Distribution System Restoration Considering Critical Infrastructures Interdependencies

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***Abstract*—Resource management is a crucial task during natural or man-made disasters to maximize the number of saved lives. As part of the vital resources, electricity restoration has a high priority for increasing the performance of critical infrastructures such as hospitals and water pumping stations. Also, the interdependencies between these infrastructures have to be taken into account in the process of service restoration to ensure their maximum operational capacity. In this paper, the problem of service restoration in a distribution system to maximize the operational capacity of a hospital is formulated. Different case studies are considered to show the effectiveness of the proposed method under a variety of emergency situations.**

I. Introduction

Many people have been victims of natural or man-made disasters all over the world. These types of disasters are almost inevitable in nature and, therefore, preparation for such events is the only way to minimize the damages. The preparation considerations are usually applied to the designing and implementation of key infrastructures to increase their resiliency against unpredictable catastrophes. Amongst those key infrastructures are electric power supply, water supply, natural gas supply, telecommunication system, and transportation system. Regardless of how resilient these infrastructures are, natural disasters will still cause damages. When a disaster happens, the most important factor is to save as many lives as possible. In order to do this, a strategic plan is needed to redistribute the limited supplies amongst the most critical demands. An understanding of the interdependencies among critical infrastructures is the key in the emergency planning [1].

There have been some studies on resources management after a disaster to minimize the fatalities. In [2], a framework for disaster response is proposed that starts with clustering the region into three major groups: suppliers, distribution centers, and affected areas. Starting from the suppliers to the distribution centers, it should be possible to deliver the needs [2]. The problem of optimal management of transportation resources during a disaster is studied in [3]. The problem is formulated as a large-scale, multi-commodity, multi-modal network flow problem and is solved using heuristic algorithms. The importance of time delays due to transportation or search-and-rescue processes in the number of saved lives is studied in [4]. The main focus of [4] is to optimally allocate the transportation to the hospital and rescue resources to minimize the number of fatalities. However, they did not consider the impacts of other recourses such as electricity and water on the performance of a hospital. The power distribution system directly supplies the consumers and, therefore, it has a great influence on the operation of critical infrastructures. In fact, reports have shown that most of power outages are due to problems in distribution networks [5]. Also, it has been reported that damages in distribution networks due to natural disasters are higher than damages in generation and transmission systems [6].

During an emergency, many patients are transferred, by different means, to hospitals for immediate care. The hospital, therefore, is considered a critical entity in saving lives. For a typical hospital to operate, there are some basic requirements such as electricity, water, doctors, medical supplies, transportations, etc. After a natural disaster happens, there are usually limits imposed on the amount of available power, water, transportation facilities, etc. Besides, some of these inputs have interdependencies. For instance, in order to have water, the water pumping station needs power. Considering these factors, the problem of optimally allocating the available resources to maximize the number of saved lives has to be carefully addressed.

In this paper, the problem of optimal resources allocation to a hospital for maximizing the number of saved lives is mathematically formulated as a mixed-integer programming (MIP) problem. The performance indicator of a hospital is defined as a function of its inputs, i.e. amount of available power and water supplies. It is assumed that other resources such as doctors, transportation system, and medical supplies are not highly dependent on the available power supply and, therefore, are considered as known values. The power restoration in an electricity distribution system is formulated considering faulty feeders, amount of available capacity at the substations, feeders ampacities, and topological configuration constraints. The linear power flow (LPF) model proposed in [7] is employed to reduce the computational complexity of the MIP problem.

The problem of distribution system restoration has been visited by several researchers aiming at restoring the power to all the customers after a total blackout, e.g. [8]. The restoration problem considered in this paper is, however, different from those studies in several aspects. There are faulty branches, limited available power, interdependencies, and priorities in restoring as much power as possible to critical loads. This problem is slightly similar to the problem of service restoration in shipboard power distribution systems [9].

The paper is organized as follows. In Section II, the assumptions and mathematical formulations of the optimization problem are discussed. Simulation results for a case study are given in Section III. The concluding remarks of the proposed scheme are provided in Section IV.

II. Problem Formulation

The optimization problem considered in this paper has several subsystems. The modeling assumptions for each subsystem is given in the following.

A. Hospital

A hospital is modeled using Infrastructure Interdependencies Simulator ("i2Sim") input-output model, as described in [10]. i2Sim is a multiple infrastructures simulator for modeling physical interdependencies among critical infrastructures [1]. The inputs are power (P_h) and water (W_h) and the output is the operational capacity of the hospital (H). The function that describes $H(W_h, P_h)$ is obtained by fitting a piecewise linear curve to i2Sim input-output hospital model which is described by Human-Readable Tables (HRT). Data used in this paper were collected from a real hospital and used to construct the i2Sim hospital model. It should be noted that the operation capacity of a hospital also depends on other resources such as doctors, medical supplies, gas supply, transportation (ambulances), telecommunication, etc. In this paper, we only focus on two resources that depend on power system operations, i.e. power and water.

B. Power Distribution System

The power distribution system is modeled here assuming several substations, controllable sectionalizing and tie switches, radial topology, and limits on feeders ampacities. Also, it is assumed that there are limits on the available power from the substations and there exist faulty feeders all due to the disaster. The linear power flow (LPF) formulation of [7] is employed to reduce the computational complexity of the restoration problem. In the LPF formulation, loads are modeled as voltage-dependent elements with both active and reactive components. The radiality of the final topology is enforced to ensure the safe operation of the protection system.

C. Water Pumping Station

A water pumping station is modeled using i2Sim input-output model [10]. This model describes the water flow rate (W_h) available to the hospital as a function of available power (P_w) to the water pumping station. Data used for this model is based on an i2Sim model for a realistic water pumping station. It is assumed to have several electric motors to pump the water through the water distribution pipes. The more power available to the station, the higher flow rate of water to the hospital. It is also assumed that water pipelines delivering water to the hospital are available and not affected by the disaster.

The water flow rate delivered to the hospital (W_h) is described by a discrete function of the power available to the pumping station (P_w). This function can be written as

$$W_h = \alpha P_w = \alpha \sum_{i=1}^{N_m} z_i p_i^m \tag{1}$$

where p_i^m stands for the power rating of each motor; α is a scaling factor to convert the power to the corresponding water flow rate; and z_i is a binary variable indicating the connection/disconnection of the i^{th} motor.

D. Load Modeling

The loads' voltage dependencies are modeled using a quadratic function, as described in [7]. Using this load model, each load will have quadratic and linear components defined as:

$$G = \frac{P_0 C_Z}{V_0^2}, \quad B = -\frac{Q_0 C'_Z}{V_0^2} \tag{2a}$$

$$I_p = \frac{P_0 C_I}{V_0}, \quad I_q = -\frac{Q_0 C'_I}{V_0} \tag{2b}$$

where P_0 and Q_0 are the active and reactive demands at the nominal voltage V_0; C_Z/C'_Z and C_I/C'_I are the quadratic and linear portions of the loads' active/reactive powers, respectively. The network admittance matrix $\bar{Y}$ will then have G and B at its diagonal elements. The two current sources appear at the right-hand side of the nodal equations. The power flow equations, assuming the load models in (2), are then written as [7]:

$$\sum_{j=1}^{N_l} \left(\bar{G}_{ij} V_j^{\text{re}} - \bar{B}_{ij} V_j^{\text{im}}\right) = I_{p,i} \tag{3}$$

$$\sum_{j=1}^{N_l} \left(\bar{G}_{ij} V_j^{\text{im}} + \bar{B}_{ij} V_j^{\text{re}}\right) = I_{q,i} \tag{4}$$

where N_l is the number of aggregated nodes; i and j are the nodes at the two ends of the line connecting aggregated nodes i and j; V^{re} and V^{im} are the real and imaginary parts of the nodal voltages; $\bar{G}$ and $\bar{B}$ are the real and imaginary parts of $\bar{Y}$, respectively. In order to consider the status of each branch, elements of $\bar{Y}$ are defined as:

$$\bar{Y}_{ij} = \begin{cases} u_{ij} y_{ij} & i \neq j \\ -\sum_{j=1}^{N_l} u_{ij} y_{ij} - Y_i & i = j \end{cases} \tag{5}$$

where Y_i is the load admittance obtained from (2a); u_{ij} and y_{ij} are the status and admittance of Branch i-j, respectively.

E. Optimization Problem

The objective of the study is to maximize the operational capacity of the hospital to increase the number of saved lives. By negating the function, the maximization can be replaced by minimization. This can be mathematically stated as

$$\text{Minimize} \quad -H(W_h, P_h) \tag{6}$$

where $H(W_h, P_h)$ is derived based on hospital and water pumping station models discussed earlier. Let us define $z = -H(W_h, P_h)$. Based on the practical data, z can be modeled as the maximum of two piece-wise linear functions [11], i.e.

$$z = \max\{a_1 W_h + b_1 P_h, a_2 W_h + b_2 P_h\} \tag{7}$$

where a_1, a_2, b_1, and b_2 are parameters to be determined. There is no constant value in the linear pieces since $H(0,0) = 0$. To minimize over the maximum of a set of linear functions, the method used in [11] can be applied to avoid the $\max\{\}$ operator. By adding the following two constraints, minimizing over z suffices:

$$z \geq a_1 W_h + b_1 P_h \tag{8a}$$

$$z \geq a_2 W_h + b_2 P_h \tag{8b}$$

It is worthwhile to emphasize that there are variable demands at the water pumping station node ($P_w = \sum_{i=1}^{N_m} z_i p_i^m$) and the hospital node (P_h), which will be determined by the optimization routine.

Assuming a known value for the voltage at the substation(s), say V_s, (3)-(4) for $i = 1$ gives the total power supplied by the substation(s), i.e.:

$$P_s + jQ_s = V_s(I_{p,1} - jI_{q,1}) \tag{9}$$

where P_s and Q_s are the active and reactive power supplied by the substation. The total available power from each substation is limited due to the disaster. This may happen due to damages to substation transformers or other equipments, power availability from higher levels (MV/HV), etc. In mathematical form, this can be written as:

$$P_s^2 + Q_s^2 \leq S_{\max}^2 \tag{10}$$

where $S^{\max}$ is the maximum apparent power available at substation.

The nodal voltages have to meet the operation limits:

$$V_{\min} \leq \sqrt{V_i^{\text{re}\,2} + V_i^{\text{im}\,2}} \leq V_{\max} \tag{11}$$

Since the voltage angles are small in distribution systems [7], this constraint can be linearized as:

$$V_{\min} \leq V_i^{\text{re}} \leq V_{\max} \tag{12}$$

The feeders' ampacities have to be respected. The current through each branch connecting Nodes i and j is calculated as:

$$I_{ij}^2 = \left[G_{ij}^2 + B_{ij}^2\right]\left[(V_i^{\text{re}} - V_j^{\text{re}})^2 + (V_i^{\text{im}} - V_j^{\text{im}})^2\right] \tag{13}$$

The feeders' ampacities is then enforced as:

$$I_{ij}^2 \leq u_{ij} I_{\max}^2 \tag{14}$$

where $I_{\max}$ is the maximum possible current flowing through the feeders; u_{ij} is the branch's status.

Each load section which has switches at both ends is modeled as a spot load, with the total impedances of that section at the far end, with respect to the substation. This may result in a small error in the power flows, which is negligible for type of analysis conducted here. A binary variable (u_{ij}) is defined to to show the status ("on"/"off") of the switch connecting the aggregated spot loads i and j. The radiality of the network topology has to be enforced. To do this, a formulation proposed in [12] is employed here, as:

$$\beta_{ij} + \beta_{ji} = u_{ij} \tag{15a}$$

$$\sum_{i \in S_j} \beta_{ij} = 1, \quad j \geq 2 \tag{15b}$$

$$\beta_{1j} = 0, \quad j \in S_1 \tag{15c}$$

$$\beta_{ij} \in \{0, 1\} \tag{15d}$$

in which β_{ij} is 1 if node i is the parent of node j, and vice versa; S_j is the set of nodes directly connected to node j, with $j = 1$ representing the substation.

III. Simulation Results

A case study is presented here to show the application of the proposed frame. The 70-node system of [13] is chosen as the distribution system. This system has 70 nodes, 4 feeders, and 2 substations. Substation A has two transformers, 6 MVA each, and Substation B has two transformers too, 4 MVA each, which gives a total of 20 MVA capacity. All the nodal loads are scaled up 3 times. All the line impedances are also reduced by 70%. The physical layout of this system is shown in Fig. 1. The hospital is assumed to be located at Node 62 and the water pumping station at Node 38. The initial configuration of the network before the disaster is shown by open switches. It is also assumed that all the branches are equipped with controllable switches. The problem to solve is to find the best radial configuration that supplies as much power as possible to the hospital and the water pumping station to maximize the operational capacity of the hospital.

For simplicity, the hospital and water pumping station are modeled with the following voltage dependency characteristics: $C_I = C_I' = 1$. For other loads, the voltage dependency characteristics are described as $C_I = C_I' = 0.5$. With these assumptions, the variables P_h and P_w will only appear on the right-hand side of (3)-(4). The nominal demand of the hospital and the water station are 2MW and 1MW [14], respectively with a power factor of 0.9 lag. A piece-wise linear function shown in Fig. 2 is fitted to the data available in the form of HRTs for a realistic hospital [10]. There are two linear pieces in this concave function. By negating the values of H, a convex piece-wise linear function is obtained and the optimization is converted to a minimization problem [11]. The values for the fitted linear pieces in (8) are given in Table I. For the water pumping station, it is assumed that there are 10 motors, 100 kW each. Based on realistic data, α in (1) is assumed to be 51 [15].

A. First Scenario

In the first scenario, it is assumed that due to the disaster, branches 54-55 and 35-36 are faulty. The circuit breaker on

Feeder 4 opened the circuit and, therefore, all the loads on Feeder 4 (Nodes 51 to 61) up to the hospital (Node 62) are disconnected. A fuse located on Branch 35-36 disconnected the fault from the rest of the feeder up to the water pumping station, i.e. Nodes 36 and 37 are disconnected. One of the transformers at Substation A is out of service due to the disaster and the total available power is reduced to 15 MVA (75% of the full capacity). With this scenario, considering the initial configuration of the network, both the hospital and the water pumping station have no power.

The goal is to reconfigure the network and restore as much power as possible to the two critical loads. The problem is formulated as a mixed-integer programming problem with the lines' status and number of online motors at the water pumping station as discrete variables, and the rest of the variables as continuous. Table II shows the results obtained using the simulations. There is 1 MW available for the motors in the water pumping station, which is sufficient to run all 10 motors. The power available to the hospital is 1.96 MW which is about 2% less than the full capacity requirement of the hospital. The overall operational capacity of the hospital has retrieved to 97.83%. Table III shows the final radial configuration of the network. Only open switches are reported.

B. Second Scenario

In the second scenario, the assumption is that the fault on Branch 34-35 is cleared by the feeder breaker (Feeder 3) at the substation and all the nodes on this feeder up to the water pumping station, i.e. Nodes 30 to 34 and Nodes 39 to 46, are disconnected. The connection of the water station to Node 34 is also removed. The fault on Branch 57-58 caused the operation of the circuit breaker on Feeder 4 and, therefore, all the nodes on this feeder up to the hospital, i.e. Nodes 51 to 61 are disconnected. The connection of the hospital to Feeder 4 is also removed. With these assumptions, there is no feeder supplying power from Substation B and, therefore, the only source of power is Substation A, with a total capacity of 12 MVA.

Under a critical shortage of available power, it is critical to maneuver on the configuration of the network to provide as much power as possible to the hospital and water pumping

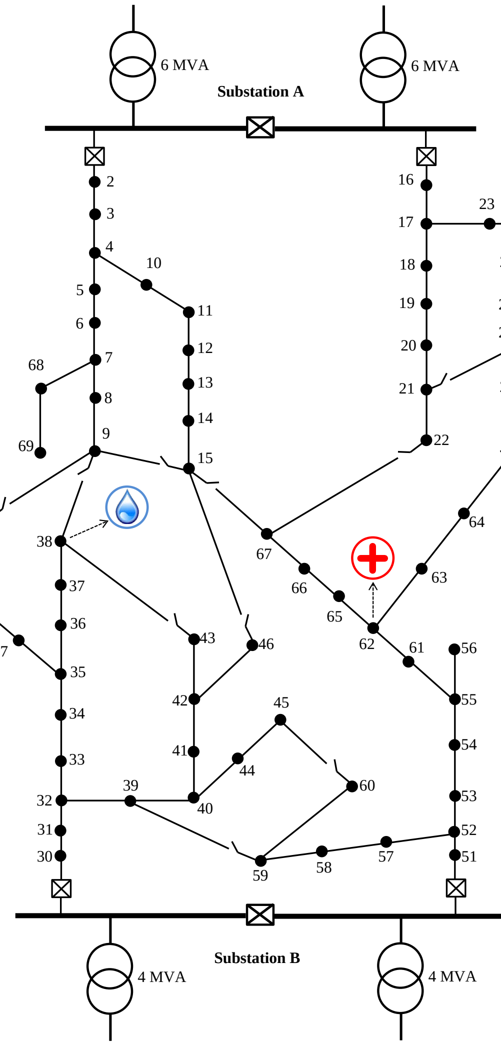

Figure 1. The 70-node system of [13].

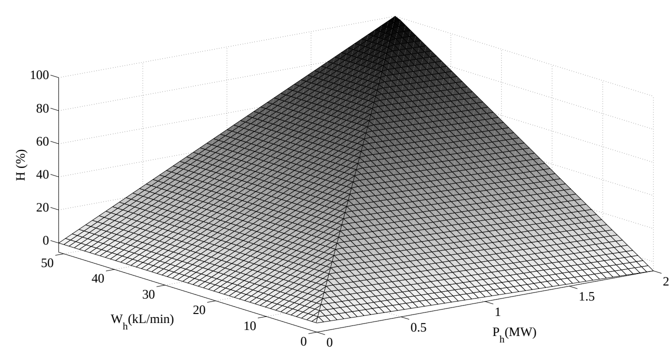

Figure 2. Operational performance of a hospital as a function of available power P_h and water W_h [10].

Table I
PARAMETERS FOR A PIECE-WISE LINEAR FITTING TO $H(W_h, P_h)$

a_1	a_2	b_1	b_2
0	2.0834	50	-2.7778

Table II
SIMULATION RESULTS

Scenario	P_w(MW)	P_h(MW)	W_h(kL/min)	$H(\%)$
1	1.0	1.96	51	97.83
2	0.7	1.41	35.7	70.46

Table III
NETWORK CONFIGURATION

Scenario	Open Switches
1	9-38,14-15,15-67,21-27,39-40,39-59,42-46,45-60,47-48, 61-62,62-63
2	9-15,11-12,15-46,21-27,34-35,38-43,39-59,45-60,48-49, 61-62,62-63

station. The solutions to this scenario are given in Table II. The best possible configuration supplies 7 motors in the water pumping station and 1.41 MW to the hospital. With these input values, the hospital operation is retrieved to 70.46% of its full operational capacity. Table III shows the final radial configuration of the network. Only open switches are reported.

IV. Conclusion

In a state of emergency, it is shown that resource management is vital to increase the rate of saved lives. In this process, the power distribution system can play a crucial role as many emergency health care facilities are highly dependent on the availability of electricity. The interdependencies between the critical infrastructures are taken into account to make the analysis more realistic. There are other factors that affect the performance of a hospital such as transportation, natural gas distribution, medical supplies, doctors, etc. As a future work, the authors will consider the Infrastructure Interdependencies Simulator (i2Sim) developed within their research group as a powerful tool to analyze more sophisticated situations and find optimum solutions.

References

[1] J. R. Marti, J. A. Hollman, C. Ventura, and J. Jatskevich, "Dynamic recovery of critical infrastructures: real-time temporal coordination," *Int. Jour. Critical Infrastructures*, vol. 4, no. 1, pp. 17–31, 2008.

[2] J.-B. Sheu, "An emergency logistics distribution approach for quick response to urgent relief demand in disasters," *Transportation Research Part E: Logistics and Transportation Review*, vol. 43, no. 6, pp. 687–709, 2007.

[3] A. Haghani and S.-C. Oh, "Formulation and solution of a multi-commodity, multi-modal network flow model for disaster relief operations," *Transportation Research Part A: Policy and Practice*, vol. 30, no. 3, pp. 231–250, 1996.

[4] F. Fiedrich, F. Gehbauer, and U. Rickers, "Optimized resource allocation for emergency response after earthquake disasters," *Safety Science*, vol. 35, no. 1, pp. 41–57, 2000.

[5] R. E. Brown, *Electric power distribution reliability*. CRC Press, 2002.

[6] J. C. Araneda, H. Rudnick, S. Mocarquer, and P. Miquel, "Lessons from the 2010 chilean earthquake and its impact on electricity supply," in *Int. Conf. Power Syst. Tech.* IEEE, 2010, pp. 1–7.

[7] J. R. Marti, H. Ahmadi, and L. Bashualdo, "Linear power-flow formulation based on a voltage-dependent load model," *IEEE Trans. Power Del.*, vol. 28, no. 3, pp. 1682–1690, 2013.

[8] R. Pérez-Guerrero, G. T. Heydt, N. J. Jack, B. K. Keel, and A. R. Castelhano, "Optimal restoration of distribution systems using dynamic programming," *IEEE Trans. Power Del.*, vol. 23, no. 3, pp. 1589–1596, 2008.

[9] K. L. Butler, N. D. R. Sarma, and V. Ragendra Prasad, "Network reconfiguration for service restoration in shipboard power distribution systems," *IEEE Trans. Power Syst.*, vol. 16, no. 4, pp. 653–661, 2001.

[10] H. A. Rahman, M. Armstrong, M. DeTao, and J. R. Marti, "I2sim: A matrix-partition based framework for critical infrastructure interdependencies simulation," in *IEEE Electric Power Conference*, 2008, pp. 1–8.

[11] H. Ahmadi, J. R. Marti, and A. Moshref, "Piecewise linear approximation of generators cost functions using max-affine functions," in *IEEE PES GM*, 2013, pp. 1–5.

[12] R. A. Jabr, R. Singh, and B. C. Pal, "Minimum loss network reconfiguration using Mixed-Integer convex programming," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 1106–1115, May 2012.

[13] D. Das, "A fuzzy multiobjective approach for network reconfiguration of distribution systems," *IEEE Trans. Power Del.*, vol. 21, no. 1, pp. 202–209, Jan. 2006.

[14] N. Ozog, "Power systems modeling for multiple infrastructure damage and repair simulations," Ph.D. dissertation, University of British Columbia, 2008.

[15] M. Bai, "Optimization decision maker algorithm for infrastructure interdependencies with i2sim applications," Master's thesis, University of British Columbia, 2012.