Sensitivity Factors for Distribution Systems

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***Abstract*—The changes in branch flows and nodal voltages with respect to a change in current injection at a particular node, defined as current transfer distribution factors (CTDF), are useful sensitivity factors for distribution system analysis. In addition, the changes in branch flows due to the outage of a particular branch, defined as branch outage distribution factors (BODF), are useful in many distribution system analysis and optimization applications. This paper uses a set of linear power flow (LPF) equations to derive both the CTDFs and BODFs for distribution systems. Applications of the proposed sensitivity factors are discussed through illustrative examples in optimal placement of distributed generation/capacitor banks as well as minimum-loss network reconfiguration.**

I. Introduction

The idea of sensitivity analysis in power systems has been widely used to avoid recalculation of the power flow solution. In transmission systems, the parameters used in these analyses are the power transfer distribution factors (PTDF) and the line outage distribution factors (LODF). PTDFs are defined as the changes in the line power flows due to a change in power injection at a particular bus. LODFs are defined as the changes in the line power flows due to the disconnection of a particular line [1]. The calculation of these sensitivity factors has gained more interest recently due to the need for fast on-line readjustments in modern power systems.

There are generally two approaches to calculate sensitivity factors for power systems. Considering a more realistic model, power flow equations form a nonlinear system of equations. In order to find sensitivities, one need to find the Jacobian matrix at a particular solution of the network, which yields sensitivity factors that are only valid for small variations around the operating point [2], [3]. The second approach is to find an approximate linear model that describes the system for relatively large variations in operating point and find the sensitivity factors for the approximate linear system. The decoupled power flow equations, for instance, are used in [4] to find the sensitivity of reactive power flows to transmission line and power transformer outages. The Fast Decoupled power flow method was adopted in [5] and [6] to derive AC distribution factors for transmission systems. These factors were then used to formulate a re-dispatch optimization problem for congestion management. The assumptions of Fast Decoupled power flow are not as strongly hold for distribution systems as they are for transmission systems (X/R ratio is smaller in distribution systems). Also, those factors depend on the operating condition of the system and, therefore, are valid for small changes in power flow patterns.

A linear approximation of the power flow equations is the so-called DC power flow, in which all voltage magnitudes are assumed to be one per-unit, line resistances are ignored, and voltage angles are assumed to be small enough so that their sines are approximately equal to the angles. Based on the linear DC power flow equations, the PTDFs and LODFs can be derived for a transmission system [1]. The assumptions of DC power flow are not valid for distribution systems since the voltage magnitude plays an important role and cannot be dismissed. Besides analytical studies, there have been some efforts recently to derive the sensitivity factors based on real-time data provided by phasor measurement units. For example, in [7], sensitivity factors are derived without the need for a power flow model.

Sensitivity factors have many applications in transmission systems, such as optimal transmission switching [8], $(N-1)$ security assessment [9], congestion management [10], generation rescheduling, etc. Using sensitivity factors, it is possible to calculate the changes in other system quantities such as losses and generation cost.

Despite broad discussions on derivations and applications of sensitivity factors at the transmission level, there is rather limited work on the distribution system counterpart. Distribution systems have high R/X ratios, radial configurations, a mixture of cables and overhead lines, and unbalanced loads. Due to these unique features, the power flow algorithms and sensitivity analysis derived for transmission systems are not always valid for distribution systems. Power flow algorithms specifically designed for distribution systems have been proposed in the literature, e.g., the linear power flow (LPF) [11]. Using the adjoint network method, which is based on the application of Tellegen's theorem to power systems, the authors of [12] derived the sensitivities of power losses and voltage magnitudes with respect to power injection at any node in the system. However, this method is only valid for radial distribution systems. Also, it does not consider the voltage dependence of the loads, which is an important consideration in distribution systems. These factors cannot be used to calculate the branch outage distribution factors (BODF).

Similar to the case of transmission systems, there are many applications for sensitivity factors in distribution systems. The problem of distributed generation (DG) placement usually requires a knowledge of the impact of power injection at each node on certain quantities of the network, e.g., power losses or voltage profile. Capacitor placement, as an example of reactive power sources, also relies highly on the sensitivity of nodal voltages and system losses to the reactive power injection at each particular node. Network reconfiguration also benefits from the BODFs, as shown later in this paper.

II. Calculation of Sensitivity Factors

In this section, the linear power flow (LPF) formulation proposed in [11] is used to derive the sensitivity factors. Loads are modeled as voltage-dependent elements, based on curve-fitting routine described in [11]. Based on this load model, and assuming small voltage angles in distribution systems, the LPF can be formulated as

$$\bar{Y}\,V = I \tag{1}$$

in which $\bar{Y}$ is the modified admittance matrix. All elements are complex numbers.

A. Current Transfer Distribution Factors

Suppose the voltages are obtained for an initial case using (1). Suppose also that a change occurs in the current injection at Node k. The aim here is to find the changes in the branch flows without having to solve (1) again. Since (1) is linear, one can write

$$\Delta V = \bar{Z}\Delta I \tag{2}$$

where $\bar{Z}$ is the inverse of $\bar{Y}$. It should be noted that $\bar{Z}$ is independent of the system loading condition and only depends on the system configuration and branch impedances. Now, suppose it is desired to find the changes in the branch currents. The current flowing through the branch connecting Node i to Node j is calculated as

$$f_{ij} = (v_i - v_j)y_{ij} \tag{3}$$

Therefore, the changes in f_{ij} due to the changes in the current injection at Node k can be calculated as

$$\Delta f_{ij} = (\Delta v_i - \Delta v_j)y_{ij} \tag{4}$$

Substituting Δv_i and Δv_j from (2) into (4), one has

$$\Delta f_{ij} = d_{ij,k}\Delta I_k \tag{5}$$

where $d_{ij,k}$ is the CTDF given as

$$d_{ij,k} = (\bar{z}_{ik} - \bar{z}_{jk})y_{ij} \tag{6}$$

The CTDFs are calculated off-line and stored in a m by n matrix $D \in \mathbb{C}^{m\times n}$, where m is the number of branches and n is the number of nodes. The concept can be easily extended to multiple injections. For instance, if two current injections at two different nodes are considered simultaneously, say Nodes k and l, then the changes in the voltage at Node i is calculated as:

$$\Delta v_i = \bar{z}_{ik}\,\Delta I_k + \bar{z}_{il}\,\Delta I_l \tag{7}$$

substituting (7) into (4), the changes in the current of Branch i-j is:

$$\Delta f_{ij} = \Big((\bar{z}_{ik} - \bar{z}_{jk})\Delta I_k + (\bar{z}_{il} - \bar{z}_{jl})\Delta I_l\Big)y_{ij} \tag{8}$$

This analysis shows that superposition can be applied to the CTDFs. In other words:

$$\Delta f_{ij} = d_{ij,k}\Delta I_k + d_{ij,l}\Delta I_l \tag{9}$$

B. Branch Outage Distribution Factors

Assume a branch is disconnected from a network. In order to find the new power flow solution, (1) has to be solved with a modified $\bar{Y}$. In a radial distribution system, disconnection of a particular branch leads to the disconnection of all the subsequent nodes and branches that are connected to the substation through that particular branch. In order to find the power flow solution under such circumstances, a careful modification is required to form a new admittance matrix for the rest of the network that is still connected to the substation.

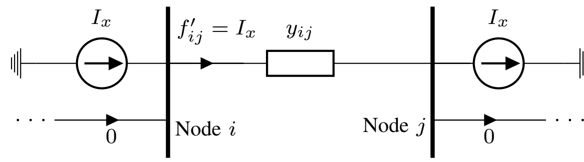


Figure 1. Modeling of a branch outage using fictitious nodal current injections.

In a weakly-meshed network, a branch disconnection may not always lead to two isolated networks. Sensitivity factors can be used to calculate a new solution with respect to the initial solution, in order to avoid rebuilding $\bar{Y}$, for both radial and weakly-meshed networks.

In order to model the outage of Branch i-j, an appropriate fictitious current (I_x) can be injected into Node i and be drawn from Node j, such that the resulting current through Branch i-j (f'_{ij}) is only circulating between the two added current sources, i.e. $I_x = f'_{ij}$. Under these conditions, this branch is considered as "disconnected" from the rest of the network, which can be verified by applying the Kirchhoff's Current Law (KCL) to Nodes i and j. This concept is illustrated in Fig. 1. It is also important to note that by adding the new current sources to Nodes i and j, the flows in the rest of the network are also affected, which is equivalent to the disconnection of Branch i-j. The new flow in Branch i-j due to the added current sources can be calculated as

$$f'_{ij} = (v'_i - v'_j)y_{ij} = f_{ij} + \Delta f_{ij} \tag{10}$$

in which the variables with a prime stand for the new quantities in the modified network. Substituting the value of Δf_{ij} from (9) resulting from the two current injections shown in Fig. 1, the new flow becomes

$$f'_{ij} = f_{ij} + (d_{ij,i} - d_{ij,j})I_x \tag{11}$$

Remember that the aim is to have $f'_{ij} = I_x$. Therefore, the appropriate fictitious current injections at Nodes i and j are

$$I_x = \frac{1}{1 - (d_{ij,i} - d_{ij,j})} f_{ij} \tag{12}$$

By injecting the calculated I_x at both ends of Branch i-j, this branch can be considered as isolated from the rest of the network. Finally, the change in the current flowing through the other branches l-r is calculated using (9) as

$$\Delta f_{lr} = h_{ij,lr} f_{ij} \tag{13}$$

where $h_{ij,lr}$ is the branch outage distribution factor (BODF) given as

$$h_{ij,lr} = \frac{d_{lr,i} - d_{lr,j}}{1 - (d_{ij,i} - d_{ij,j})} \tag{14}$$

The new flows in the remaining branches after the outage of a particular branch are calculated by adding the changes obtained in (13) to the initial branch currents. For a network with m branches, all the $h_{ij,lr}$ form a $m \times m$ matrix, $H \in \mathbb{C}^{m\times m}$, with -1 in the diagonal elements.

C. Power Injections and Power Losses

In distribution systems, determining the location for installing distributed generation (DG) or capacitor banks has been of interest for both academia and industry. CTDFs derived in Section II-A are not directly applicable when active/reactive power injections are to be considered instead of current injections. Nevertheless, it is possible to derive equivalent current injections that closely mimic the active/reactive power injections. When a current I_k is injected to Node k, the resulting power injection is calculated as:

$$S_k = V_k\, I_k^* = (V_k^{\text{re}}\, I_k^{\text{re}} + V_k^{\text{im}}\, I_k^{\text{im}}) + j(V_k^{\text{im}}\, I_k^{\text{re}} - V_k^{\text{re}}\, I_k^{\text{im}}) \quad (15)$$

in which the superscripts indicate real and imaginary. The equivalent active and reactive power injections are:

$$P_k = V_k^{\text{re}}\, I_k^{\text{re}} + V_k^{\text{im}}\, I_k^{\text{im}},\ \ Q_k = V_k^{\text{im}}\, I_k^{\text{re}} - V_k^{\text{re}}\, I_k^{\text{im}} \quad (16)$$

Assume now that an active power P_k is injected at Node k with zero reactive power $Q_k = 0$. The equivalent current injections can be found from (16) as:

$$I_k^{\text{re}} = \frac{V_k^{\text{re}}}{|V_k|^2} P_k,\ \ I_k^{\text{im}} = \frac{V_k^{\text{im}}}{|V_k|^2} P_k \quad (17)$$

The voltages in (17) are the final voltages after injecting I_k. By applying I_k the values of V_k^{re} and V_k^{im} will change from their initial values. Since the final values are not known beforehand, an estimation is used here. Assuming $V_k^{\text{re}} \approx 1$ and $V_k^{\text{im}} \approx 0$ gives a good approximation for the final values of the voltages at the nodes where a power injection is introduced. The accuracy of these approximations is illustrated in Section III.

A similar analysis can be done for reactive power injection Q_k with zero active power $P_k = 0$. From (16), the equivalent current injections can be calculated as:

$$I_k^{\text{re}} = \frac{V_k^{\text{im}}}{|V_k|^2} Q_k,\ \ I_k^{\text{im}} = -\frac{V_k^{\text{re}}}{|V_k|^2} Q_k \quad (18)$$

The previous assumptions for the final values of voltages, i.e. $V_k^{\text{re}} \approx 1$ and $V_k^{\text{im}} \approx 0$, apply equally here.

The total active power losses in a distribution system when a new current injection is applied can be calculated as:

$$P'_L = \sum_{i,j} R_{ij}\, |f'_{ij}|^2 \quad (19)$$

in which R_{ij} and $|f'_{ij}|$ are the resistance and the new current magnitude of Branch i-j, respectively. When the changes in branch currents are due to a change in power injection at Node k, the proposed CTDFs can be readily used to find the new branch flows without requiring a new power flow solution. Using (5), the power losses due to the new injection at Node k, $I_k^{\text{re}} + j\, I_k^{\text{im}}$, can be calculated as:

$$P'_{L,k} = \sum_{i,j} R_{ij}\, |f_{ij} + d_{ij,k}\, (I_k^{\text{re}} + j\, I_k^{\text{im}})|^2 \quad (20)$$

where f_{ij} is the initial flow in Branch i-j.

A similar analysis can be conducted to find the losses when a branch outage occurs, using the proposed BODFs. This is very useful when trying to find the best radial network from an initially meshed network that generates the minimum losses.

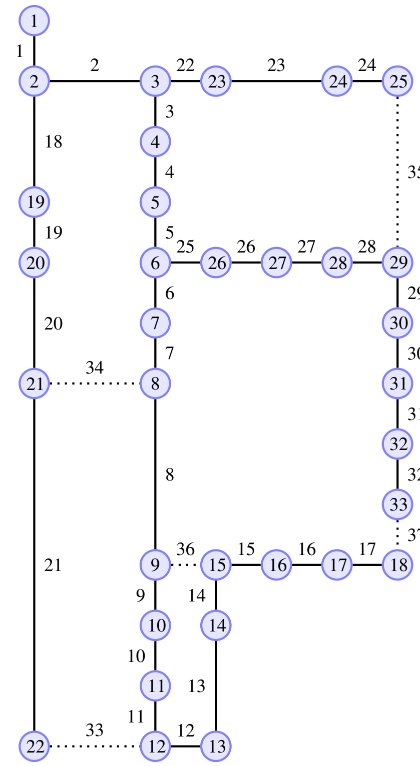


Figure 2. Initial configuration of the 33-node system [13]. The dotted-lines indicate open switches.

III. Simulation Results

Multiple studies are done in this section to show different applications of the proposed sensitivity factors. In Section III-A, the problem of DG/capacitor placement in a distribution system for loss reduction/voltage profile improvement is considered. In Section III-B, the problem of network reconfiguration for loss reduction/voltage profile improvement is considered. For the purpose of illustration, the 33-node system of [13] is used here. The topology of this system is reproduced here, for reference, in Fig. 2. In order to bring the simulation results close to the constant-power load model, for the sake of comparison with traditional methods, and without loss of generality, the parameters defining the load voltage dependence in [11] are chosen as $C_Z = C'_Z = -1$ and $C_I = C'_I = 2$. These parameters basically define a voltage-dependent load which is very insensitive to voltage variations within the normal operating ranges ($\pm 10\%$). Using these parameters, the voltages obtained by the LPF and the Newton-Raphson method have a maximum relative difference of 0.08%. All the simulations are done in MATLAB platform.

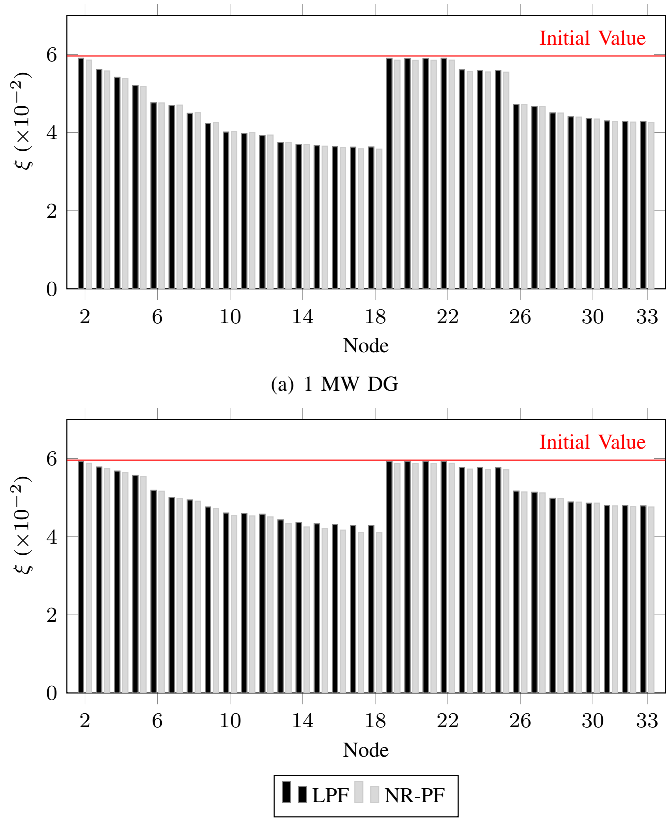
A. DG/Capacitor Placement

Placing a DG/capacitor in a radial network has two aspects that need to be determined: location and size. To optimally locate a DG/capacitor is usually more sophisticated than its sizing since, in a general formulation, it involves discrete variables representing its location. One consideration in determining the best location for new DG/capacitor is its effect on losses and/or voltage profile. The voltage profile improvement is quantified here by calculating the variance of voltage magnitudes with respect to 1 p.u., represented by ξ.

The 33-node system shown in Fig. 2 has a total load of $3.715 + j2.3$ MVA with initial losses of 202.7 kW and $\xi = 0.0596$. Assume now that a 1MW DG is to be installed at one of the nodes in the network. The appropriate current injection at each node is calculated using (17), and its effect on voltage profile is shown in Fig. 3(a). The best nodes to place a DG in this case are Nodes 17 and 18 since they lead to the smallest ξ. The results obtained by the proposed method and by performing 32 Newton-Raphson power flow solutions give the same conclusion.

Assume a 1 MVAR capacitor bank is to be installed in a node. The values of ξ obtained using the proposed method and by performing 32 Newton-Raphson power flow solutions are given in Fig. 3(b). The two candidate nodes are again Nodes 17 and 18. It is important to notice here that adding 1 MW active power or 1 MVAR reactive power source have very similar effects on the system voltage profile.

The impacts of installing a DG/capacitor in losses is studied here. The power losses are calculated using (20) and by conducting 32 Newton-Raphson power flow solutions. The results for 1 MW DG and 1 MVAR capacitor bank placement at each node are shown in Figs. 4(a) and 4(b), respectively. The best candidate node for installing either a DG or a capacitor is Node 29, which reduces the losses to 127 kW in the case of a DG and 146 kW in the case of a capacitor bank. It should be noted that installing extra active/reactive power supply at Nodes 20, 21, and 22 increases the losses. This increase in the losses occurs since the generated power at the mentioned nodes needs to travel through a longer electric path to reach the other loads.



(b) 1 MVAR capacitor

Figure 3. Voltage profile variances (ξ) obtained using the LPF and Newton-Raphson power flow solutions when a 1 MW/MVAR DG/capacitor is added.

B. Network Reconfiguration

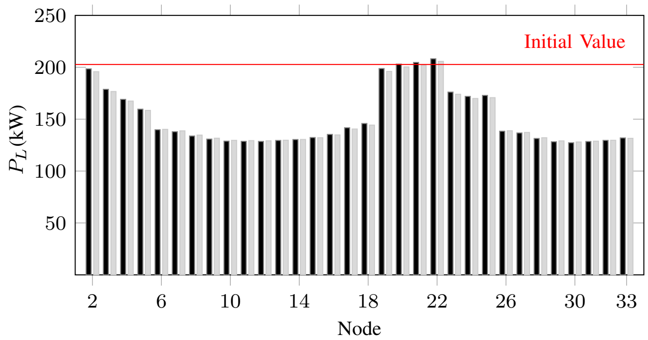
One common approach for optimal network reconfiguration is to first close all the tie switches, forming a meshed network. The switches are then opened one at a time based on the reduction in losses [14]. We apply this procedure to the 33-node system. All the switches are first closed and the BODFs are calculated for this meshed system. The active power losses upon disconnection of each branch can then be calculated using (19), while the new flows are calculated using (13).

In the first iteration, the changes in the total power losses due to the outage of each branch, i.e. $P_L^{\text{new}} - P_L^{\text{initial}}$, is calculated and reported in Table I, second column. Note that disconnection of Branch 1 is equivalent to disconnecting all the nodes in the network and, therefore, is not feasible. Disconnection of Branches 9 and 10 reduces the losses in the meshed network. Therefore, Branch 9 is selected as the first candidate to be opened. By removing Branch 9 from the network, a new set of BODFs are calculated and the process is repeated to find the next candidate. It is important to note that in the second iteration, disconnecting Branches 10 and 11 reduces the losses. However, checking this with Fig. 2 reveals that disconnecting Branches 10 or 11 leads to isolating part of network. The next candidate is Branch 14. In the second iteration, Branches 10-13, 21, and 35 reduce the losses while isolating part of the network. The next accepted candidate is Branch 32 in this iteration. In the fourth iteration, opening any of Branches 8, 10-13, 15-17, 21, 29-31, and 34-36 leads to an isolated network. The next candidate is, therefore, Branch 7. In the fifth and final iteration, Branch 37 is the best candidate.

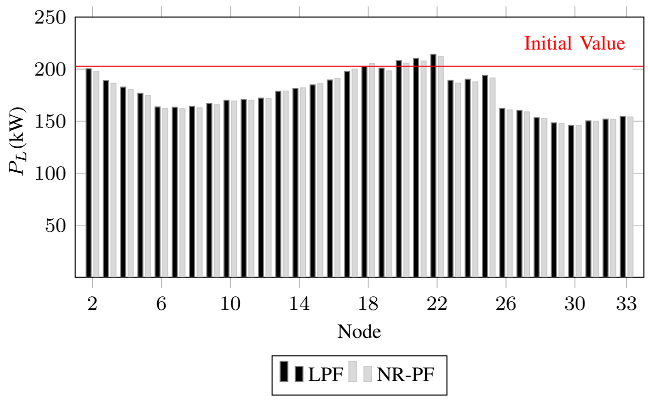
According to the above analysis, the final configuration has the following open switches: 9, 14, 32, 7, and 37. This is the global optimal solution for this network, which reduces the total losses from 202.7 kW down to 139.6 kW, i.e. 31.1% reduction. If the BODFs are not used, one needs to run $36 \times 35 \times 34 \times 33 \times 32 = 45,239,040$ power flow routines, if no intelligent search method is used. Recalculation of the BODFs at each iteration for the new network requires only one linear power flow (LPF) solution, which in total requires 5 solutions.

IV. Conclusion

Sensitivity factors for distribution systems analysis were derived based on a linear power flow formulation. It was shown that the sensitivity factors derived here can be advantageous in many applications such as DG placement, capacitor placement, network reconfiguration, etc. Another application, which was not discussed in the paper, is in the optimal near-real-time adjustment of active/reactive power sources, which requires an understanding of the sensitivity of the objective (losses/voltage profile/generation costs/emissions) to the power injection from that particular source. In distribution systems where many DGs are present, a meshed network is more preferable than the radial one to increase reliability and efficiency of the network. In such networks, branch outages should be studied, similar to the $N-1$ security criteria in transmission systems, to ensure the static security limits such as nodal voltages and branch



(a) 1 MW DG



(b) 1 MVAR capacitor

Figure 4. Total power losses obtained using the LPF and Newton-Raphson power flow solutions when a 1 MW/MVAR DG/capacitor is installed.

ampacities. The branch outage distribution factors (BODF) derived here can be directly applied to such problems.

Table I. CHANGES IN POWER LOSSES (KW) DUE TO A BRANCH OUTAGE

Branch	Iter.#1	Iter.#2	Iter.#3	Iter.#4	Iter.#5
2	445.5	458.0	536.5	560.9	-84.7
3	51.7	52.2	54.1	54.6	75.4
4	40.9	41.3	43.0	43.3	58.3
5	36.4	36.7	38.2	38.5	51.2
6	4.5	4.6	4.7	6.5	-10.0
7	0.5	0.5	0.6	**0.8**	-
8	5.5	7.4	14.9	-24.1	-24.5
9	**-0.04**	-	-	-	-
10	-0.03	-3.6	-3.3	-3.3	-3.6
11	0.2	-6.7	-6.1	-6.0	-6.5
12	3.5	5.2	-11.0	-11.0	-11.6
13	2.1	3.0	-7.9	-7.8	-8.3
14	0.3	**0.2**	-	-	-
15	8.1	8.1	7.9	-18.7	-19.6
16	5.5	5.5	5.2	-15.5	-16.4
17	3.4	3.4	3.2	-11.9	-12.8
18	72.7	75.1	80.3	82.6	-44.5
19	62.1	64.2	68.4	70.3	-44.2
20	52.5	54.3	57.5	59.1	-43.7
21	13.4	20.8	-20.9	-20.8	-21.7
22	102.3	102.3	107.7	121.2	123.1
23	88.8	88.8	93.6	105.3	106.5
24	40.8	40.7	43.3	49.0	48.5
25	17.2	17.4	18.1	23.4	26.6
26	14.2	14.3	14.9	19.5	22.3
27	11.5	11.6	12.1	16.0	18.4
28	9.2	9.3	9.7	13.1	15.1
29	59.5	59.7	73.5	-54.6	-51.5
30	9.6	9.6	12.6	-28.2	-26.8
31	2.9	2.9	4.2	-17.6	-16.8
32	0.3	0.3	**0.3**	-	-
33	6.8	7.4	11.4	11.6	-31.1
34	3.3	5.2	10.9	-21.4	-22.1
35	9.1	14.2	-17.9	-17.8	-18.7
36	1.2	1.2	1.0	-5.4	-5.9
37	12.0	12.0	13.0	15.2	**14.6**

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