ELEC 344
1st Tutorial

Maxwell's Equations

(What's the Electric & Magnetic field?)

September 9, 2016
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Summary of electromagnetism to Maxwell (1860)

- Two Concepts:
  - The electric field produced by a changing magnetic field (Faraday)
  - The magnetic field produced by a changing electric field (Maxwell)

I. Maxwell Equation No.1

a) Gauss's Law

$$\Phi_{E\,\text{closed surface}} = \frac{q_{inside}}{\varepsilon_0}$$

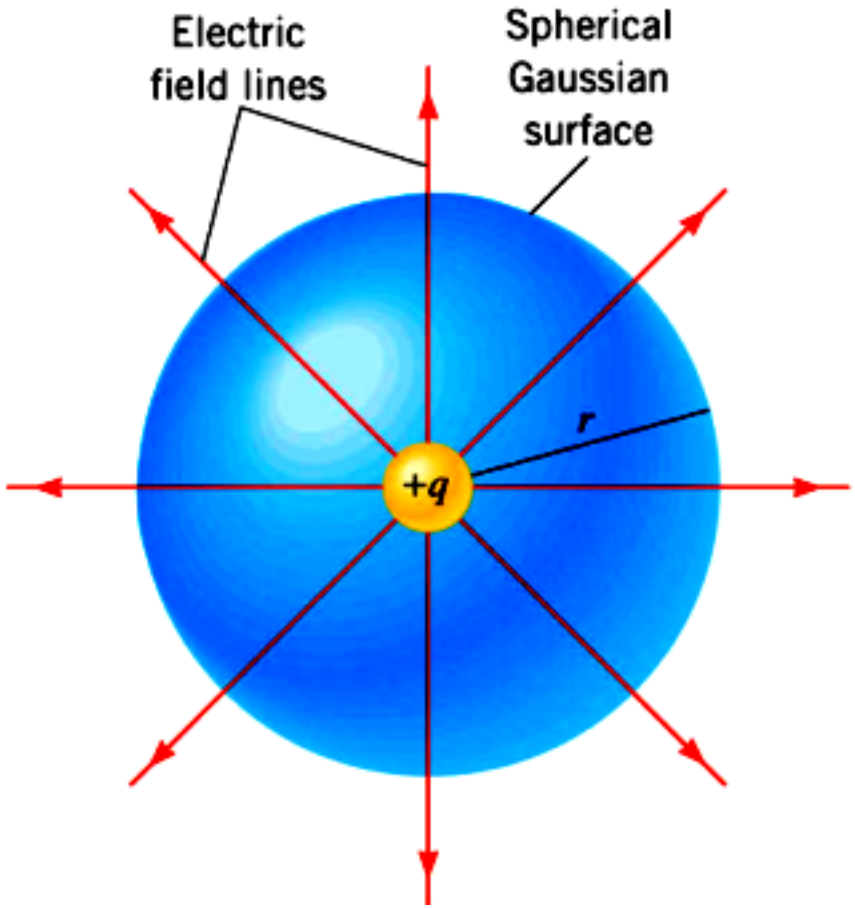
Electric charge produces an electric field, and the flux of that field passing through any closed surface is proportional to the total charge contained within that surface.

I. Maxwell Equation No.1

a) Gauss's Law

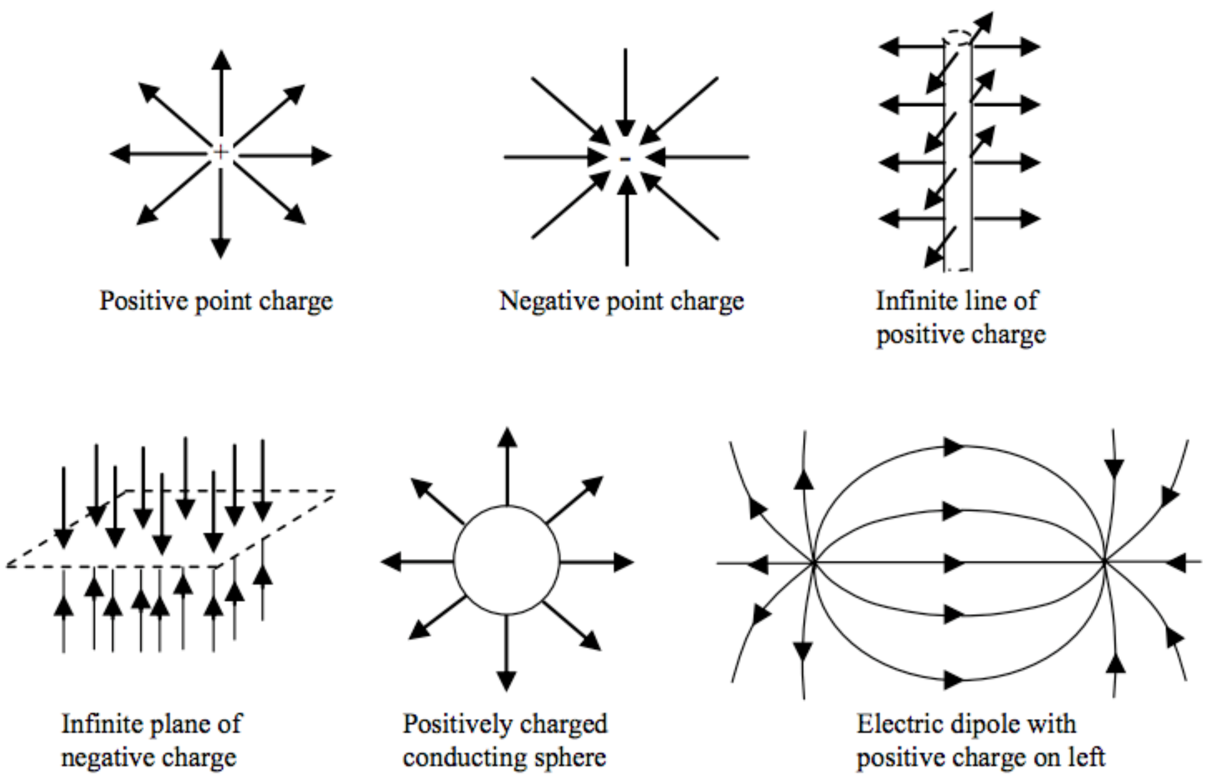
$$\Phi_E \; = \frac{q_{inside}}{\varepsilon_0}$$
closed surface

A line that doesn't start or finish inside must leave if it enters.



I. Maxwell Equation No.1

a) Gauss's Law



<Example of Electric Fields>

I. Maxwell Equation No.1

a) Gauss's Law

$$\Phi_{E\,\text{closed surface}} = \frac{q_{inside}}{\varepsilon_0}$$

$$\boxed{\Phi_{E\,\text{closed surface}} = 0}$$

(in free space)

Gauss's law expresses Coulomb's law, but valid for moving charges

II. Maxwell Equation No.2

b) Gauss's Law for magnetism

$$\Phi_M \underset{\text{closed surface}}{} = 0$$

(since there are no magnetic monopoles)

The total magnetic flux passing through any closed surface is zero.

II. Maxwell Equation No.2

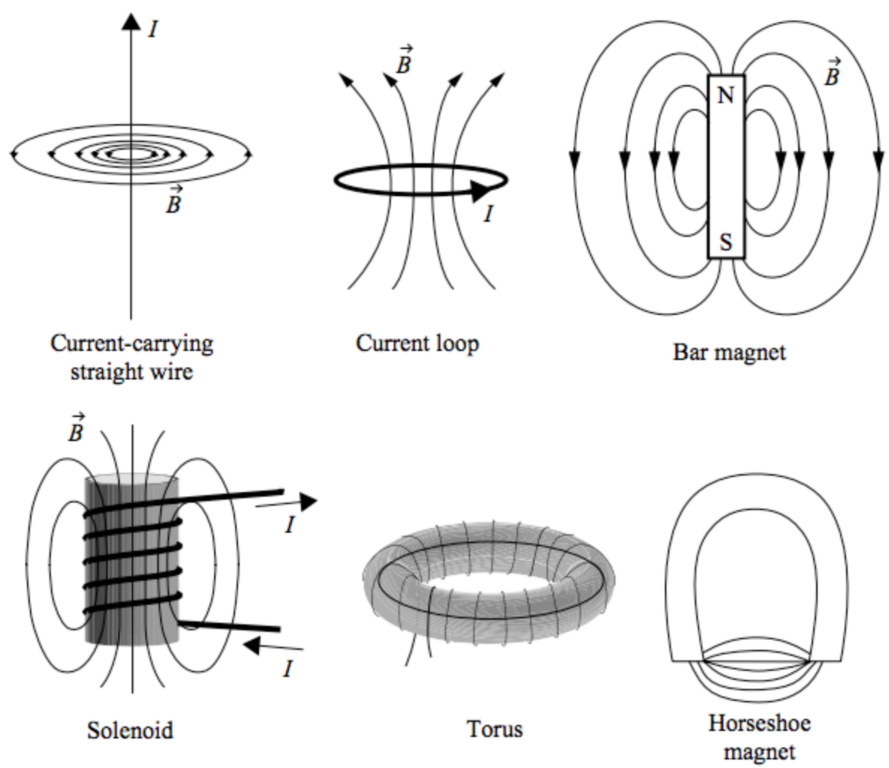
b) Gauss's Law for magnetism

$$\Phi_M \underset{\text{closed surface}}{} = 0$$

It means that for every magnetic field line that enters the volume enclosed by the surface, there must be a magnetic field line leaving that volume. Thus the inward (negative) magnetic flux must be exactly balanced by the outward (positive) magnetic flux.

II. Maxwell Equation No.2

b) Gauss's Law for magnetism

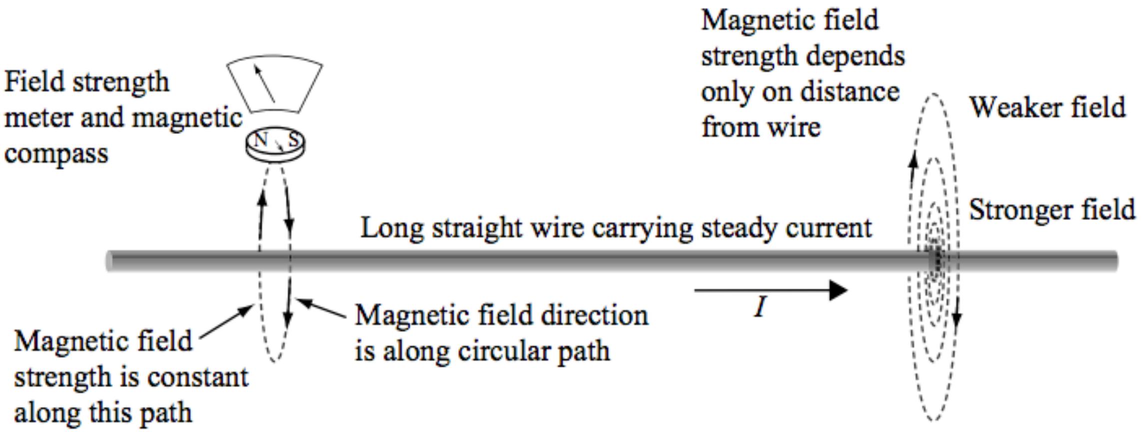


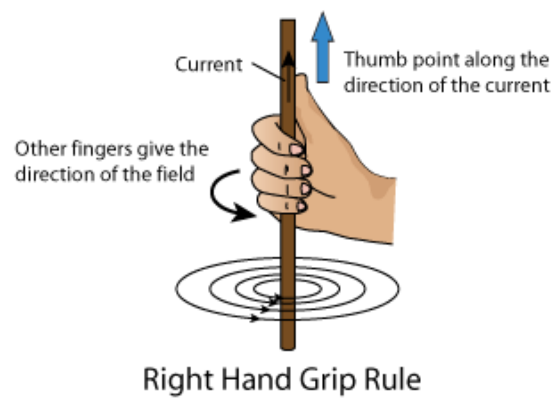
<Example of Magnetic Fields>

III. Maxwell Equation No.3

c) Ampere's Law

An electric current or a changing electric flux through a surface produces a circulating magnetic field around any path that bounds that surface.





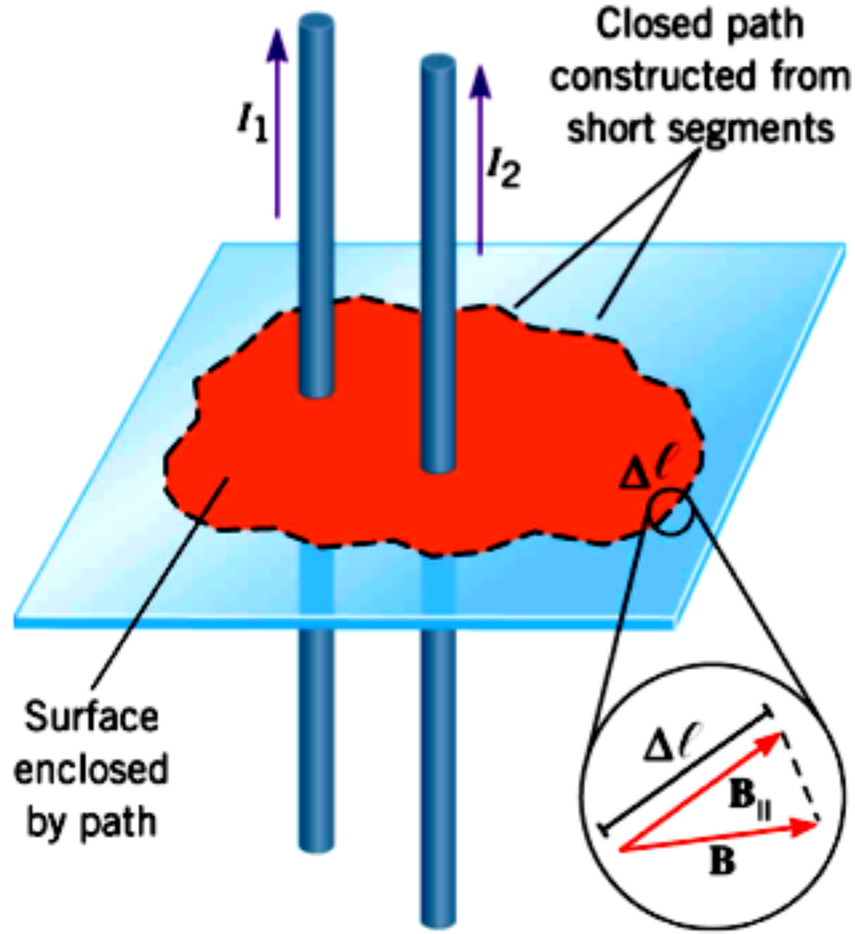
III. Maxwell Equation No.3

c) Ampere's Law

$$\sum_{loop} B_{//}\Delta\ell = \mu_0 I$$

$$\boxed{\sum_{loop} B_{//}\Delta\ell = 0}$$

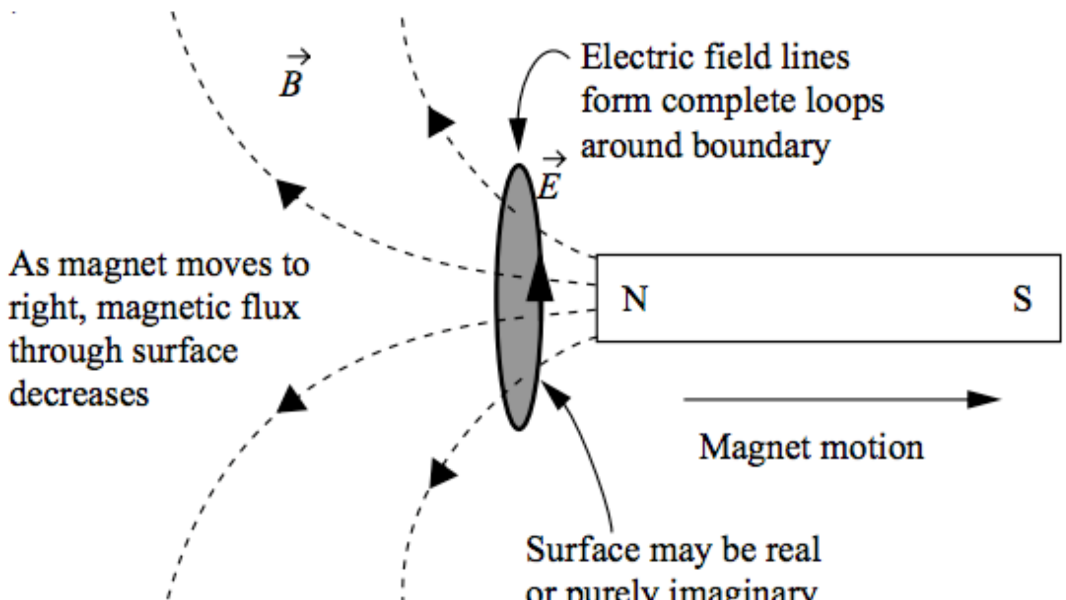
(in free space)



IV. Maxwell Equation No.4

d) Faraday's Law

Changing magnetic flux through a surface induces an emf in any boundary path of that surface, and a changing magnetic field induces a circulating electric field.



IV. Maxwell Equation No.4

d) Faraday's Law

$$\mathcal{E} = -\frac{\Delta\Phi_M}{\Delta t}$$

Lenz's Law (the minus sign)

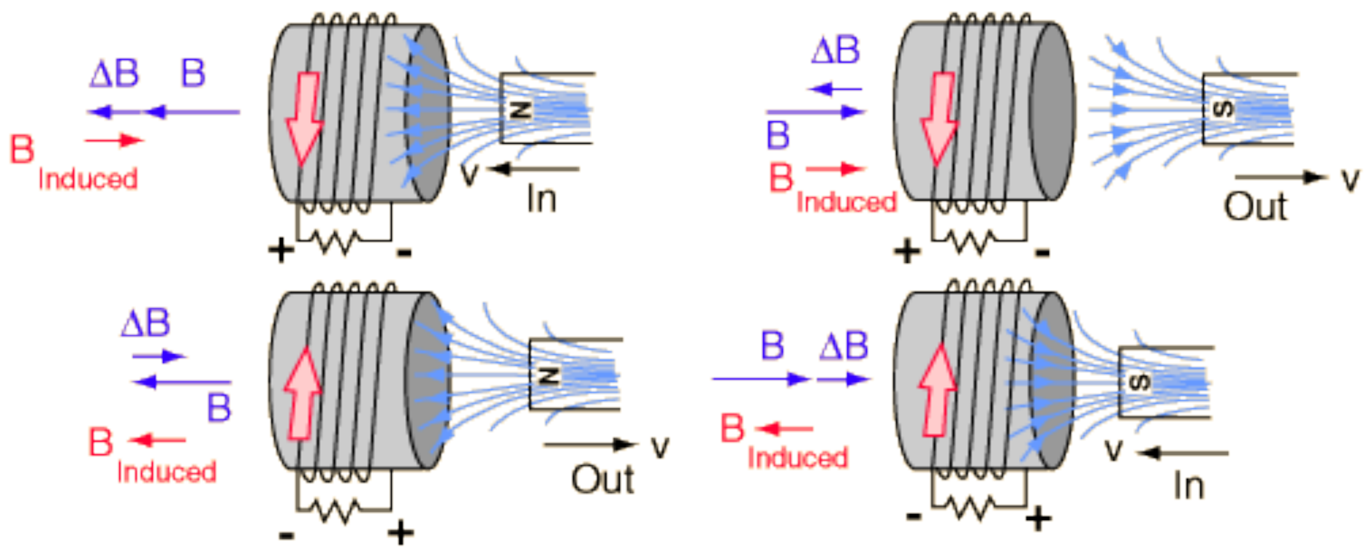
Recall $E_x = -\frac{\Delta V}{\Delta x}$ so around a loop, $\mathcal{E} = \sum_{loop} E_{//}\Delta\ell$

$$\boxed{\sum_{loop} E_{//}\Delta\ell = -\frac{\Delta\Phi_M}{\Delta t}}$$

IV. Maxwell Equation No.4

Lenz's Law

When an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it. The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. In the examples below, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.



Summary of Maxwell Equations

(Free Space)

a)
$$\underset{\text{closed surface}}{\Phi_E} = 0$$

c)
$$\sum_{loop} B_{//} \Delta \ell = 0$$

b)
$$\underset{\text{closed surface}}{\Phi_M} = 0$$

d)
$$\sum_{loop} E_{//} \Delta \ell = -\frac{\Delta \Phi_M}{\Delta t}$$

Reference

- *Lecture note of Physics2610 class, U of Manitoba*
- *A Student's Guide to Maxwell's Equations, Daniel Fleisch, Wittenberg University*