A WAVELET BASED DE-NOISING TECHNIQUE FOR OCULAR ARTIFACT CORRECTION OF THE ELECTROENCEPHALOGRAM

Tatjana Zikov, Stéphane Bibian, Guy A. Dumont, Mihai Huzmezan
Department of Electrical and Computer Engineering, The University of British Columbia, BC, CANADA

Abstract – **This paper investigates a wavelet based de-noising of the electroencephalogram (EEG) signal to correct for the presence of the ocular artifact (OA). The proposed technique is based on an over-complete wavelet expansion of the EEG as follows: i) a stationary wavelet transform (SWT) is applied to the corrupted EEG; ii) the thresholding of the coefficients in the lower frequency bands is performed; iii) the de-noised signal is reconstructed. This paper demonstrates the potential of the proposed technique for successful OA correction. The advantage over conventional methods is that there is no need for the recording of the electrooculogram (EOG) signal itself. The approach works both for eye blinks and eye movements. Hence, there is no need to discriminate between different artifacts. To allow for a proper comparison, the contaminated EEG signals as well as the corrected signals are presented together with their corresponding power spectra.**

***Keywords* - stationary wavelet transform (SWT), electroencephalogram (EEG), electrooculogram (EOG), ocular artifact (OA)**

I. INTRODUCTION

The electroencephalogram (EEG) gives researchers a non-invasive insight into the intricacy of the human brain. It is a valuable tool for clinicians in numerous applications, from the diagnosis of neurological disorders, to the clinical monitoring of depth of anesthesia. For awake healthy subject, normal EEG amplitude is in the order of 20-50μV. The EEG is very susceptible to various artifacts, causing problems for analysis and interpretation. In current data acquisition, eye movement and blink related artifacts are often dominant over other electrophysiological contaminating signals (e.g. heart and muscle activity, head and body movements), as well as external interferences due to power sources. Eye movements and blinks produce a large electrical signal around the eyes (in the order of mV), known as electrooculogram (EOG), which spreads across the scalp and contaminates the EEG. These contaminating potentials are commonly referred to as ocular artifact (OA).

The rejection of epochs contaminated with OA usually leads to a substantial loss of data. Asking subjects not to blink or move their eyes, or to keep their eyes shut and still, is often unrealistic or inadequate. The fact that the subject is concentrating on fulfilling these requirements might itself influence his EEG. Hence, devising a method for successful removal of ocular artifacts (OAs) from EEG recordings has been and still is a major challenge.

Widely used time-domain regression methods involve the subtraction of some portion of the recorded EOG from the EEG [1, 2]. They assume that the propagation of ocular potentials is volume conducted, frequency independent and without any time delay. However, Gasser *et al.* in [3] argued that the scalp is not a perfect volume conductor, and thus, attenuates some frequencies more than others. Consequently, frequency-domain regression was proposed. In addition, no significant time delay was found, which was in consistency with the EOG being volume conducted.

In [4] it was reported that, in reality, the frequency dependence does not seem to be very pronounced, while the assumption of no measurable delay was confirmed. Thus, while some researchers support the frequency domain approach for EOG correction [3, 5], others disputed its advantages [4, 6, 7]. However, neither time nor frequency regression techniques take into account the propagation of the brain signals into the recorded EOG. Thus a portion of relevant EEG signal is always cancelled out along with the artifact. Further, these techniques mainly use different correction coefficients for eye blinks versus eye movements. They also heavily depend on the regressing EOG channel.

In addition, Croft and Barry [7] demonstrated that the propagation of the EOG across the scalp is constant with respect to ocular artifact types and frequencies. They proposed a more sophisticated regression method (the aligned-artifact average solution) that corrects blinks and eye movement artifacts together, and made possible the adequate correction for posterior sites [6]. They claim that the influence of the EEG-to-EOG propagation has been minimized in their method.

In an attempt to overcome the problem of the EEG-to-EOG propagation, a multiple source eye correction method has been proposed by Berg and Scherg [8]. In this method, the OA was estimated based on the source eye activity rather than the EOG signal. The method involves obtaining an accurate estimate of the spatial distribution of the eye activity from calibration data, which is a rather difficult task.

Due to its decorrelation efficiency, the principal component analysis (PCA) has been applied for OA removal from the multi-channel EEG and it outperformed the previously mentioned methods. However, it has been shown that PCA cannot fully separate OAs from the EEG when comparable amplitudes are encountered [9].

Recently, independent component analysis (ICA) has demonstrated a superior potential for the removal of a wide

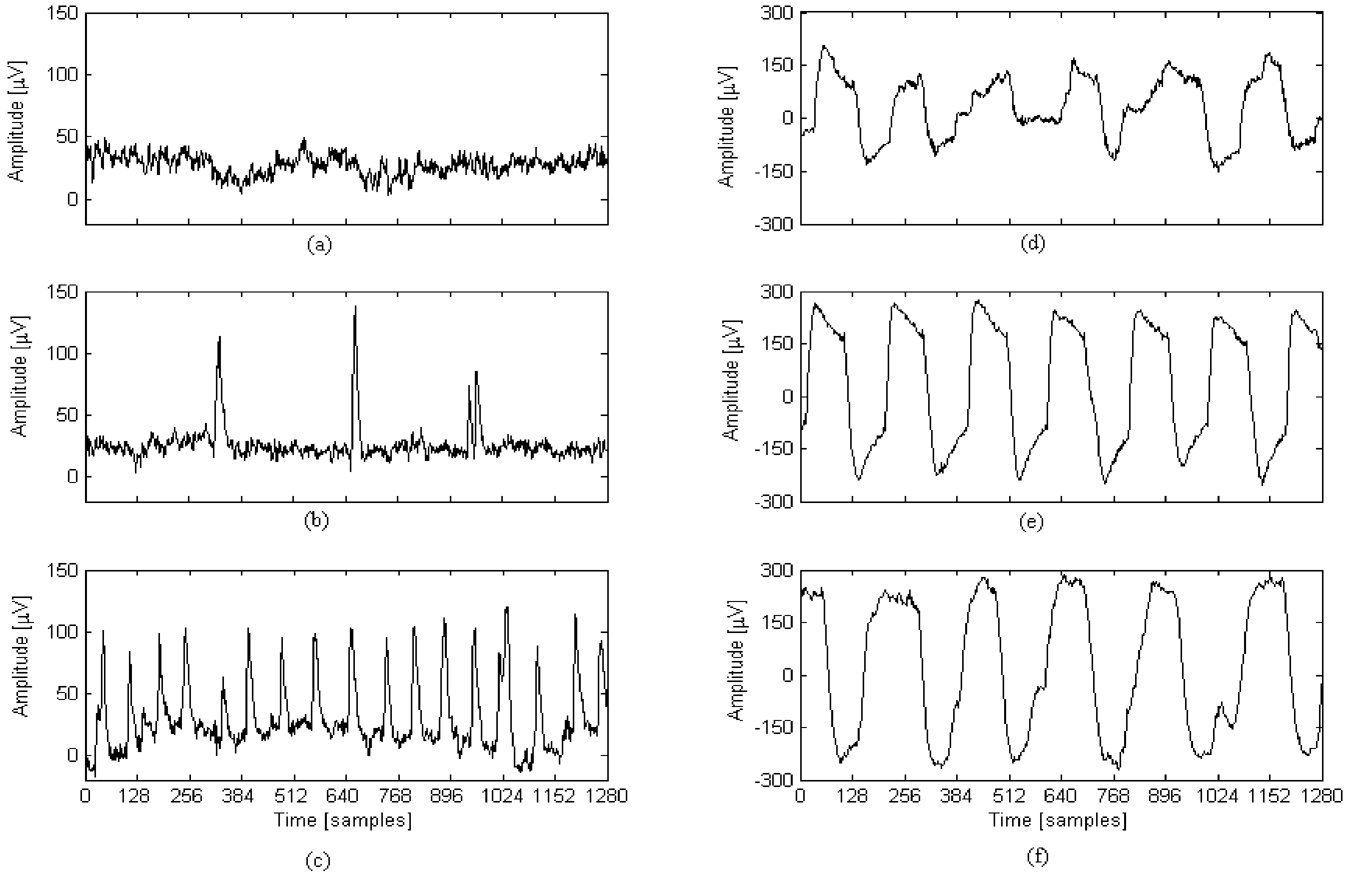


Fig.1 Uncontaminated Baseline EEG and Various Artifacts
(a) Uncontaminated baseline EEG (b) EEG contaminated with slow blink artifact (c) EEG contaminated with fast blink artifact (d) EEG contaminated with vertical eye movement artifact (e) EEG contaminated with horizontal eye movement artifact (f) EEG contaminated with round eye movement artifact

variety of artifacts from the EEG [10, 11], even in a case of comparable amplitudes. ICA simultaneously and linearly unmixes multi-channel scalp recordings into independent components, that are often physiologially plausible. Also, there is no need for a reference channel corresponding to each artifact source. However, ICA artifact removal is not yet fully automated and requires visual inspection of the independent components in order to decide their removal.

Other attempts have been based on different adaptive signal processing techniques [12-16]. The performance of these methods also relies on a cerebral activity to minimally contaminate the EOG reference.

The EEG may contain pathological signals, which resemble OAs. Thus, these signals are most likely to be removed from the EEG recordings, leading to erroneous diagnosis. Therefore, it is important to distinguish between artifacts and pathological EEG signals prior to artifact removal. Artificial intelligence techniques prove to be somehow helpful in achieving this goal [17].

Our aim is to present a real-time OA removal technique, based on stationary wavelet transform (SWT) de-noising of a single frontal channel EEG. The proposed technique is based on an over-complete wavelet expansion of the EEG as follows: i) a stationary wavelet transform is applied to the corrupted EEG; ii) the thresholding of the coefficients in the lower frequency bands is performed; iii) the de-noised signal is reconstructed. No reference EOG channel is needed and the same approach is used for both the blinks and eye movement artifacts.

The time and frequency characteristics of OAs are addressed in Section II, while Section III discusses the proposed method. Results are further presented in Section IV.

II. Ocular Artifacts

There are two different originating phenomena for ocular potentials [1, 18, 19]. There is a potential difference of about 100 mV between a positively charged cornea and negative retina of the human eye, thus forming an electrical dipole (i.e. corneo-retinal dipole). Firstly, the rotation of the eyeball results in changes of the electrical field across the skull. Secondly, eye blinks are usually not associated with ocular rotation; however, the eyelids pick up the positive potential as they slide over the cornea. T his creates an electrical field that is also propagated through the skull.

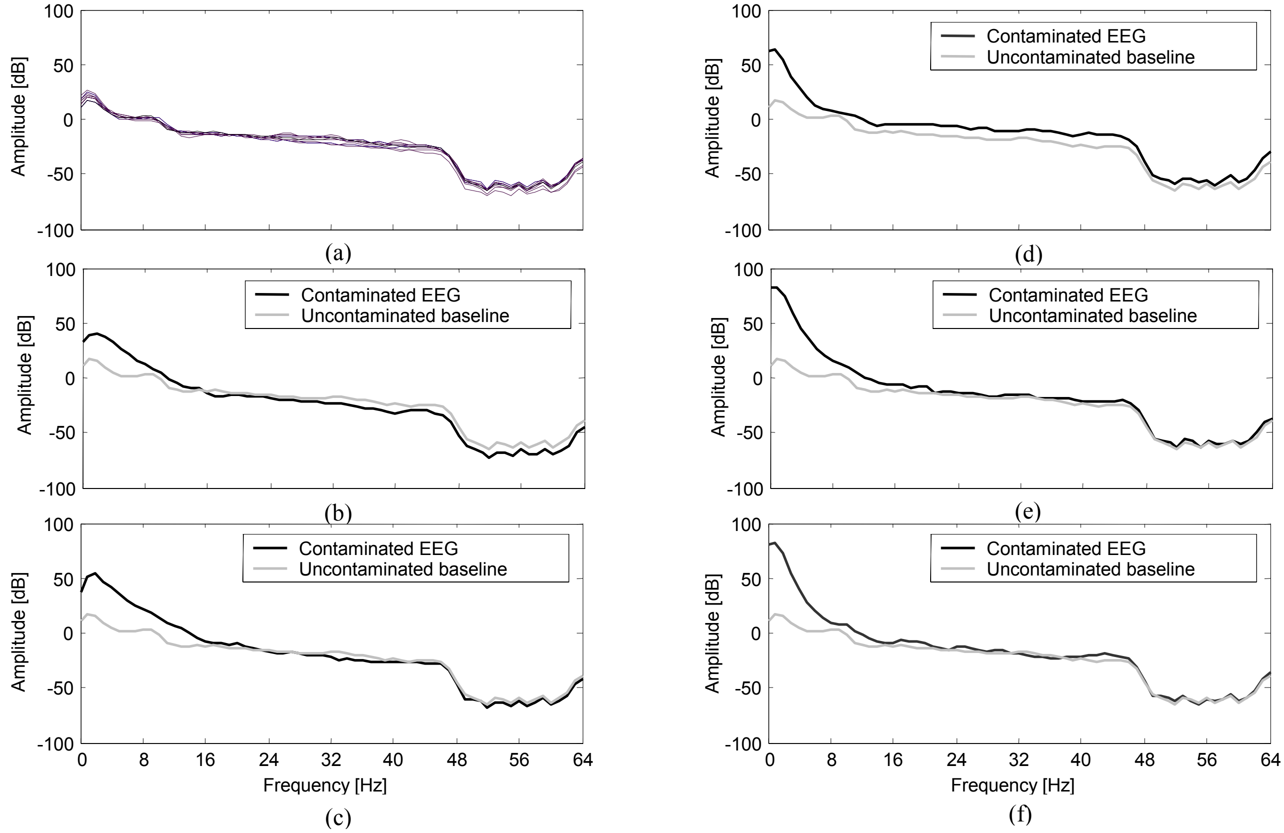


Fig.2 Power Spectra of Uncontaminated Baseline EEG and Various Artifacts
(a) Uncontaminated baseline EEG and other resting periods (b) EEG contaminated with slow blink artifact (c) EEG contaminated with fast blink artifact (d) EEG contaminated with vertical eye movement artifact (e) EEG contaminated with horizontal eye movement artifact (f) EEG contaminated with round eye movement artifact

Hence, ocular potentials spread across the scalp and superimpose on the EEG.

The mechanism of origin (eye movements versus eye blinks) and the direction of eye movements determine the resulting EOG waveshape. Vertical, horizontal and round eye movements usually result in square-shaped EOG waveforms, while blinks are spike-like waves.

Ocular artifacts decrease rapidly with the distance from the eyes [18]. Therefore, the most severe interference occurs in the EEG recorded by the electrodes placed on the patient's forehead. Yet, this is the most convenient region for their placement. Thereto, the frontal and prefrontal lobes, which are at the origin of higher cognitive functions, are located directly behind the forehead. Therefore, the task of EOG correction for frontal channels is challenging.

For the purpose of this paper, we have acquired EEG data from an awake, healthy male subject. A single frontal channel was recorded, corresponding to the F_{pz} electrode placement in the nomenclature of the International 10-20 System. The baseline EEG and five various artifacts were recorded in the following fashion. For each artifact, the subject was first instructed to keep his eyes shut and still. Sixty seconds of presumably uncontaminated baseline EEG was thus recorded. Then, for the next 60 seconds, the subject was instructed to blink or move his eyes in a predetermined fashion. Finally, another resting period of 60 seconds with no EOG activity was recorded. Five ocular artifacts were recorded in this fashion: slow blinks (1 blink per 2 seconds), fast blinks (2 blinks per second), vertical, horizontal and round eye movements. The signal was notch filtered at 50-60 Hz and sampled at 128 Hz.

The waveforms in Fig. 1 present 10 seconds of each signal; i.e. the uncontaminated baseline EEG and the EEG contaminated with different artifacts. The corresponding power spectra are presented in Fig. 2 along with the average power spectrum of the baseline EEG (see Fig. 2a), and each individual power spectra of the EEG recorded during resting periods.

Fig. 2 clearly shows that OAs are large, transient slow waves. They occupy the lower frequency range; from 0 Hz up to 6-7 Hz for the eye movement artifacts, and typically up to the alpha band (8-13 Hz), excluding very low frequencies, for the eye blinks. This is a well-known and documented result [3], which our experiments proved consistent with. Clearly, OA amplitudes are of a much higher order than those of the uncontaminated EEG and have a characteristic pattern of changes. Vertical eye movement artifacts (Fig. 2.d) also seem to produce a rise in the higher frequencies. However, this is most likely due to the increased muscle activity, and it is also present to a lesser degree for the other two eye movements (horizontal - Fig. 2.e, and round - Fig. 2.f).

III. Wavelet-based De-Noising

A. Problem Statement

As previously mentioned, the EOG is the non-cortical activity that contaminates the EEG recordings. Thus, since the brain and eye activities have physiologically separate sources, we will assume the recorded EEG is a superposition of the true EEG and some portion of the EOG signal. Thus, we have:

$$EEG_{rec}(t) = EEG_{true}(t) + k \cdot EOG(t) + dc_{offset} \qquad (1)$$

where EEG_{rec} is the recorded contaminated EEG, EEG_{true} is due to the cortical activity, and $k \cdot EOG$ is the propagated ocular artifact at the recording site. The dc_{offset} takes into account the zero mean value of the cortical EEG, since this might not be true for the recorded EEG due to the process of data acquisition.

We are interested in estimating the ocular artifact based on the analysis of the recorded EEG. By subtracting it from the contaminated EEG, we will then obtain a corrected EEG, which minimizes the effect of the ocular artifact and gives an appropriate representation of the true cortical EEG.

The true EEG is a noise-like signal. We can not observe any clear patterns within it, nor can we simply correlate the particular underlying events with its waveshape [20]. Furthermore, in the awake, conscious state, neurons are firing in a more independent fashion. As a result of this desynchronization, the awake EEG signal is even more random-apearing. The EOG removal can be approached by recovering the regression function ($k \cdot EOG$) from the recorded data. For this purpose, in the last decade, wavelet thresholding has emerged as a simple, yet effective technique for de-noising [21].

B. Wavelet Thresholding

The main statistical application of wavelet thresholding is a nonparametric estimation of the regression function f, based on observations s_i at time points t_i. The s_i observations are modelled as:

$$s_i = f(t_i) + \varepsilon_i, \qquad i = 1, 2, \ldots N \; (N=2^n) \qquad (2)$$

where ε_i are independent and identically distributed $N(0,\sigma^2)$ random variables at equally spaced time points t_i.

Due to the orthogonality of the wavelet transform, we are alowed to perform filtering in the space of wavelet coefficients. The procedure for supressing the noise involves: i) finding the coefficients of the wavelet transform of $\{s_i\}$; ii) comparing each wavelet coefficient against an appropriate threshold; iii) keeping only those coefficients larger than a threshold; and iv) applying an inverse wavelet transform to obtain an estimate of f. The assumption is that large coefficients kept after thresholding belong to the function to be estimated, and those discarded belong to the noise. This is a fair assumption due to the good energy compaction of the wavelet transform. Some of the function coefficents might eventually be discarded since they are of the same level as the noise coefficients. Thus, this de-noising technique works well for functions whose wavelet transform results in only a few nonzero wavelet coefficients, like e.g. functions that are smooth almost everywhere, except for only a few abrupt changes [22].

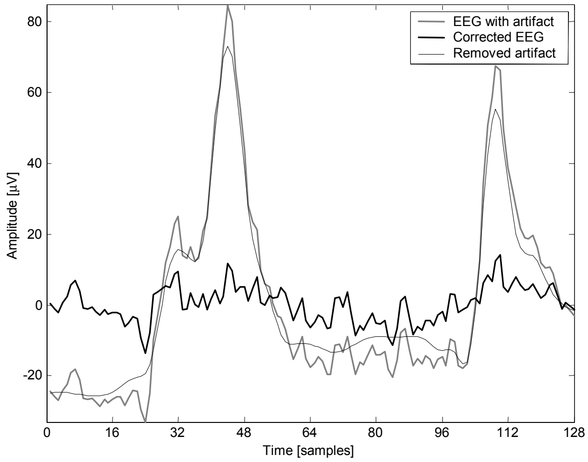
Special care has to be taken when chosing an appropriate threshold, which always involves the estimation of the noise variance σ^2 based on the data.

C. Stationary Wavelet Transform

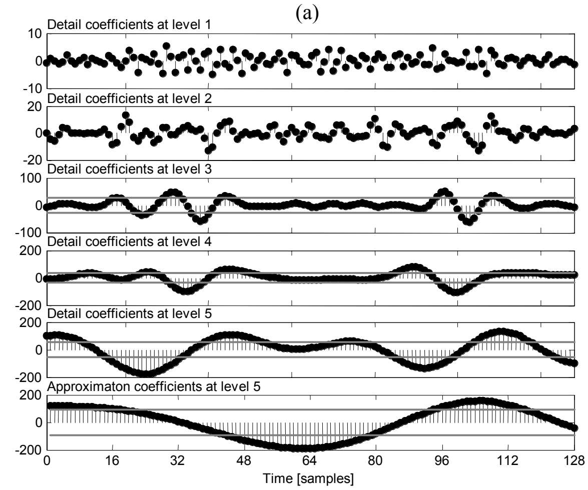
The discrete wavelet transform (DWT) is not translation-invariant, meaning that in general, if we apply a DWT to a shifted version of a signal x, we do not get the shifted version of the DWT of x. Due to this drawback, de-noising with standard DWT often suffers from artificial additional artifacts, e.g. ringing effects in the vicinity of discontinuity, depending on its actual location [23]. This produces wavelet estimators of irregular visual appearance.

In practice, we obtain the DWT of a sequence $\{x_i\}$ by applying a pair of low-pass and high-pass filters, which must comply with certain conditions, such as orthogonality [24, 25]. Then, the resulting sequences are decimated (i.e. only every even member of a sequence is kept). By feeding down the low-pass sequence to the next level and repeating the filtering and decimation, we obtain the approximation and the detail of the sequence $\{x_i\}$ at that level. This procedure is repeated until we reach the desired level of decomposition. The decimation steps can be equally carried out by selecting every odd member of each sequence instead of even ones. Furthermore, we can choose to select even members at some levels and the odd members at others. Obviously, the final result will be different, but the orthogonality of the transformation is kept, hence the process can be easily inverted to obtain the perfect reconstruction.

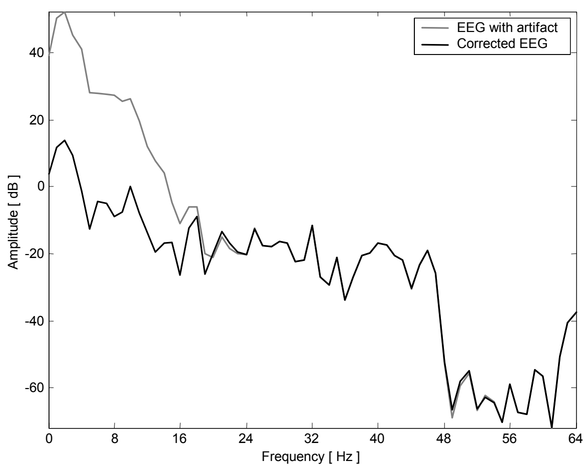
Different wavelet transforms corresponding to various selections of decimation steps are referred to as the ε-decimated discrete wavelet transforms, and they are all shifted versions of the ordinary DWT applied to the shifted sequence $\{x_i\}$ [26]. Any particular ε-decimated DWT defines a particular set of wavelet bases and their time positions, and consequently, the grid of integers for each level at which the wavelet coefficients are localized. Various misalignments between the features in the signal and those of wavelet basis are leading to more or less pronounced



(a)



(b)



(c)

Fig.3 Fast Blink Artifact
(a) Contaminated EEG and corrected EEG
(b) Stationary wavelet decomposition of contaminated EEG
(c) Power spectra of contaminated EEG and corrected EEG

artifacts when de-noising. To minimize these artifacts, we apply ε-decimated DWTs with different shifts, followed by the averaging over the obtained results [23, 26].

The stationary wavelet transform (SWT) of sequence $\{x_i\}$ is equivalent to applying each possible ε-decimated DWT, and then averaging over the results [26]. In practice, SWT is easily obtained in a similar fashion as DWT, except that the decimation step is not performed. This leads to over-complete (redundant) representation of the original signal, with great potential for statistical applications. The approximation and detail sequences at each level of decomposition are of the same length as the original sequence, rather than becoming shorter by a factor of 2 as the level increases; the complexity at level L is increased from $O(2^L)$ to $O(L2^L)$.

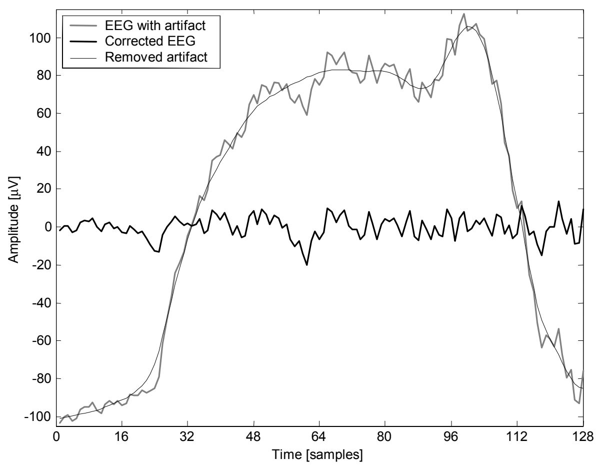
D. Methodology

Since OAs occupy lower frequency bands and have significantly larger amplitudes than the noise-like awake EEG, a multi-level wavelet decomposition of EEG_{rec} allows the detection of the presence of artifacts, as they generate much larger coefficients. Thresholding these coefficients (i.e. setting them to zero), and then recomposing the signal will thus correct the EEG. We will use the stationary wavelet transform and its inverse, since it has better sampling rates in the lower frequency bands, hence leading to smoother results.

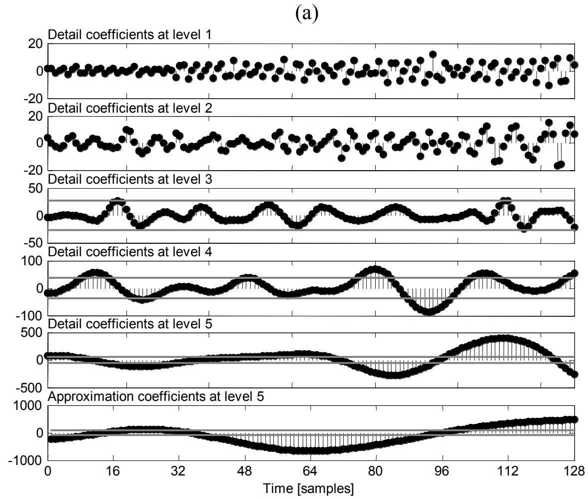
The analysis is based on a 1-second epoch of EEG signal (128 samples). To overcome boundary effects, epochs are extended on both ends, with the samples from the previous epoch at the beginning and fliped samples of the current epoch at the end. The length of the epoch extensions has to be greater or equal to the wavelet filter length. For acceptable computational complexity, the analysis has been carried out by performing a 5 level decomposition (frequency bands 0-2 Hz, 2-4 Hz, 4-8 Hz, 8-16 Hz, 16-32 Hz and 32-64 Hz). A Coiflet 3 wavelet filter has been chosen, since the shape of its mother wavelet resembles the shape of the eye blink artifact. This maximizes the amplitude of coefficients corresponding to the eye blink artifacts in the lowest band of decomposition. Furthermore, this choice minimizes the spread of artifacts to higher frequency bands. The filter length keeps the computational complexity low. It has turned out that it works properly for the eye movement artifacts as well.

Wavelet coefficients have been thresholded only in the lower frequency bands (i.e. up to 16 Hz). The thresholding procedure sets all coefficients larger than a threshold to zero. This one step procedure is equivalent to estimating the OA with standard wavelet de-noising technique, and then subtracting it from the corrupted EEG.

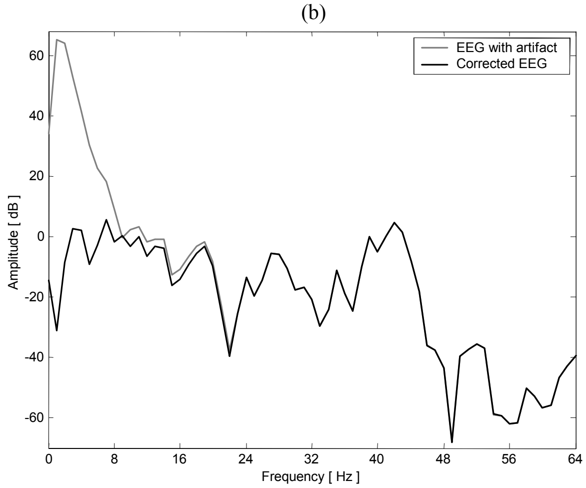
Although in overall the EEG signal is non-stationary, we can assume its stationarity during the period of one epoch. Furthermore, it is true that the variance of the awake EEG signal can significantly change from one second to another, but this variation is small in comparison with the variance of the epochs corrupted by artifacts. Hence, the



(a)



(b)



(c)

Fig.4 Horizontal Eye Movement Artifact
(a) Contaminated EEG and corrected EEG
(b) Stationary wavelet decomposition of contaminated EEG
(c) Power spectra of contaminated EEG and corrected EEG

threshold could be estimated by simple statistical analysis of the baseline EEG, which is presumably artifact-free. Thus, the aquired 60 seconds of the baseline EEG was analyzed and each second was decomposed by 5 level SWT with Coiflet 3 wavelet filter. For each second, the maximum absolute value M_k of wavelet coefficents has been calculated for each band k of decomposition below 16 Hz. The threshold T_k for the band k has been calculated as

$$T_k = mean(M_k) + 2 \cdot std(M_k) \tag{3}$$

The proposed methodology enables real time EEG correction in the presence of OAs.

IV. Results

Figures 3 and 4 show the results of the de-noising for a single epoch of fast blink and horizontal eye movement. The contaminated and corrected EEG epochs and their power spectra are shown, together with the removed artifacts. In addition, the stationary wavelet coefficients of the contaminated EEG epochs are presented along with the applied thresholds for details at levels 3-5 (frequency bands 18-16 Hz, 4-8 Hz and 2-4 Hz), and the approximation at level 5 (frequency band 0-2 Hz). Note the exact preservation of the high frequency content of the original signal, both in amplitude and phase. Note also how the dc bias is automatically removed, which is the result of thresholding in the approximation band.

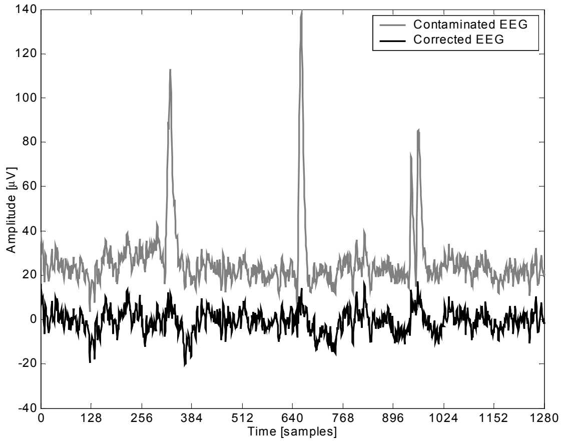
Fig. 5 shows 10 seconds of contaminated and corrected EEG for slow blink, while Fig. 6 presents the de-noising result obtained from a 5-second EEG contaminated with vertical eye movement. Since the per second analysis is based on extended epochs, we are able to cancel out the boundary distortion originating from the convolution of the wavelet filter with the sampled data.

V. Conclusion

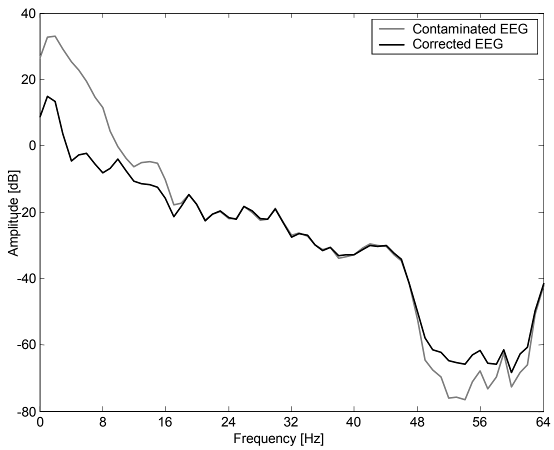
Ocular artifact correction can be a challenging task. A multitude of techniques have been proposed in the literature. However, assumptions concerning the propagation of these artifacts across the scalp are still actively discussed. There is no general consensus as of which of these techniques offer the best choice.

In this work, we have investigated a simple wavelet-based de-noising technique. Compared with previous methods, this technique neither relies on prior measurements of the EOG, nor makes the distinction between eyeball movement and eye blink artifact correction. Since in real life applications both artifacts are present simultaneously, a method based on such discrimination is then unreliable.

Using the time-frequency localization property of the wavelet transform, and the high sampling feature of SWT in all frequency bands, the proposed technique has clearly shown its potential in correcting for low frequency artifacts, such as OAs, while preserving the phase and magnitude of higher frequency components.

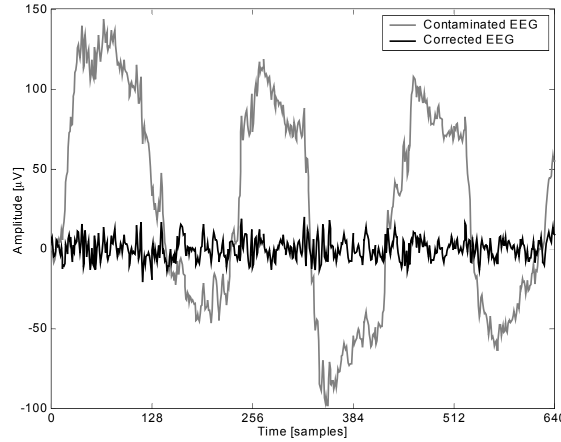


(a)

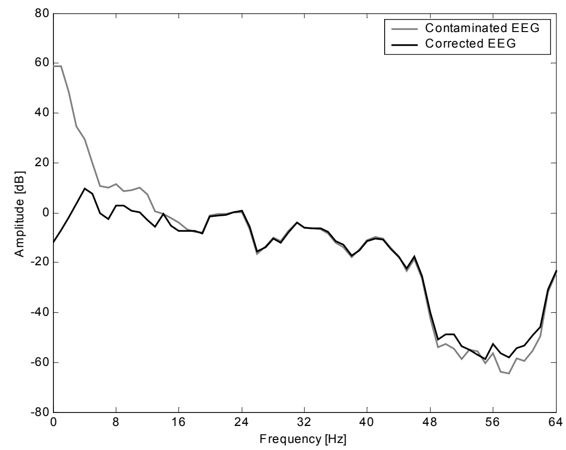


(b)

Fig.5 Slow Blink Artifact – 10 seconds
(a) Contaminated EEG and corrected EEG
(b) Power spectra of contaminated EEG and corrected EEG



(a)



(b)

Fig.6 Vertical Eye Movement Artifact – 5 seconds
(a) Contaminated EEG and corrected EEG
(b) Power spectra of contaminated EEG and corrected EEG

The selection of the threshold is still under investigation. A more conservative threshold leads to a stronger filtering in lower frequency bands. Adaptive filtering techniques might be used to fine tune threshold values. Further, when computational complexity is not of a concern, a higher level of decomposition (e.g. 7 levels) can produce even smoother results.

The technique presented in this paper represents a natural evolution towards a real-time reliable wavelet based algorithm for estimation of the hypnosis level during clinical anesthesia. In conjunction with our work presented in [27], the proposed EOG removal technique is paving a path for clinical trials of a new hypnosis monitor.

Acknowledgments

The authors wish to acknowledge the financial support from the Advanced Systems Institute of British Columbia, the Canadian Institutes of Health Research, and the Jean Hugill Templeton Chair in Anesthesia.

We also thank our colleagues from the Division of Control Systems in Pharmacology and Therapeutics (CSPT) for their continuous support.

References

[1] G. Gratton, M. G. Coles, E. Donchin, “A new method for off-line removal of ocular artifact,” *Electroenceph. Clin. Neurophysiol.,* 55(4), 468-484, 1983.

[2] R. Vergler, T. Gasser, and J. Mocks, “Correction of EOG artifacts in event-related potentials of the EEG: aspects of reliability and validity,” *Psychophysiol.,* 19, 472-480, 1982.

[3] T. Gasser, L. Sroka, and J. Mocks, “The transfer of EOG activity into the EEG for eyes open and closed,” *Electroenceph. Clin. Neurophysiol.,* 61, 181-193, 1985.

[4] J. L. Kenemans, P. C. Molenaar, M. N. Verbaten, and J. L. Slangen, “Removal of the ocular artifact from the EEG: a comparison of time and frequency domain methods with simulated and real data,” *Psychophysiol.,* 28(1), 114-121, 1991.

[5] J. C. Woestenburg, M. N. Verbaten, and J. L. Slangen, “The removal of the eye-movement artifact from the EEG by regression analysis in the frequency domain,” *Biol. Psychol.,* 16(1-2), 127-147, 1983.

[6] R. J. Croft, and R. J. Barry, “Removal of ocular artifact from the EEG: a review,” *Neurophysiol. Clin.,* 30(1), 5-19, 2000.

[7] R. J. Croft, and R. J. Barry, "EOG correction: a new aligned-artifact average solution," *Electroenceph. Clin. Neurophysiol.,* 107(6), 395-401, 1998.

[8] P. Berg, and M. Scherg, "A multiple source approach to the correction of eye artifacts," *Electroenceph. Clin. Neurophysiol.,* 90, 229-241, 1994.

[9] T. D. Lagerlund, F. W. Sharbrough and N. E. Busacker, "Spatial filtering of multichannel electroencephalographic recordings through principal component analysis by singular value decomposition," *J. Clin. Neurophysiol.,* 14(1), 73-82, 1997.

[10] R. N. Vigario, "Extraction of ocular artifacts from EEG using independent component analysis," *Electroenceph. Clin. Neurophysiol.,* 103, 395-404, 1997.

[11] T.-P. Jung, S. Makeig, C. Humphries, T.-W. Lee, M. J. McKeown, V. Iragui, and T. J. Sejnowski, "Removing electroencephalographic artifacts by blind source separation," *Psychophysiol.,* 37, 163-178. 2000.

[12] E. C. Ifeachor, B. W Jervis, E. L. Morris, E. M. Allen, and N. R. Hudson, "A new computer-based online ocular artifact removal (OAR) system," *IEE Proceedings A* , 133, 291-300, 1986.

[13] K. D. Rao, and D. C. Reddy, "On-line method for enhancement of electroencephalogram signals in presence of electro-oculogram artefacts using non-linear recursive least squares technique," *Med. Biol. Eng. Comput.,* 33, 488-491, 1995.

[14] P. K. Sadasivan, and D. N. Dutt, "Adaptive technique for the minimization of EOG artefacts from EEG signals using TDL structure and nonlinear, estimation model," *Tencon '91. IEEE Region 10 International Conference on EC3-Energy, Computer, Communication and Control systems* , 3, 251 –25, 1991.

[15] S. Selvan, and R. Srinivasan, "Removal of ocular artifacts from EEG using an efficient neural network based adaptive filtering technique," *IEEE Signal Processing Letters* , 6(12), 330 –332, 1999.

[16] R. Ksiezyk, K. J. Blinowska, P. J. Durka, W. Szelenberger, and W. Androsiuk, "Neural network with wavelet preprocessing in EEG artifact recognition," *Medicon'98,* Lemessos, Cyprus, CD-ROM ISBN 9963-607-13-6, 1998.

[17] E. C. Ifeachor, M. T. Hellyar, D. J. Mapps, and E. M. Allen, "Knowledge-based enhancement of human EEG signals," *Radar and Signal Processing, IEE Proceedings F,* 137(5), 302 –310, 1990.

[18] D. A. Overton, and C. Shagass, "Distribution of eye movement and eye blink potentials over the scalp," *Electroenceph. Clin. Neurophysiol.,* 27, 546, 1969.

[19] T. Eelbert, W. Lutzenberger, B. Rockstroh, and N. Birbaumer, "Removal of ocular artifacts from the EEG - a biophysical approach to the EOG," *Electroenceph. Clin. Neurophysiol.,* 60(5), 455-463, 1985.

[20] I. Rampil, "A primer for EEG signal processing in anesthesia," *Anesthesiol.,* 89(4), 980-1002, 1998.

[21] D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation by wavelet shrinkage ," *Biometrika,* 81, 425-455, 1994.

[22] The Mathworks Inc., MA, "Matlab user's guide. Wavelet toolbox", 1997.

[23] R. R. Coifman, and D. L. Donoho, "Translation invariant de-noising," *Lecture Notes in Statistics,* 103, 125-150, 1995.

[24] C. S. Burrus, H. Guo, and R. A. Gopinath, "Introduction to wavelets & wavelet transforms: a primer," *Prentice Hall,* Englewood Cliffs, NJ, 1997.

[25] G. Strang, and T. Nguyen, "Wavelets and filter banks," *Wellesley-Cambridge Press,* Wellesley, MA, 1997.

[26] G. P. Nason, and B. W. Silverman, "The stationary wavelet transform and some statistical applications," *Lecture Notes in Statistics,* 103, 281-29, 1995.

[27] S. Bibian, T. Zikov, G. Dumont, C. Ries, E. Puil, H. Ahmadi, M. Huzmezan, and B. Macleod, "Estimation of the anesthetic depth using wavelet analysis of electroencephalogram," *In Proceedings of EMBC 2001, 23rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society,* Istanbul, Turkey, 2001.