AUTOMATIC TUNING FOR MODEL BASED PREDICTIVE CONTROL DURING RECONFIGURATION

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Abstract. When major failures are encountered in a missile controlled by a nominal Model Based Predictive Control ($MBPC$) these are reflected in the internal model via a relatively sophisticated mechanism involving high fidelity models written in the quasi-LPV form. At this stage an optimisation will provide the tuning parameters – the controller cost function weights– which will enable the controller to match, on-line, a reference closed loop. The algorithm is applied to provide the $MBPC$ tuning while reconfiguring on-line a missile for major structural impairments.

Keywords. Reconfigurable Flight Control, Predictive Control, Optimisation

1. INTRODUCTION

Control reconfiguration is a critical technology with particular applications in the aerospace industry. A reconfigurable controller possesses the capability during a system failure to redistribute and coordinate the control effort among the the aircraft's remaining effective control surfaces such that satisfactory flight performance is still retained.

Reconfigurable control regarded as an adaptive algorithm can be achieved in a multiple model framework using model based predictive control ($MBPC$) strategy. This control strategy has a special ability to handle constraints (Rawlings and Muske, 1993; Huzmezan and Maciejowski, 1997) and an inherent self-reconfiguration property (Huzmezan and Maciejowski, 1998) which led us to consider it as a potential control tool for reconfigurable flight control. This is our definition for reconfiguration comparative with the one for on-line design which involves always a healthy plant and its corresponding model.

In (Maciejowski, 1997; Huzmezan and Maciejowski, 1998) a particular combination of high fidelity models, fault detection and isolation (FDI), model approximation/simplification and constrained predictive control which enables a generic solution to reconfiguration and adaptation was described.

When major failures are encountered in a missile controlled by a nominal $MBPC$ controller these are reflected in the internal model via a relatively sophisticated mechanism involving high fidelity models written in the quasi-LPV form (Huzmezan and Maciejowski, 1998).

As shown in (Huzmezan and Maciejowski, 1998) it is not too simplistic to believe that it is enough to change the internal model and the constraints limits in order to reflect a fault for which the $MBPC$ controller has to provide satisfactory control inputs. But, in the most general case the controller needs to be re-tuned in order to achieve satisfactory performance.

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"Tuning" here means the adjustment of the $MBPC$ cost function weights and possibly the relaxation of some of the constraints in order to make the problem feasible.

Therefore, in this paper, on-line optimisation to provide the tuning parameters that will enable the reconfigured controller to provide good performance is investigated. In Section 5 a practical problem of reconfiguring a missile on-line in the presence of major impairments is solved.

2. THE PARAMETER DEPENDENT $MBPC$ CLOSED LOOP REALISATION

The $MBPC$ algorithm (Rawlings and Muske, 1993) provides the future control movement in the unconstrained case based on the repeated minimisation of the following performance objective (1):

$$\begin{aligned} J_i(k) = \sum_{j=1}^{N_y} \|(C_i \hat{x}_i(k+j) - s_i(k+j))\|^2_{Q_i(j)} + \\ + \sum_{j=0}^{N_u-1} \|\Delta u_i(k+j)\|^2_{R_i(j)} \end{aligned} \tag{1}$$

Hence the control variable $u(k+j)$ is manipulated, over the control horizon (N_u), to drive the predicted output $\tilde{y}(k+j)$, over the prediction horizon (N_y), towards the set-point $s(k+j)$. This optimisation problem can be re-cast in the following form:

$$\min_{\Delta U_i(k)} \Delta U_i(k)^T \mathcal{A}_i \Delta U_i(k) + \left[\hat{x}_i(k)^T, y_i(k)^T, \mathcal{S}(k)^T\right] \mathcal{B}_i \Delta U_i(k) \tag{2}$$

where

$$\begin{aligned} \mathcal{A}_i &= \mathcal{G}_i^T \mathcal{C}_i^T \mathcal{Q}_i \mathcal{C}_i \mathcal{G}_i + \mathcal{R}_i \\ \mathcal{B}_i &= 2\left[\mathcal{C}_i \mathcal{F}_i \;\; \mathcal{C}_i \mathcal{H}_i \;\; -I_{N_y p}\right]^T \mathcal{Q}_i \mathcal{C}_i \mathcal{G}_i \\ \mathcal{Q}_i &= diag\left[Q_i(1), \ldots, Q_i(N_y)\right] \\ Q_i(k+j) &= diag\left[\Theta_1, \ldots, \Theta_p\right] \\ \mathcal{R}_i &= diag\left[R_i(1), \ldots, R_i(N_u)\right] \\ R_i(k+j) &= diag\left[\Theta_{p+1}, \ldots, \Theta_{p+m}\right] \\ \mathcal{C}_i &= diag\left[\hat{C}_i, \ldots, \hat{C}_i\right] \\ \mathcal{S}_i(k) &= \left[s_i(k+1)^T, \ldots, s_i(k+N_y)^T\right]^T . \end{aligned} \tag{3}$$

The augmented system matrices will be referred to by the realization $\left[\begin{array}{c|c} \hat{A}_i & \hat{B}_i \\ \hline \hat{C}_i & 0 \end{array}\right]$ where it is assumed that the plant models are defined by $\left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array}\right]$ where the index i represents the model in use:

$$\begin{aligned} \begin{bmatrix} \Delta x_i(k+1) \\ y_{x_i}(k+1) \end{bmatrix} &= \hat{A}_i \begin{bmatrix} \Delta x_i(k) \\ y_{x_i}(k) \end{bmatrix} + \hat{B}_i \Delta u_i(k) \\ y_i(k) &= \hat{C}_i \begin{bmatrix} \Delta x_i(k) \\ y_{x_i}(k) \end{bmatrix} + \nu(k) \end{aligned}$$

Using the following simplifications $H_(0) = K$ (the observer matrix), $F_i(0) = \hat{A}_i - H_i(0)\hat{C}_i$, $F_i(1) = F_i(2) = \ldots = F_i(N_y - 1) = \hat{A}_i$, $G_i = \hat{B}_i$ which assume that the same model is used across the prediction horizon we are able to define the matrices involved in equation (3):

$$\mathcal{F}_i(k) = \begin{bmatrix} F_i(0) \\ F_i(1)F_i(0) \\ \vdots \\ \prod_{j=N_y-1}^{0} F_i(j) \end{bmatrix} \quad \mathcal{H}_i(k) = \begin{bmatrix} H_i(0) \\ F_i(1)H_i(0) \\ \vdots \\ \prod_{j=N_y-1}^{1} H_i(0) \end{bmatrix}$$

$$\mathcal{G}_i = \begin{bmatrix} G_i & \ldots & 0 \\ F_i(1)G_i & \ldots & 0 \\ \vdots & \ddots & \vdots \\ \left[\prod_{j=N_u-1}^{1} F_i(j)\right] G_i & \ldots & G_i \\ \vdots & \ddots & \vdots \\ \left[\prod_{j=N_y-1}^{1} F_i(j)\right] G_i & \ldots & \left[\prod_{j=N_y-1}^{N_u} F_i(j)\right] G_i \end{bmatrix}$$

where $\prod_{\lambda=\epsilon}^{\delta} R(\lambda) = R(\epsilon)R(\epsilon-1)R(\epsilon-2)\ldots R(\delta)$, $\epsilon > \delta$

Using results from (Fletcher, 1991) and assuming the availability of the state for measurement the optimal solution is given by the first control move $\Delta u_i(k) = \Sigma_1 \Delta U_i^*(k)$ where:

$$\Delta U_i^*(k) = -\frac{1}{2} \cdot \Sigma_1 \cdot \mathcal{A}_i^{-1} \cdot \mathcal{B}_i^T \cdot \Sigma_2 \left[\Delta x_i(k)^T, y_i(k)^T, s_i(k)^T\right]^T$$

for $\Sigma_1 = \left[I_{m\times m} \;\; 0_{m\times m} \;\; \ldots \;\; 0_{m\times m}\right]$ and, assuming a step type of set-point, Σ_2 is defined as:

$$\Sigma_2 = \begin{bmatrix} I_{n\times n} & 0_{n\times p} & 0_{n\times p} \\ 0_{p\times n} & I_{p\times p} & 0_{p\times p} \\ 0_{p\times n} & I_{p\times p} & 0_{p\times p} \\ 0_{p\times n} & 0_{p\times p} & I_{p\times p} \\ \ldots & \ldots & \ldots \\ 0_{p\times n} & I_{p\times p} & 0_{p\times p} \end{bmatrix}$$

Hence, the open loop is closed, as shown in Figure 1, with the following feedback:

$$\Delta u_i(k) = K\left(\left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array}\right], \Theta\right) \begin{bmatrix} \Delta x_i(k) \\ y_i(k) \\ s_i(k) \end{bmatrix} \tag{4}$$

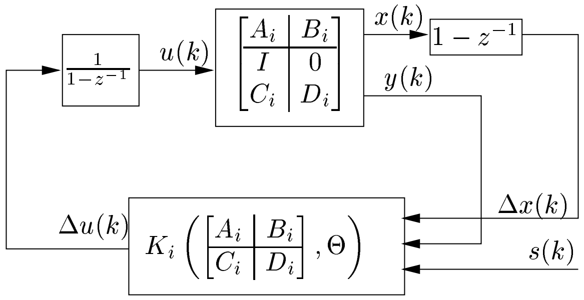

Fig. 1. The unconstrained $MBPC$ closed loop

This loop can be restructured in the manner depicted in Figure 2 where the augmented plant, based on the model realisation $\left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array}\right]$, is: $\left[\begin{array}{c|c} \bar{A}_i & \bar{B}_i \\ \hline \bar{C}_i & \bar{D}_i \end{array}\right] = \left[\begin{array}{cc|c} A_i & 0 & B_i \\ C_iA_i & I & C_iB_i + D_i \\ \hline I & I & 0 \end{array}\right]$.

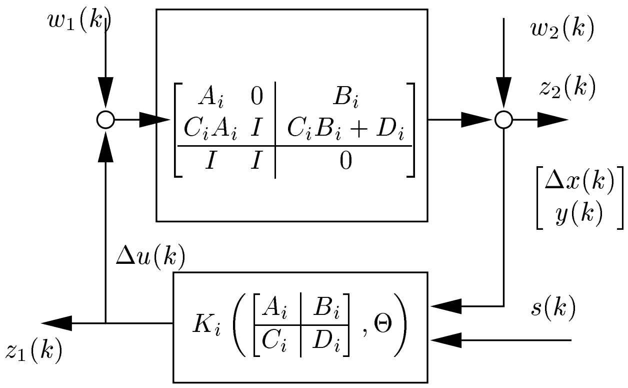

Fig. 2. The unconstrained $MBPC$ restructured closed loop

For a zero reference vector the feedback law is:

$$\Delta u_i(k) = \bar{K}_i \begin{bmatrix} \Delta x(k) \\ y(k) \end{bmatrix}$$

where $\bar{K}_i$ has the following form:

$$\bar{K}_i = \Sigma_1 \mathcal{A}_i^{-1} \mathcal{G}_i^T \mathcal{C}_i^T \mathcal{Q}_i^T \left[\mathcal{C}_i \;\; \mathcal{F}_i\right]$$

Considering the transfer matrix from the input $w_1(k)$ and output $w_2(k)$ disturbance signals, respectively, to the controller $z_1(k)$ and plant $z_2(k)$ output signals, respectively, we can picture the following structure, see Figure 3:

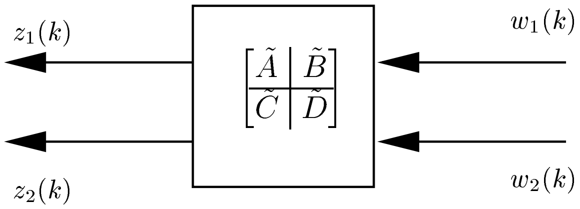

Fig. 3. The four-block transfer matrix

Where matrices $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ are defined as follows:

$$\tilde{A} = \bar{A} + \bar{B}(I - \bar{K}\bar{D})^{-1}\bar{K}\bar{C}$$
$$\tilde{B} = \left[\bar{B}\left[I + (I - \bar{K}\bar{D})^{-1}\bar{K}\bar{D}\right] \;\; \bar{B}(I - \bar{K}\bar{D})^{-1}\bar{K}\right]$$
$$\tilde{C} = \begin{bmatrix} (I - \bar{K}\bar{D})^{-1}\bar{K}\bar{C} \\ \bar{C}\left[I + \bar{D}(I - \bar{K}\bar{D})^{-1}\bar{K}\right] \end{bmatrix}$$
$$\tilde{D} = \begin{bmatrix} (I - \bar{K}\bar{D})^{-1}\bar{K}\bar{D} & (I - \bar{K}\bar{D})^{-1}\bar{K} \\ \bar{D}\left[I + (I - \bar{K}\bar{D})^{-1}\bar{K}\bar{D}\right] & \left[I + (I - \bar{K}\bar{D})^{-1}\bar{K}\right] \end{bmatrix}$$

3. MATCHING THE REFERENCE CLOSED LOOP EIGENVALUES VIA ON-LINE OPTIMISATION

Our main goal is to have in the case of the impaired system the same responses as the ones provided by the healthy plant and its corresponding controller. Before discussing about the optimisation problem involved to solve the matching of the two closed loops we would like to point out two assumptions made, valid for both Section 3 and 4.

First one assumes the horizons as fixed parameters for both the nominal and the impaired system cost functions. In fact horizon dimensions are defined according with the general guide lines (Soeterboek, 1992) used in tuning $MBPC$ controllers. The control horizon was chosen to be the same as the number of model states which will ensure stability even in the worst case when all the plant modes are unstable (Rawlings and Muske, 1993). The prediction horizon is determined by the sampling time and the performance to be reached with the closed loop.

The second assumption refers to the order of the impaired system. Because in our case the impaired model is obtained employing the high fidelity model written in the quasi-LPV form (Huzmezan and Maciejowski, 1998) the number of states is considered to be the same in spite of major structural failures.

Authors like (Gao and Antsaklis, 1991) applying pseudo-inverse reconfiguration methods, translate the matching of the healthy and impaired system responses into the goal of keeping the closed loop eigenvalues of the impaired system in the same region as in the nominal case, if possible.

Initially, we matched the $MBPC$ controller by performing a Frobenius norm optimisation of the form:

$$\min_{\Theta} \|K_f(\Theta) - K_n\|_F$$

The results obtained drove us to the conclusion that a matching of the closed-loop eigenvalues with the ones corresponding to the nominal closed-loop will be more appropriate. As result the following optimisation was employed:

$$\min_{\Theta} \|\bar{A}_f(\Theta) - \bar{A}_n\|_F$$

The minimisation was carried out using the `fminu` function from the Matlab Optimisation Toolbox, a nonlinear optimisation algorithm based on quasi-Newton method using a mixed quadratic and cubic search procedure. This was employed due to the particular nonlinear dependency of A_f on Θ.

The form of the cost function (1) depending on Q and R explains the existence of a non-unique solution. In other words, for the SISO case, the non-uniqueness of the solution can be explained observing that the actual parameter in the optimisation problem is the ratio between the weights Q and R.

The reason of choosing the Frobenius norm as a measure for the distance between the two closed loop matrices is not only related to the optimisation feasibility but as well to stability guarantees that we provide when the reconfigured controller is employed. In the case of a small perturbation upon the eigenvalues of the closed loop (A_f) from the nominal closed loop (A_n) the Bauer-Fike theorem gives a condition for the location of the eigenvalues of the reconfigured closed loop. We include the proof because in (Golub and Loan, 1996) the theorem was stated and proved only for p-norms. The stability guarantees are restricted to the assumption that A_n is non defective in the sense that it can be reduced to a diagonal form by a similarity transformation.

Theorem 3.1. If μ is an eigenvalue of the matrix $A_f \in \mathbb{R}^{q\times q}$, $X^{-1}A_nX = diag([\lambda_1, \dots, \lambda_q]) = D$ where X is the matrix of the A_n eigenvectors, $k_2(X)$ is the X matrix condition number and λ is an eigenvalue of A_n ($\lambda \in \lambda(A_n)$) then:

$$\min_{\lambda\in\lambda(M_n)} |\lambda - \mu| \le k_2(X) \parallel E \parallel_F \tag{5}$$

where $E = A_f - A_n$

Proof It is enough to consider the case when μ is not in $\lambda(A_n)$. The matrix $X^{-1}(A_f - \mu I)X$ is singular, then applying the matrix inversion lemma so is $I + (D - \mu I)^{-1}X^{-1}(A_f - A_n)X$. Note that if $I + M$ is singular at least one of the eigenvalues of M is -1 which means that $\|M\|_F \ge \|M\|_2 \ge \bar{\sigma}(M) \ge |-1|$. Thus, using the modified sub multiplicative property for the Frobenius norm written in its two different forms $\|AB\| \le \|A\|_2\|B\|_F$ and $\|AB\| \le \|B\|_2\|A\|_F$ we have:

$$\begin{aligned}
1 &\le \|(X^{-1}A_nX - \mu I)^{-1}(X^{-1}EX)\|_F \\
&\le \|(X^{-1}A_nX - \mu I)^{-1}\|_2\|(X^{-1}EX)\|_F \\
&\le \min_{\lambda\in\lambda(A_n)} \frac{1}{|\lambda-\mu|}\|X^{-1}\|_2\|EX\|_F \\
&\le \min_{\lambda\in\lambda(A_n)} \frac{1}{|\lambda-\mu|}\|X^{-1}\|_2\|X\|_2\|E\|_F \\
&\le \min_{\lambda\in\lambda(A_n)} \frac{1}{|\lambda-\mu|}k_2(X)\|E\|_F
\end{aligned}$$

□

Basically the theorem defines a disk, centred on the old eigenvalue, in which a new eigenvalue will be located once the optimisation performed. This bound is given by the condition number of the eigenvector matrix of A_n and the value of the cost function.

The eigenvalues of A_f lie in the union of disks $|\lambda - \mu|$ having a radius $k_2(X) \parallel E \parallel_F$. We recommend a nominal design that will place the nominal closed loop eigenvalues centred within the open unit disk ensuring an increased level of robust stability to the system. Using the above procedure, for a good conditioning of eigenvectors of the nominal A_n, the minimisation performed provides a closed loop behaviour of the reconfigured system close to the nominal one. The increased control effort used, that requires enough actuator authority, might be considered as a disadvantage of the current design procedure which made us to consider the solution presented in the next section.

If $k_2(X)$ is large then small changes in A_f can induce large changes in the closed loop eigenvalues. In the case of a practical problem a poor conditioning of the matrix of eigenvectors of A_n (X) is possible. This is a problem located at the level of the nominal design. Further research has to be carried out in order to provide a good conditioning for the $k_2(X)$ number when designing the $MBPC$ controller for the nominal plant.

Despite the stability guarantees and a relatively simple optimisation procedure the above approach suffers from the fact that only the closed-loop poles are matched but not the zeros, which are known to have as well a big effect on the system response.

4. CONTROL RECONFIGURATION AUTOMATIC DESIGN PROCEDURE

The natural evolution was to solve a minimisation that by matching the system four-block transfer matrix which accounts for both closed-loop poles and zeros.

The automatic tuning procedure employed as part of the overall control mechanism follows the following steps:

(1) A nominal controller is designed for the given performance specifications having the nominal realisation $\left[\begin{array}{c|c} A_n & B_n \\ \hline C_n & D_n \end{array}\right]$ as the internal model.
(2) Given a plant model $\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right]$ reflecting an impairment a closed loop realisation is produced depending on the elements of the cost function weighting matrices Q and R which we denote by the parameter vector Θ.
(3) A minimisation procedure is performed with respect to Θ of the ∞-norm of the difference between the transfer matrix defining the nominal reference closed loop $\hat{M}_n\left(\left[\begin{array}{c|c} A_n & B_n \\ \hline C_n & D_n \end{array}\right]\right)$ and the transfer matrix $\hat{M}_f\left(\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right], \Theta\right)$ which is defined on the basis of the impaired model controlled by the $MBPC$, depending on the parameter Θ.

The procedure can be frequency weighted via W_i and W_o, as common in model matching setups, leading to the following optimisation problem with respect to the parameter Θ:

$$\min_{\Theta} \left\| W_o \left[\hat{M}_f\left(\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right], \Theta\right) - \hat{M}_n\left(\left[\begin{array}{c|c} A_n & B_n \\ \hline C_n & D_n \end{array}\right]\right)\right] W_i \right\|_{\infty} \quad (6)$$

The algorithm chosen to perform this minimisation is based on a fairly brute force procedure but in exchange it provides feasibility together with good control of the level of matching. This procedure proved successful when severe impairments were encountered.

Therefore here we include the steps of the algorithm which produces a solution for Θ that gives the best matching:

(1) Perform a gridding of the parameter space Θ with an appropriate step.
(2) Compute for the current value of the parameter the value of the cost function from (6).
(3) Store the value of Θ and the corresponding norm if smaller than figures previously obtained.
(4) Iterate Steps 2 and 3 for all the grid vertices.

5. SOLVING A PRACTICAL PROBLEM

This work represent a natural extension of the idea described in (Huzmezan and Maciejowski, 1998). In this paper we concentrate on the same missile model of which complete details are given in (Shamma and Cloutier, 1993). In flight scheduling for the nominal loop and reconfiguration were performed using quasi-LPV high-fidelity models and $MBPC$ control.

The $MBPC$ internal model is based on the plant having the approximated discrete time transfer function (7) obtained for a sampling time of $T_s = 0.01$s. This model was defined at $\alpha = 0$, even though α varies during the manoeuvre.

$$\frac{n_Z}{\nu} = \frac{(z-1.3732)(z-0.073)}{(z-1)(z-0.9417)(z-1.055)} \quad (7)$$

The structural failure was simulated by manipulating the nonlinear model dynamics. The failure model (8) was obtained for values of the parameters k_Z and k_m, see (Huzmezan and Maciejowski, 1998), reflecting a failure that modified the aerodynamic characteristics. The resulting plant model is:

$$\frac{n_Z}{\nu} = \frac{(z-0.77)(z-1.42)}{(z-1)(z-1.12)(z-0.975)} \quad (8)$$

As seen from (7) and (8) major failures lead to a change in the linear internal model. The severity of these impairments require the controller re-tuning.

The automatic tuning procedure keeps the horizons fixed but adjusts the weights in (1) via the optimisation (6) performed with respect to Θ. The control horizon was chosen to be the same as the number of states ($N_u = 3$ in this case) which will ensure stability even in the worst case, namely when all the modes are unstable, should that ever occur (Rawlings and Muske, 1993). The small sampling time has determined a long prediction horizon of $N_y = 30$ in order to predict 0.3s ahead. This was enough to ensure the performance criteria required in (Shamma and Cloutier, 1993). The nominal values chosen for Q and R were 0.005 and 1, respectively.

The new weights obtained from the optimal Θ were obtained using the procedure described in the Section 4. The value for the grid step was in this case 0.01 but for other cases exhibiting a large ration between Q and R it has to be much finer. Of course the grid step influences the solution speed.

We show in Figure 4 the behaviour of the closed loop with the different controllers when the system is subjected to a step of 0.1 in the non-dimensional controlled variable n_Z.

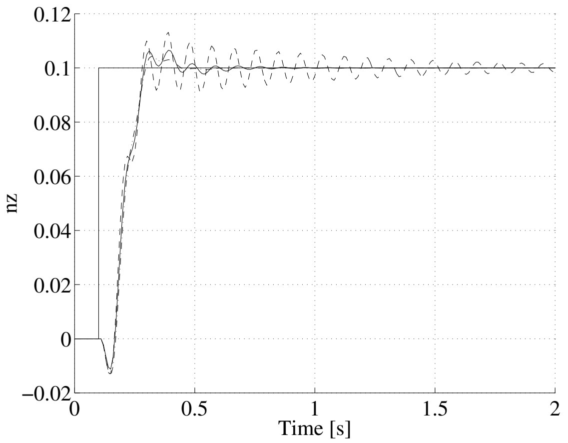

Fig. 4. Evaluation of the $MBPC$ automatic tuning

In Figure 4 the dash-dotted line depicts the behaviour of the nominal plant with an $MBPC$ controller based on a nominal internal model and nominal tuning. The broken line is showing the impaired plant controlled by the $MBPC$ having information about the failure via the internal model but still using the cost function nominal tuning. The solid line shows the behaviour of the impaired plant this time controlled by the re-tuned $MBPC$ using the newly developed automatic procedure.

6. CONCLUSIONS

The automatic tuning procedure represents an extension to a general reconfiguration strategy described in (Huzmezan and Maciejowski, 1998). Depending on the bounds defined by the ∞-norm computation we can state for a practical problem how big the failure can be to have the method coping with it.

Together with the ability of this method to provide automatic tuning for the $MBPC$ controller while reconfiguring in case of failures we consider other features included. For instance using MIL type of specifications for the closed loop it is possible to employ this method to tune the controller in such a way as to match a reference model. The approximation of other state feedback controllers with an $MBPC$ regulator is possible in a similar manner.

7. ACKNOWLEDGEMENTS

We would like to acknowledge Pembroke College Cambridge, Lundgren Fund, ORS scholarship, Cambridge Overseas Trust and CT Taylor Fund by which Mihai Huzmezan is supported. We acknowledge as well the support given by the DERA – Bedford in support of the work covered in this paper.

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