBPC$ controller does not seem to interfere with the $\mathcal{H}_\infty$ controller.

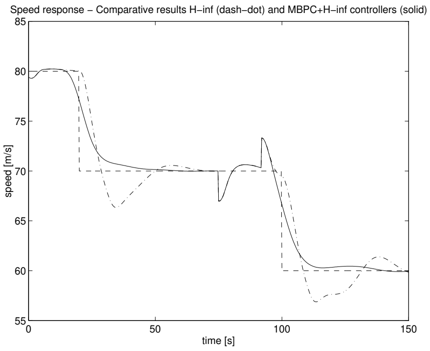

Figure 6: Speed response

4 The design procedure

4.1 Flight control objectives

The RCAM design challenge involves designing a control law that is able to perform an approach and landing in the presence of turbulence, wind-shear and an engine failure, whilst remaining robust to miss modelling [6]. A view of the entire trajectory is presented in Figure 7.

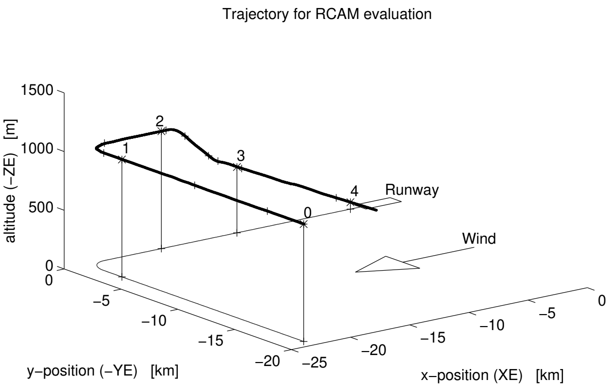

Figure 7: RCAM model trajectory response

In this paper only the longitudinal controller will be presented. The tracking requirements on the airspeed and flight path angle are as follows:

V_{air} (airspeed)	γ_f (flight path angle)
rise time $< 12sec$	rise time $< 5sec$
settling time$< 45sec$	settling time $< 20sec$
overshoot$< 5\%$ for height $> 305m$	
overshoot$< 30\%$ for height $< 305m$	

Note that the climb rate $\dot{z} = V_{air} \sin \gamma_f$. Good disturbance rejection requires that a $13m/s$ wind step should not induce a deviation in airspeed greater than $2.6m/s$ for more than $15s$. No cross-coupling requirements are defined between V_{air} and $\dot{z}$ [6].

Disturbance rejection, robustness, tracking and output decoupling will be addressed by the $\mathcal{H}_\infty$ loop-shaping controller. The flight path optimisation, safety, ride quality and control activity will be achieved by the $MBPC$ controller.

4.2 The controller structure

The control structure of the longitudinal part of the autopilot is given in Figure 8.

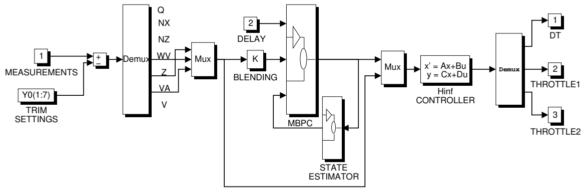

Figure 8: The controller structure

A single controller was designed for both stability augmentation and guidance even though it is conventional to design an inner loop to provide stability and outer loop for tracking. The loop-shapes were from tailplane and throttle to $p_1\dot{z} + p_2 z$ and V_{air}. The advantage of this structure is that the designer obtains insight in how robustness is traded off for good performance in altitude following. The greater the value of p_2 the faster the altitude tracking but the less robust the design is. As a result we have to include a filter that translates a climb rate demand to the blended variable. Actuator states and loop delays to simulate computational delays were also used.

Moreover, a single controller for both SAS and GS has the added advantage of reducing the number of states of the $\mathcal{H}_\infty$ controller. This becomes very important when employing an $MBPC$ outer loop as the $MBPC$ uses an internal model of the closed loop for prediction. The higher the complexity of the closed loop the longer the optimisation problem will take to be solved. The internal model provided to the $MBPC$, to reduce the complexity of the problem, has 7 states in total. The number of states was chosen in such a way that the ν-gap of the full state closed loop and model reduced one was small (the ν-gap is a measure of how close two plants are with respect to how they perform in feedback with the same controller [14]).

4.3 Design cycle

1. The open loop plant must be scaled according to the desired output decoupling and actuator usage. This is because the open loop singular values can not be associated with any one input or output. A badly scaled plant is equivalent to a badly formulated problem.
2. Choose the weights W_1 and W_2. W_1 is chosen for disturbance rejection, robust stability and speed of response. W_2 is chosen for noise rejection.
3. Choose the position of the $\mathcal{H}_\infty$ loop-shaping controller in the inner closed loop. Produce a low order state-space model of the inner closed loop that will serve as the internal model.
4. Define the constraints related to inputs, rates of change of the inputs, outputs and states and construct matrices that represent these over the control and prediction horizons.

5. Choose appropriate dimensions for control and prediction horizons and the cost function weighting matrices.
6. Tune the performance of the closed loop system via closed loop linear and nonlinear simulations. This involves iteration of steps 4 and 5.
7. Perform a stability and robustness analysis.

5 The evaluation procedure

To attest the idea of the combined controller structure the most relevant segment of the approach manoeuvre (segment 2–3 in Figure 7) is extracted. This segment, that represents the final descend to land, requires the capture of $-6°$ and $-3°$ glide-slopes.

In Figure 9 the longitudinal response of the aircraft and bounds that specify acceptable behaviour are plotted. Both controllers, $MBPC$ plus $\mathcal{H}_\infty$ (dotted) and $\mathcal{H}_\infty$ on its own (dash-dot), behave in an acceptable manner. The difference arises when considering the vertical deviation from the desired glide slope. While the $\mathcal{H}_\infty$ controller tries to follow as close as possible the descent reference trajectory resulting in overshoots, the $MBPC$ plus $\mathcal{H}_\infty$ controller takes advantage of the a priori known trajectory, optimising and improving comfort and safety.

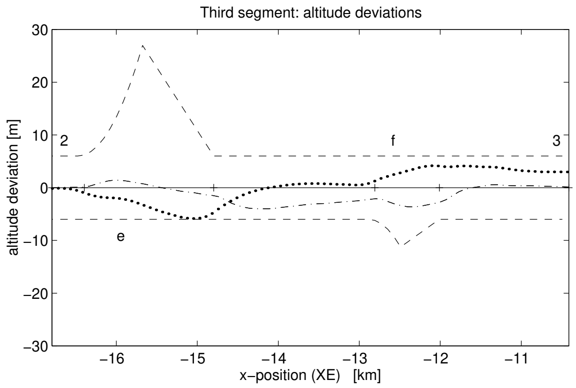

Figure 9: Vertical deviations while capturing the $-6°$ and $-3°$ glide-slopes

Table 1 gives comparative numerical results based on the two distinct simulations.

Criteria	$MBPC/\mathcal{H}_\infty$	$\mathcal{H}_\infty$
Comf.	0.5137	1.3316
Safety	0.0075	0.0084
Power	0.0155	0.0150

Table 1: Comparative numerical results

The smaller the numbers the better the design. The motivation and calculation principles of these figures can be found in [6].

6 Conclusions

We have presented an automatic pilot with a novel architecture. An $\mathcal{H}_\infty$ loop-shaping controller will provide the stability augmentation and guidance functions and a $MBPC$ controller will act as a flight manager and overall supervisor. The design method has been used to design a longitudinal autopilot for the RCAM. Non-linear time simulations have shown that this combined $MBPC/\mathcal{H}_\infty$ autopilot yields satisfactory performance. Our present research is concentrated on investigating the robustness and stability issues of this multi-rate controller and exploring in more depth whether including this inner loop is a way of parametrising all stabilising cost functions. We will also consider whether the $MBPC$ controller cancels the action of the $\mathcal{H}_\infty$ controller.

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