Probabilistic Optimal Power Flow Incorporating Wind Power Using Point Estimate Methods

Hamed Ahmadi, Hassan Ghasemi, *Member, IEEE*
School of Electrical and Computer Engineering
University of Tehran
Tehran, Iran
h.ahmadi@ut.ac.ir, h.ghasemi@ut.ac.ir

Abstract—**Global tendency toward environmental-friendly sources of energy with lower generation costs led to increased penetration of renewable energies in power systems, especially wind power. Stochastic nature of wind speed has introduced new challenges to the conventional power system studies including optimal power flow (OPF) analysis, unit commitment, etc. In this paper, a Weibull distribution for wind speed with actual data is assumed and system loads are considered as random variables (RV) with normal distributions. Point estimate methods (PEM) are used here in order to handle the probabilistic OPF problem for a 6-Bus test system. The PEMs are compared with respect to Monte Carlo simulation (MCS) results. It is shown that these methods give acceptable results with significant reduction in computation burden. However, under certain conditions (e.g. RVs with high values of skewness and high order of nonlinearity in the OPF problem), PEMs may give inaccurate results. This inefficiency has not been observed in previous studies and is demonstrated in the test case used here.**

Keywords- Point estimate methods; wind power; Weibull distribution; Monte Carlo simulation; probabilistic OPF

I. INTRODUCTION

Wind farms (WF) penetration into the existing power systems is rapidly increasing. Compared to the other renewable energy resources such as solar, geothermal etc., wind energy has received more attention with higher growth. The remarkable advancements which have been reached during the past decade in wind power generation technology, along with the environmental issues pertaining to the fuel-consuming power plants, have made the wind power one of the most economical resources for replacing the conventional power plants.

Wind power integration, in turn, will introduce new issues to the existing power systems from several aspects, especially when the level of penetration is significant [1]. Among these aspects, market operation and power flow analysis are now becoming a concern.

Probabilistic nature of wind speed makes the output power of a WF a random variable (RV) as well. Although several methods are available for wind speed forecasting during a period of time, e.g. day ahead forecasting, all these methods provide an average value for the forecast as well as an estimation error [2]. Therefore, these forecasts are RVs with certain mean values and variances.

Several probability distribution functions (PDF) for wind speed have been developed in the literature [3]. The most widely accepted PDF is the two-parameter Weibull distribution [4]. This model has been used frequently in power flow analysis and other power system studies [5], [6]. In order to incorporate this model in an optimal power flow problem, we need to employ the probabilistic optimal power flow (P-OPF) techniques. The solution of P-OPF problem will be also a PDF of variables such as voltage magnitudes, voltage angles, locational marginal prices (LMP), etc. Based on these results, a power system operator will be able to generate a short-term operation plan, e.g. unit commitment, status of energy storage systems and other system components in order to use available wind power as much as possible without jeopardizing system security.

Various methods have been proposed for solving the P-OPF problem such as truncated Taylor series expansion method [7], the first-order second-moment method (FOSMM) [8], the cumulant method [9], the point estimate method (PEM) [10], [11], etc. The Monte Carlo Simulations (MCS) are used as a reference to determine the accuracy of all mentioned methods. Some deficiencies of these methods are reported in [12]. A computationally more efficient method, called 2-point estimation method (2PEM) has been introduced originally in [13] is used for analyzing a P-OPF problem in [12]. It is shown that 2PEM may give inaccurate results in case of input RVs with large variances. On the other hand, for normally distributed RVs with small variances, this method gives acceptable results. Following the proposed method in [12], Hong's PEM [13] with higher orders ($2m+1$ and $4m+1$ schemes) is employed in power flow analysis in [21] for input RVs with binominal and normal distributions and the results are compared to 2PEM (also called $2m$ in [21]) and MCS, where m is the number of RVs. It is shown that the 2PEM is not as accurate as $2m+1$ scheme. However, these methods are not evaluated for input RVs having PDF with high values of skewness and kurtosis, or RVs with different PDFs. Besides, when the OPF problem exhibits high nonlinearity, especially when the limits on line flows have been reached, PEMs may lead to inaccurate results. These deficiencies have not been

978-1-4244-8782-0/11/$26.00 ©2011 IEEE

observed previously and hence are studied in the present paper.

In this paper, the Weibull distribution (which has relatively large skewness) is assumed for wind speed and normal PDF is assumed for loads in order to investigate the P-OPF problem. Weibull parameters obtained from real wind speed data in [23] are used here for more realistic study. Weibull distribution of wind speed and energy conversion equations of wind turbines are explained in Section II. The 2PEM and *2m+1* scheme employed to solve the P-OPF problem are briefly described in Section III. In Section IV, P-OPF outputs are obtained and compared using 2PEM, 3PEM and MCS in a 6-Bus test system. The paper is concluded by listing the main findings of the study.

II. WEIBULL DISTRIBUTION FOR WIND SPEED

General structure of the two-parameter Weibull PDF is [14]:

$$f_v(v) = \frac{\alpha}{\beta}\left(\frac{v}{\beta}\right)^{\alpha-1} e^{-\left(\frac{v}{\beta}\right)^{\alpha}} \tag{1}$$

in which β and α are referred to as scale factor and shape factor, respectively and v is the wind speed. Parameter estimation methods for determining the values of β and α from the measured data of wind speed are reported in the literature [15]-[17]. These parameters depend on the geographical characteristics of a region, thus their values varies from area to area. Considering the findings in those studies, in this paper, the values listed in Table I are assumed for β and α.

Wind speed is converted to electrical power by different types of wind turbine generators. The mostly used type is variable speed structure. The simplified energy conversion equation used in [5], [6], [18] for this type is:

$$P_w = \begin{cases} 0 & v < v_{in}\ or\ v > v_{out} \\ P_{w,rated} & v_r < v < v_{out} \\ \dfrac{(v - v_{in})}{v_r - v_{in}} P_{w,rated} & v_{in} < v < v_r \end{cases} \tag{2}$$

A more accurate equation is a cubic relation between the output power and wind speed for $v_{in}<v<v_r$ [17], [19]:

$$P_w = 0.5 \rho v^3 A \quad (W) \tag{3}$$

in which A is the cross-section swept by the wind turbine blades in m^2 ($A = \pi l^2_{blade}$), ρ is the air density. A variable speed 1.41MW wind turbine is used here with parameters as listed in Table I. Fig. 2 compares the calculated output power of a 30MW WF composed of 21 wind turbine by (7) and (8) from the wind speed distribution shown in Fig. 1. Observe that the PDF of two curves are somewhat different.

III. POINT ESTIMATE METHOD AND P-OPF FORMULATION

A. Point Estimate Methods [10]

Suppose that X is a vector of m RVs and $Y = G(X) = G([x_1,x_2,\ldots,x_m])$ is a nonlinear function of X. We are going to find the statistical characteristics of Y, with the assumption that PDF of X is given. Using PEM, we need to numerically calculate $G(X)$ $2m$ times. Let $M_i(x_k)$, μ_k and σ_k represent the i^{th} order central moment, mean, and standard deviation of k^{th} RV (x_k), respectively. Define $\lambda_{k,i}$ for k^{th} RV:

$$\lambda_{k,i} = \frac{M_i(x_k)}{\sigma_k^i} \tag{4}$$

Using Taylor series expansion of $G(X)$ about μ_x, and neglecting the i^{th} derivatives of $G(X)$ with respect to X for i higher than 3, the following procedure leads to the desirable quantities for 2PEM:

$$\xi_{k,i} = \frac{\lambda_{k,3}}{2} + (-1)^{3-i}\sqrt{m + \left(\frac{\lambda_{k,3}}{2}\right)^2} \tag{5}$$

$$p_{k,i} = \frac{1}{m}\frac{(-1)^i \xi_{k,3-i}}{\zeta_k}, i = 1,2 \tag{6}$$

TABLE I. PARAMETERS FOR WEIBULL DISTRIBUTION AND WIND ENERGY CONVERSION EQUATIONS (7) AND (8)

α (m/s)	*β* (m/s)	ρ (kg/m³)	v_{in} (m/s)	v_{out} (m/s)	v_r (m/s)	A (m²)
1.6	9	1.225	4.5	40	15	706.8

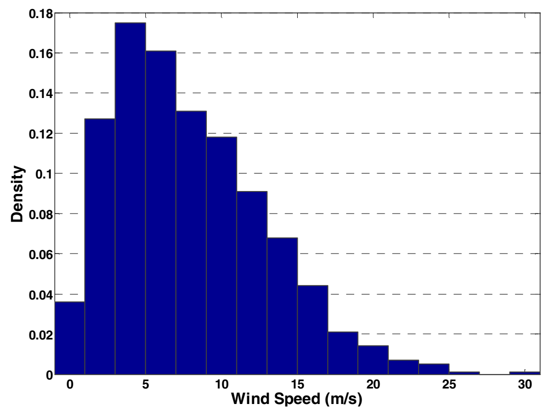

Figure 1. Random wind speed generated by Weibull distribution using parameters given in Table I (1000 samples).

$$\zeta_k = 2\sqrt{m + \left(\frac{\lambda_{k,3}}{2}\right)^2} \tag{7}$$

$$x_{k,i} = \mu_k + \xi_{k,i}\sigma_k, k = 1,2,\ldots,m \tag{8}$$

Finally, we have:

$$E(Y^j) = \sum_{i=1}^{2}\sum_{k=1}^{m} p_{k,i} G([\mu_1, \mu_2, \ldots, x_{k,i}, \ldots, \mu_{m-1}, \mu_m])^j \tag{9}$$

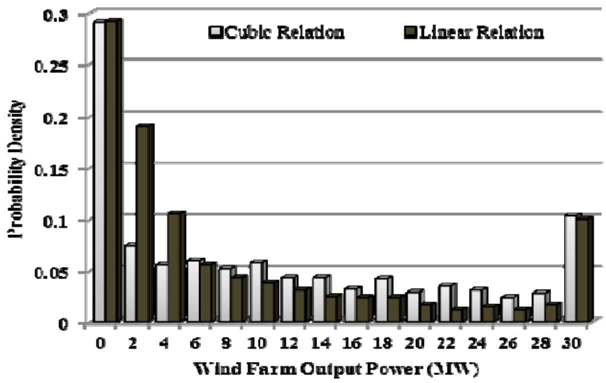

Figure 2. Comparing the results of wind energy conversion equations with wind speed shown in Fig. 1; linear relations (7) and cubic relation (8).

In order to derive the mean values μ and variances σ^2 of the outputs, the following formulas are used:

$$\mu = E(Y),\sigma^2 = E(Y^2) - \mu^2 \tag{10}$$

More explanations are available in [10].

Standard deviations (σ) in (10) sometimes become complex quantities due to the approximations made in obtaining Taylor series expansion. Because the imaginary parts are very small, using the absolute value of σ would solve the problem.

B. P-OPF Formulation

General formulation for security constrained OPF maximizing social welfare assumed in this study [20] is as follows:

$$\begin{aligned}
&Min. \ -\left(\sum C_{Di}P_{Di} - \sum C_{Si}P_{Si}\right) \\
&s.t. \ \ G\left(\delta, V, Q_G, P_S, P_D\right) = 0, P_{S_{min}} \le P_S \le P_{S_{max}} \\
&P_{D_{min}} \le P_D \le P_{D_{max}} \ , Q_{G_{min}} \le Q_G \le Q_{G_{max}} \\
&V_{min} \le V \le V_{max} \ , \left|I_{ij}\right| \le I_{ij_{max}}
\end{aligned} \tag{11}$$

where C_S and C_D are vectors of supply and demand bids in \$/MWh, respectively; Q_G stand for the generator reactive powers; V and δ represent the bus voltage magnitudes and angles; I_{ij} represent the currents flowing through the lines; P_S and P_D represent bounded supply and demand power bids in MW.

IV. Test Case Results

A 6-Bus test system including a WF is employed here for applying the P-OPF method. System data are given in [22].

Bus 2 is assumed to be the slack bus. A 120MW WF consisting of 85 wind turbines (1.41MW each one) is connected to Bus 6. The MCS were carried out for 1000 samples.

A. No Constraints on Line Flows

In the first scenario, we assume that there are no constraints on line flows. For loads, a normal distribution with the nominal values of demands as its mean and a variance of 0.1 p.u. is assumed. Assuming a Weibull distribution for wind speed with parameters given in Table I, P-OPF problem is solved by MCS, 2PEM and 3PEM using PSAT [20] and corresponding results are given in Figs. 3 through 6. It can be seen that all methods yield similar results. The sum of absolute relative errors ε corresponding to 2PEM and 3PEM are calculated and shown in Table II according to (12). The values of ε for each variable show that the 2PEM is more accurate than 3PEM in this scenario.

$$\varepsilon = \frac{\left|y_{PEM} - y_{MCS}\right|}{y_{MCS}} \times 100 \tag{12}$$

B. Constraints on Line Flows Are Respected

In the second scenario, the line flow limits are activated in PSAT. The results are reported in Figs. 8 through 11. Fig. 7 exhibits the density functions of current flows through some of the lines. Note that intensive skewness is observed in all flows and some lines have, at some samples, reached their limits, e.g. lines 5 and 6. In this scenario, the 2PEM leads to more accurate results when compared to the 3PEM. The errors in variances are greater than the errors in mean values for both methods.

Regarding the results of these two scenarios, it can be concluded that the 3PEM is more accurate than the 2PEM in the case that no congestion exists and the OPF problem exhibits a smooth behavior (first case). On the other hand, when the line constraints are active (Fig. 7) and the input RVs have relatively large skewness, PEMs lead to inaccurate results. Note that as opposed to 3PEM's complexity, 2PEM is simpler and yields more accurate results.

This can be explained considering the approximations made for obtaining the mean and variance of output variables in [10]. Only if *G(X)* is a function that its j^{th} derivative ($G(X)^{(j)}$) with respect to X for $j>3$ equals zero, then the 2PEM results are accurate. For instance, the mean value of Y is calculated:

$$\begin{aligned}
&E\left(Y\right) = p_1 G\left(x_1\right) + p_2 G\left(x_2\right) \\
&+\sum_{j=4}^{\infty} \frac{1}{j\,!} G^{(j)}\big|_{\mu_x} \times (\lambda_{x,j} - (p_1\xi_1^j + p_2\xi_2^j))\sigma_x^j
\end{aligned} \tag{13}$$

However, if this condition is not met, then there is no guaranty for the accuracy of results. The OPF problem is certainly a nonlinear function and when the constraints become active, the order of nonlinearity increases. Furthermore, the Weibull distribution assumed for wind speed intensifies the nonlinearity of the solution leading to inaccuracy in PEM results.

More precise results for 3PEM can be obtained by not using the mean value as one of the three concentration points. Hence, the points for 3PEM are computed as follows:

$$p_i = \frac{\xi_j \xi_k + 1}{\left(\xi_j - \xi_i\right)\left(\xi_k - \xi_i\right)}, \ i, j, k = 1,2,3, i \ne j \ne k \tag{14}$$

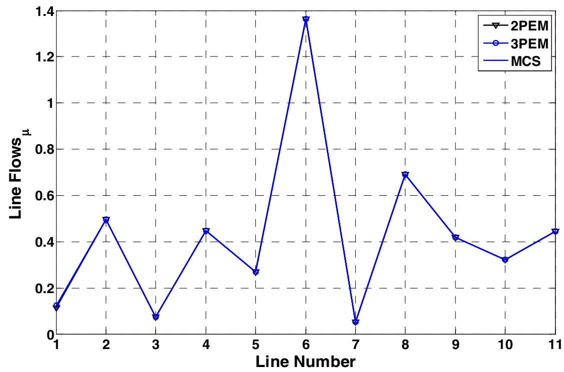

Figure 3. Mean values of line flows; No constraint on line flows

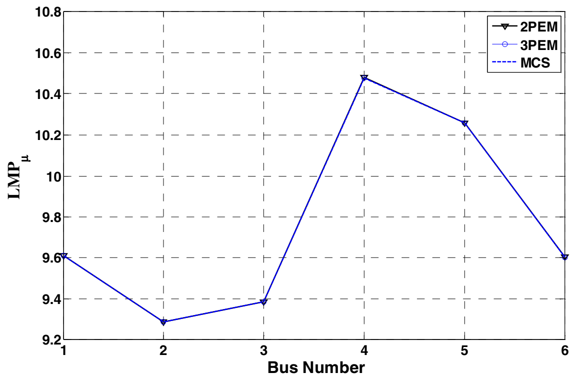

Figure 4. Mean values of LMPs; No constraint on line flows

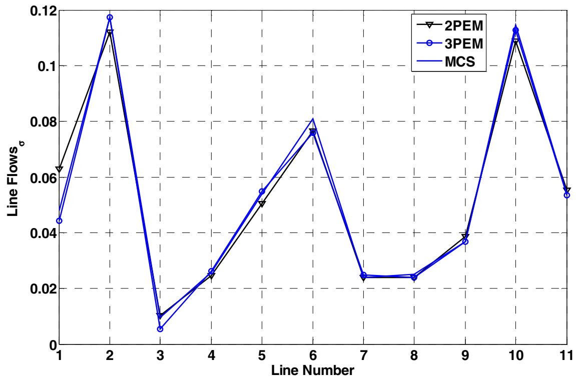

Figure 5. Variances of line flows; No constraint on line flows

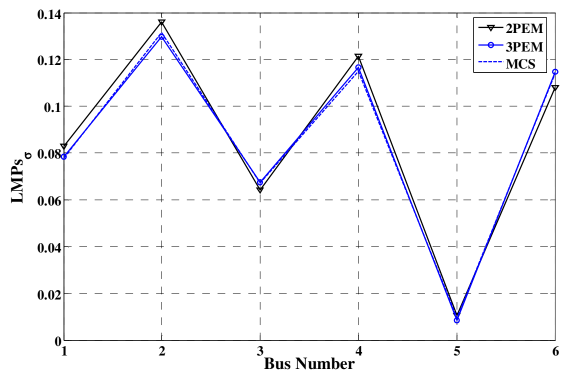

Figure 6. Variances of LMPs; No constraint on line flows

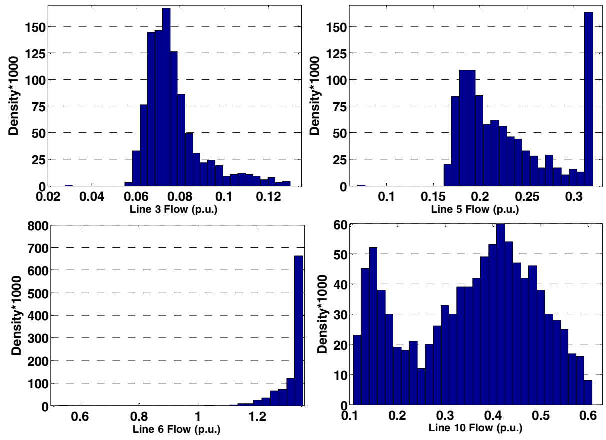

Figure 7. Probability density of selected line flows.

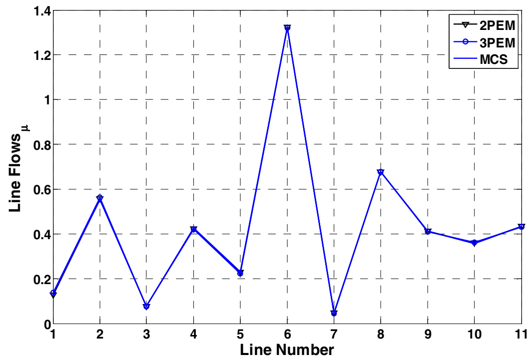

Figure 8. Mean values of line flows; Constraints on line flows are respected.

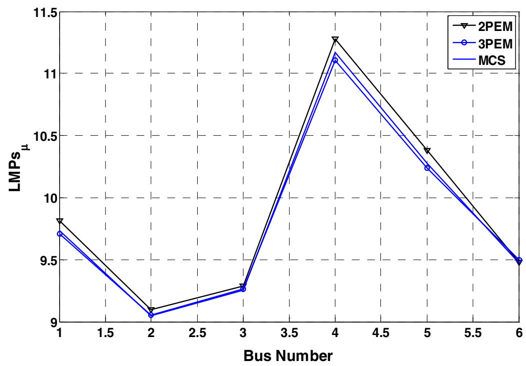

Figure 9. Mean values of LMPs; Constraints on line flows are respected.

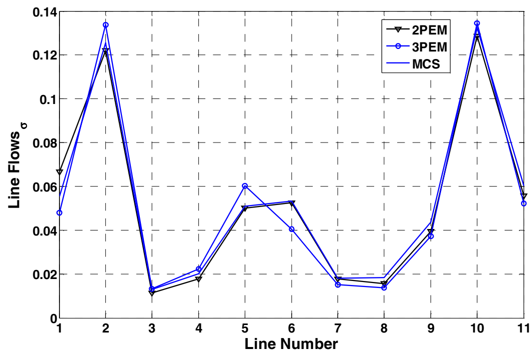

Figure 10. Standard deviations of line flows; Flow constraints are respected.

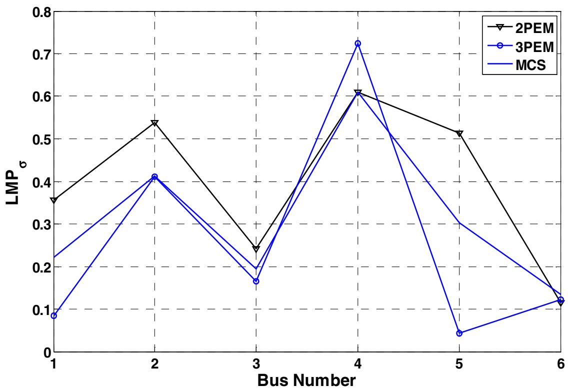

Figure 11. Standard deviations of LMPs; Constraints on line flows are respected.

TABLE II. SUM OF ABSOLUTE RELATIVE ERRORS (E) FOR MEAN AND STANDARD DEVIATION VALUES

		First Scenario		Second Scenario	
		2PEM	*3PEM*	*2PEM*	*3PEM*
Voltages	μ	0.0095	0.0117	0.0835	0.2048
	σ	25.2689	16.2459	26.47	83.6568
Angles	μ	1.4082	1.914	3.7931	11.7641
	σ	9.1656	17.6645	20.9769	64.8762
Line Flows	μ	8.4218	3.9331	12.7145	19.6098
	σ	77.1187	69.9259	87.4031	142.7476
LMPs	μ	0.0112	0.0283	3.729	1.681
	σ	36.4226	14.1999	198.3308	190.544

Where ξ_i are the roots of following equation:

$$d_3\xi^3 + d_2\xi^2 + d_1\xi + d_0 = 0 \qquad (15)$$

For the present study, two roots of this equation are complex pairs and therefore, this method is not applicable. The cumulant method [9] may be more suitable for this type of study and is of interest to authors for future research.

V. CONCLUSION

A Weibull distribution for wind speed with real data is assumed in order to incorporate the stochastic nature of wind power in the OPF problem. A more realistic energy conversion formula was used instead of the linear approximation. PEMs are briefly described and two well-known schemes (2PEM and 3PEM) were used for finding the statistical characteristics of OPF results. Relatively accurate results along with significant reduction in computational burden have been achieved using PEMs. However, by respecting/ignoring line flow limits, it was shown that both 2PEM and 3PEM may give inaccurate results due to existing nonlinearities in OPF problem and high skewness of Weibull distribution. These findings show the deficiencies of previously proposed method for solving P-OPF problem and exhibit that there is no guarantee that PEMs give accurate results when the problem is highly nonlinear. However, in cases with lower skewness and nonlinearity, the results obtained by PEMs are acceptable. Therefore, these methods have to be tested for the system under study before using them on a day-to-day basis to ensure that accurate results will be obtained.

REFERENCES

[1] H. Ahmadi, H. Ghasemi, H. Lesani, "A comparative small signal analysis of PMSG and SCIG-based wind farms," *in proceeding of 25th International Power System Conference*, Tehran, Iran, 2010.
[2] B. G. Brown, R.W. Katz, and A. H. Murphy, "Time series models to simulate and forecast wind speed and wind power," *J. Clim. Appl. Meterol.*, vol. 23, pp. 1184–1195, Aug. 1984.
[3] J. A. Carta, P. Ramírez, and S. Velázquez, "A review of wind speed probability distributions used in wind energy analysis: Case studies in the Canary Islands," *Renewable Sustainable Energy Rev.*, vol. 13, no. 5, pp. 933–955, 2009.
[4] M. R. Patel, *Wind and Solar Power Systems*. Boca Raton, FL: CRC Press, 1999.
[5] X. Liu, "Economic load dispatch constrained by wind power availability: A wait-and-see approach," *IEEE Trans Smart Grid*, under publication, 2010.
[6] J. Hetzer, D. C. Yu, and K. Bhattarai, "An economic dispatch model incorporating wind power," *IEEE Trans. Energy Convers.*, vol. 23, no. 2, pp. 603–611, Jun. 2008.
[7] G. J. Hahn and S. S. Shapiro, "Statistical Models in Engineering," New York: Wiley, 1967.
[8] M. Madrigal, K. Ponnambalam, and V. H. Quintana, "Probabilistic optimal power flow," in *Proc. IEEE Can. Conf. Electrical Computer Engineering*, Waterloo, ON, Canada, May 1998, pp. 385–388.
[9] A. Schellenberg, J. Aguado, and W. Rosehart, "Introduction to cumulant-based probabilistic optimal power flow (P-OPF)," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 1184–1186, May 2005.
[10] H. P. Hong, "An efficient point estimate method for probabilistic analysis," *Reliab. Eng. Syst. Saf.*, vol. 59, pp. 261–267, 1998.
[11] E. Rosenblueth, "Point estimation for probability moments," *Proc. Nat. Acad. Sci. United States Amer.*, vol. 72, no. 10, pp. 3812–3814, Oct. 1975.
[12] G. Verbic, C. A. Canizares, "Probabilistic optimal power flow in electricity markets based on two-point estimate method," *IEEE Trans Power Systems*, vol. 21, no. 4, pp. 1883-1893, Nov. 2006.
[13] C.-L. Su, "Probabilistic load-flow computation using point estimate method," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1843–1851, Nov. 2005.
[14] H. P. Hong, "An efficient point estimate method for probabilistic analysis," *Reliab. Eng. Syst. Saf.*, vol. 59, pp. 261–267, 1998.
[15] M.J. Stevens, P.T. Smulders, "The estimation of the parameters of the Weibull wind speed distribution for wind energy utilization purposes," *Wind Eng.*, vol. 3, no. 2, pp. 132-145, 1979.
[16] S. Rehman, T. O. Halawani, and T. Husain, "Weibull parameters for wind speed distribution in Saudi Arabia," *Solar Energy*, vol. 53, no. 6, pp. 473-479, 1994.
[17] P. Ramirez, J. A. Carta, "Influence of the data sampling interval in the estimation of the parameter of the Weibull wind speed probability distribution: a case study," *Energy Conv. Manag., Elsevier*, vol. 46, pp. 2419-2438, 2005.
[18] S. Roy, "Market constrained optimal planning for wind energy conversion systems over multiple installation sites," *IEEE Trans. Energy Convers.*, vol. 17, no. 1, pp. 124–129, Mar. 2002.
[19] T. Ackerman, "Wind Power in Power Systems," *Wiley*, 2005.
[20] F. Milano, "An open source power system analysis toolbox," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1199–1206, Aug. 2005.
[21] J. M. Morales, J. Perez-Ruiz, "Point estimate schemes to solve the probabilistic power flow," *IEEE Trans Power Systems*, vol. 22, no. 4, 1594-1601, Nov. 2007.
[22] Brooke, D. Kendrick, A. Meeraus, R. Raman, R. E. Rosenthal, GAMS, a User's Guide, GAMS Development Corporation, 1217 Potomac Street, NW, Washington, DC 20007, USA, Dec. 1998, available: http://www.gams.com.