Mathematical Representation of Radiality Constraint in Distribution System Reconfiguration Problem

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Abstract

Distribution systems are most commonly operated in a radial configuration for a number of reasons. In order to impose radiality constraint in the optimal network reconfiguration problem, an efficient algorithm is introduced in this paper based on graph theory. The paper shows that the normally followed methods of imposing radiality constraint within a mixed-integer programming formulation of the reconfiguration problem may not be sufficient. The minimum-loss network reconfiguration problem is formulated using different ways to impose radiality constraint. It is shown, through simulations, that the formulated problem using the proposed method for representing radiality constraint can be solved more efficiently, as opposed to the previously proposed formulations. This results in up to 30% reduction in CPU time for the test systems used in this study.

Keywords: Distribution system reconfiguration, planar graph, dual graph, minimum spanning tree, radiality constraint, mixed-integer programming.

1. Introduction

Optimizing the operation of distribution systems (DS) has been an active topic for years with added emphasis recently with the smart gird initiatives. In many utilities, simplicity and reliability of operation has usually been given higher priority than its optimality. In order to keep operation and protection

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as simple as possible, radial configurations are usually preferred. Despite the simplicity provided by radial topologies, the continuity of power supply may suffer by having only one point of supply. To impose supply continuity for critical loads, redundant feeders are often built, while radial structure is still maintained.

In the course of DS automation, the reconfiguration of the network for a number of purposes has been vastly studied. Objectives such as service restoration, loss reduction, load balancing, and voltage profile improvement are commonly used goals in the network reconfiguration problem. There are controllable switches (automated/manual) throughout the network which allows the operator to change the topology of the system. The number of switches in real systems is relatively large and optimization routines are required to determine optimal switching actions to satisfy particular objectives. There are excellent methodologies proposed in the literature to solve the network reconfiguration problem, with pioneering work by [1]-[3]. Deterministic mathematical approaches have been proposed for this problem, e.g., Benders Decomposition [4], and mixed-integer programming [5]-[7]. Heuristic approaches have also been proposed, such as Hyper-Cube Ant Colony Optimization [8], Bacterial Foraging Optimization Algorithm [9], Particles Swarm Optimization [10], Dynamic Switches Set Heuristic Algorithm [11], Artificial Immune Systems [12], Adaptive Imperialist Competitive Algorithm [13], and Genetic Algorithms [14]. The radiality constraint is normally imposed implicitly in all of these studies. The radiality constraint, however, is difficult to represent mathematically, as is also acknowledged in [15].

The term "radial" refers to a configuration that includes all the nodes but has no loops. A brief review of the different methods of imposing the radiality constraint is given in [15]. In the heuristic methods, radiality is usually dealt with implicitly, e.g., [3]. In direct mathematical models, on the other hand, a mathematical formulation for the radiality constraint is required. A few studies provide mathematical models for the radiality constraint, such as [4]-[10], [16]-[18].

In this paper, the authors follow the concept that a distribution network can be modeled as an undirected graph, taking its nodes as vertices and its branches as edges. In order to establish a radial configuration as a subgraph (which translates into a spanning tree in a graph), two conditions must be satisfied:

1. All nodes are inside the subgraph
2. The subgraph is connected and has no loops (simple cycles)

The first condition ensures the subgraph spans all the nodes, and the second condition ensures the subgraph is a tree. These two are necessary conditions, and together are also sufficient. However, this fact has not been paid enough attention to in the literature. A brief review of the existing approaches for imposing the radiality constraint follows.

In [10], [16], and [17], a simple constraint is used to impose the radiality. That constraint requires the ultimate configuration to have $n-1$ branches, where n is the number of nodes. However, it is shown later in this paper, and was also shown in, e.g., [19], that this is not a sufficient condition to guarantee radiality.

In recent work of [6] and [5], the radiality constraint is imposed by the following statement: "every node except the root has exactly one parent". However, the formulations provided may not represent a spanning tree. This fact is shown by a counterexample in this paper. In fact, the constraints provided in [5], for example, does not guarantee a connected graph.

In [8] and [18], radiality is imposed using the branch-to-node incidence matrix. The elements of this matrix are 0, 1 or -1, and its size is m by n (m is the number of branches). A necessary and sufficient condition for having a spanning tree is that the determinant of the incidence matrix must be non-zero. Although this is a strong condition, conventional optimization routines are not capable of handling determinant constraints. In other words, the determinant calculation cannot be stated as a closed-form mathematical formulation.

In this paper, DS is modeled as a planar graph, which allows the enforcement of the radiality constraint in a very simple and effective way as compared to a regular graph. A planar graph is a graph that can be drawn on a two-dimensional plane such that the edges of the graph only meet at the vertices [20]. In other words, even if there are intersections between edges, rearrangement of the vertices will make it possible to redraw the graph as a planar graph. Power distribution networks usually possess this property. A useful feature of a planar graph is its dual graph, which allows for an efficient mathematical representation of the radiality constraint. Using the primal and dual graphs, the author of [21] has shown that an efficient formulation is possible for finding minimum spanning trees (MST).

A mixed-integer quadratically constrained formulation for the network reconfiguration problem is proposed in [7]. It is found by the authors that the currently available formulations for radiality constraint are not efficient to be solved by the state-of-the-art solvers, e.g., GUROBI. One of the possible reasons is that those formulations produce infeasible subproblems in the branch-and-cut algorithm, the algorithm used for solving mixed-integer programming problems. The infeasible subproblems slow down the whole process unnecessarily. To clarify this fact, it should be noted that, for example, it takes four iterations to solve a feasible quadratic programming problem, whereas it takes ten iterations to prove an infeasible one. Another reason for the proposed formulation to be more efficient is that it admits tighter quadratic programming relaxations which enhances the convergence speed of the branch-and-cut routine by adding more constraints to the problem and reducing the search space. By doing this, reductions of up to 30% are achieved in CPU time for the systems used in this paper.

The rest of this paper is organized as follows. In Section 2, a brief background on the graph theory is presented. In Section 3, the inadequacy of the existing methods for representing the radiality constraint is shown. A mixed-integer quadratically constrained formulation for the minimum-loss network reconfiguration is described in Section 4. Section 5 presents examples of finding radial configurations for various test systems. Finally, Section 6 concludes the paper.

2. Background

2.1. Planar Graph

A graph is called planar if, with any rearrangement of its vertices, it can be drawn on a plane without having intersecting edges. A planar graph with n vertices and m edges divides the plane into f faces. Euler's formula [20] suggests the following relation for a planar graph:

$$f = n - m + 2 \tag{1}$$

For example, consider the graph shown in Fig. 1. The faces are the regions on the plane separated by the graph's edges. The outside infinite face (shown by "E") is also counted. In Fig. 1, the faces are identified by capital letters. There are two necessary, but not sufficient, conditions for a graph to be planar [20]:

$$n \geq \frac{3}{2} f \tag{2a}$$

$$n \leq 3m - 6 \tag{2b}$$

Apart from those necessary conditions, there is a theorem in [20] that provides necessary and sufficient conditions for planarity. Before referring to that theorem, two particular graphs, known as Kuratowski's graphs, need to be introduced. The graphs shown in Fig. 2, known as K_5 and $K_{3,3}$, are Kuratowski's two graphs. Another preliminary concept is that of *homeomorphic* graphs. Two graphs are homeomorphic if one can be obtained from the other by adding new edges in series to the existing ones or by merging already-existing edges that are in series. As per [20], a necessary and sufficient condition for a graph to be planar is that it does not contain either of Kuratowski's two graphs, or any graph homeomorphic to either of them.

According to the authors' experience, all distribution systems encountered satisfy the conditions for planarity. Overhead lines are mainly built along land corridors, and because they are geographically distributed in a plane (which is the definition of a planar graph), the natural intuition is that a DS has a planar graph representation. There are also formulated algorithms to check whether an arbitrary graph is planar, e.g., [22].

2.2. Dual Graph

The dual graph G^* of a planar graph G is defined as follows [20]:

- For each face of G, there is one corresponding vertex in G^*.
- For each edge joining two neighbouring faces in G, there is a corresponding edge between the two vertices in G^*.
- For any pendent edge (an edge with only one vertex connected to it) in G, there is one self-loop at the corresponding vertex in G^*.

From the above definition, it immediately follows that if G has n vertices, m edges and f faces, then G^* has f vertices, m edges and n faces [20]. Figure 3 depicts the dual graph of the 9-node graph shown in Fig. 1. As can be seen in Fig. 3, there may be more than one edge between two vertices in the dual graph which have to be distinguished.

2.3. The Spanning Tree Constraint

The radiality constraint in a DS is identical to the spanning tree constraint in graph theory. The minimum spanning tree in a weighted undirected graph is the subgraph that is a tree and the sum of its weights is the minimum possible. This problem is well-addressed in the literature [21]. Also, a mixed-integer linear programming formulation for this problem specifically designed for planar graphs is proposed in [21]. This method is briefly explained in the following.

An undirected graph is first converted into a directed graph. Define the following variables and sets in G:

- x_{ij}: status of the edge connecting vertex i to vertex j.
- w_{ij}: weight of the edge connecting vertex i to vertex j.

- N_i: set of vertices directly connected to vertex i.

and in G^*:

- $y_{(k,l),e}$: status of the edge(s) connecting vertex k to vertex l. The index e is used to distinguish between multiple edges connecting the same two vertices.
- M_k: set of vertices directly connected to vertex k.
- $S_{k,l}$: set of multiple edges between vertices k and l.

The minimum spanning tree problem is formulated as follows [21]:

$$\text{Minimize} \quad \sum_{i,j} w_{ij} x_{ij} \tag{3}$$

Subject to:

$$\sum_{j \in N_i} x_{ij} = 1, \quad 1 \leq i \leq n-1 \tag{4a}$$

$$\sum_{l \in M_k} \sum_{e \in S_{k,l}} y_{(k,l),e} = 1, \quad 1 \leq k \leq f-1 \tag{4b}$$

$$x_{ij} + x_{ji} + y_{(k,l),e} + y_{(l,k),e} = 1, \text{ For all edges in } G \tag{4c}$$

Note that since (4c) has one equation for each edge in G, it represents m constraints. Also, if $i = n$ or $j = n$ ($k = f$ or $l = f$) in $G(G^*)$, $x_{ij}(y_{(k,l),e})$ only exists in one direction terminating in vertex $n(f)$ in $G(G^*)$, i.e. the last vertex.

The set of constraints in (4) enforce the spanning tree constraint. These constraints are used later in the network reconfiguration problem to impose radiality.

2.4. Formulation of Radiality Constraints

The formation of the radiality constraint for the 9-node system shown in Fig. 3 is discussed here in detail as a reference.

The needed sets are formed as follows:

$$N_1 = \{2,6\} \quad N_2 = \{1,3,5\} \quad N_3 = \{2,4\}$$
$$N_4 = \{3,5,8\} \quad N_5 = \{2,4,9\} \quad N_6 = \{1,7,9\}$$
$$N_7 = \{6,8\} \quad N_8 = \{4,7,9\} \quad N_9 = \{5,6,8\}$$

$$M_A = \{B,C,D,E\} \quad M_B = \{A,D,E\}$$
$$M_C = \{A,D,E\} \quad M_D = \{A,B,C,E\}$$
$$M_E = \{A,B,C,D\}$$

$$S_{A,E} = \{1,2\} \quad S_{B,E} = \{1,2\} \quad S_{C,E} = \{1,2\}$$

In the following, examples are given on how to write the problem constraints.

In the primal graph (4a) is

For $i = 1$, $x_{1,2} + x_{1,6} = 1$.

For $i = 2$, $x_{2,1} + x_{2,3} + x_{2,5} = 1$.

For $i = 3$, $x_{3,2} + x_{3,4} = 1$.

For $i = 4$, $x_{4,3} + x_{4,5} + x_{4,8} = 1$.

For $i = 5$, $x_{5,2} + x_{5,4} + x_{5,9} = 1$.

For $i = 6$, $x_{6,1} + x_{6,9} + x_{6,7} = 1$.

For $i = 7$, $x_{7,6} + x_{7,8} = 1$.

For $i = 8$, $x_{8,4} + x_{8,7} + x_{8,9} = 1$.

In the dual graph (4b) is

For $k = A$, $y_{A,B} + y_{A,C} + y_{A,D} + y_{A,E,1} + y_{A,E,2} = 1$

For $k = B$, $y_{B,A} + y_{B,D} + y_{B,E,1} + y_{B,E,2} = 1$

For $k = C$, $y_{C,A} + y_{C,D} + y_{C,E,1} + y_{C,E,2} = 1$

For $k = D$, $y_{D,A} + y_{D,B} + y_{D,C} + y_{D,E} = 1$

For (4c), each branch has one equation:

For Branch 1-2, $x_{1,2} + x_{2,1} + y_{A,E,1} + y_{E,A,1} = 1$

For Branch 2-3, $x_{2,3} + x_{3,2} + y_{B,E,1} + y_{E,B,1} = 1$

For Branch 3-4, $x_{3,4} + x_{4,3} + y_{B,E,2} + y_{E,B,2} = 1$

For Branch 4-5, $x_{4,5} + x_{5,4} + y_{B,D} + y_{D,B} = 1$

For Branch 1-6, $x_{1,6} + x_{6,1} + y_{A,E,2} + y_{E,A,2} = 1$

For Branch 6-7, $x_{6,7} + x_{7,6} + y_{C,E,1} + y_{E,C,1} = 1$

For Branch 7-8, $x_{7,8} + x_{8,7} + y_{C,E,2} + y_{E,C,2} = 1$

For Branch 8-9, $x_{8,9} + x_{9,8} + y_{C,D} + y_{D,C} = 1$

For Branch 2-5, $x_{2,5} + x_{5,2} + y_{A,B} + y_{B,A} = 1$

For Branch 6-9, $x_{6,9} + x_{9,6} + y_{A,C} + y_{C,A} = 1$

For Branch 5-9, $x_{5,9} + x_{9,5} + y_{A,D} + y_{D,A} = 1$

For Branch 4-8, $x_{4,8} + x_{8,4} + y_{D,E} + y_{E,D} = 1$

If there is a pendent node in the network, i.e. a node that has only one branch connected to it, that branch is definitely in the spanning tree since its disconnection renders the graph disconnected.

3. Inadequacy of Existing Methods in Representing the Radiality Constraint

There are several different methods proposed in the literature for representing the radiality constraint. These methods, however, may not be adequate, as is shown through examples here. The first method of representing the radiality constraint, which is the simplest, is to require the number of branches to be exactly equal to the number of nodes minus one. In other words,

$$\sum_{ij \in W} u_{ij} = n - 1 \tag{5}$$

where u_{ij} is the binary variable standing for branch i-j status (0: "open", 1: "close"); W is the set of all branches. This criterion has been used to enforce the radiality constraint in, e.g., [10], [16], [17]. However, this constraint does not guarantee the connectivity of the resulting network, as is also acknowledged in [15], [19]. As a counterexample, look at the topology shown in Fig. 4 for the network shown in Fig. 1. It is trivial to check that the network in Fig. 4 satisfies (5), but is not connected.

The second approach used in the literature for representing radiality is to model the network as a directed graph, e.g., [6], [5]. The clear statement of the constraints is as follows [5]:

$$\beta_{ij} + \beta_{ji} = u_{ij}, \quad (i,j) \in W \tag{6a}$$

$$u_{ij} = u_{ji}, \quad (i,j) \in W \tag{6b}$$

$$\sum_{j \in N_i} \beta_{ij} = 1, \quad i = 2, \ldots, n. \tag{6c}$$

$$\beta_{1j} = 0, \quad j \in N_1 \tag{6d}$$

$$\beta_{ij} \in \{0,1\}, \quad (i,j) \in W \tag{6e}$$

Note that (6b) is implicitly imposed by (6a) and is only restated for clarity. It is trivial to check that the network shown in Fig. 4 satisfies all the constraints in (6). The values for β constructing this network are given in Table 1. Only non-zero values of β_{ij} are reported in Table 1.

The reason that the mentioned constraints representing radiality still work, when embedded in a network reconfiguration problem, is discovered by the authors. The disconnected network leads to an infeasible power flow solution. In other words, the connectivity constraint is imposed by the power flow equations. This fact is also emphasized in [15] and the network flows are used to impose the connectivity of the network. However, during the process of solving a mixed-integer programming problem, infeasible configurations may be generated, which is due to insufficiency of the constraints representing the radiality. The infeasible subproblems in a branch-and-cut algorithm lead to a larger number of iterations which, in turn, increases the CPU time of the whole solution process. The most severe case is when decomposition algorithms are used to solve mixed-integer problems, e.g., [4]. When Bender's Decomposition is used [4], the master problem, which mainly deals with the integer variables, may generate infeasible configurations that would not be known to be infeasible until the subproblem is solved. Moreover, in most of the heuristic methods, many infeasible configurations are generated first and then discarded by checking the radiality constraint. The process of determining and discarding the infeasible solutions slows down the whole solution process, leading to an unnecessarily large CPU time.

Another method for representing the radiality constraint is to employ the branch-to-node incidence matrix, e.g., [8], [18]. The elements of the incidence matrix are all -1, 0, or 1. It is known that if a graph represents a spanning tree, then the determinant of the incidence matrix must be -1 or 1 [23]. In other words, if the determinant is zero, then the obtained subgraph is Not a spanning tree. One deficiency of this method is that it cannot be explicitly stated in a mathematical formulation that can be used in conventional optimization models. Another problem with this method is that it needs calculations with high computational complexity. Therefore, its direct application in mathematical models of network reconfiguration problem is impractical.

4. A Mixed-Integer Quadratically Constrained Formulation of Minimum-Loss Network Reconfiguration Problem

The minimum-loss network reconfiguration problem is, by nature, a mixed-integer nonlinear programming problem with non-convex constraints [24]. Recently, the authors proposed a linear power flow (LPF) formulation based on a voltage-dependent load model in [25]. The LPF equations are used in [7] to form a mixed-integer quadratically constrained programming (MIQCP) formulation for the minimum-loss network reconfiguration. This formulation is briefly described here.

4.1. Objective

The active power losses in a network are calculated as:

$$P_{\text{loss}} = \sum_{\substack{m,k \\ m<k}} u_{m,k} G_{m,k} \left[(V_m^{\text{re}} - V_k^{\text{re}})^2 + (V_m^{\text{im}} - V_k^{\text{im}})^2 \right] \tag{7}$$

where $G_{m,k}$ is the branch conductance; V_k^{re} and V_k^{im} are the real and imaginary parts of the nodal voltages; $u_{m,k} \in \{0, 1\}$ stands for the status of the branch connecting Nodes m and k. The products of binary and continuous variables are dealt with as described in Appendix.

4.2. Load Modeling

A voltage-dependent load model is considered in this study. The following equations describe the voltage dependence of loads at each node:

$$\frac{P(V)}{P_0} = C_Z \left(\frac{V}{V_0}\right)^2 + C_I \left(\frac{V}{V_0}\right) \tag{8}$$

$$\frac{Q(V)}{Q_0} = C'_Z \left(\frac{V}{V_0}\right)^2 + C'_I \left(\frac{V}{V_0}\right) \tag{9}$$

in which P_0 and Q_0 are the active and reactive power demand at the nominal voltage V_0; the parameters C_Z, C'_Z, C_I, and C'_I are to be determined using a curve fitting process [25]. Using the above load model, each load can be synthesized as a current injection in parallel with an admittance. The values for the current injections and admittances are determined as follows:

$$G_L = \frac{P_0 C_Z}{V_0^2}, \quad B_L = -\frac{Q_0 C'_Z}{V_0^2} \tag{10a}$$

$$I_p = \frac{P_0 C_I}{V_0}, \quad I_q = -\frac{Q_0 C'_I}{V_0} \tag{10b}$$

In these equations, G_L and B_L are the equivalent conductance and susceptance of load, respectively; I_p and I_q are the real and imaginary parts of the equivalent current injection from each load, respectively.

4.3. Power Flow Equations

The linear power flow (LPF) equations are derived in [25] based on the assumption of small voltage angles in distribution systems. The LPF equations at Node m are stated as:

$$\sum_{k=1}^{n} \left(\bar{G}_{m,k} V_k^{\mathrm{re}} - \bar{B}_{m,k} V_k^{\mathrm{im}}\right) = I_{p,m} \tag{11}$$

$$\sum_{k=1}^{n} \left(\bar{B}_{m,k} V_k^{\mathrm{re}} + \bar{G}_{m,k} V_k^{\mathrm{im}}\right) = I_{q,m} \tag{12}$$

where n is the number of nodes; $\bar{G}_{m,k}$ and $\bar{B}_{m,k}$ are, respectively, the conductance and susceptance parts of the modified network admittance matrix (load admittances are added to the diagonal elements of the admittance matrix); $I_{p,m}$ and $I_{q,m}$ are, respectively, the equivalent real and imaginary current injections of the load connected to Node m. The product of binary and continuous variables are dealt with as described in Appendix.

It should be noted that the LPF formulation is intended for a balanced distribution system analysis. This is justifiable in case of network reconfiguration since the tie/sectionalizing switches are three-phase operated units. In other words, there is no single-phase operation in the course of network reconfiguration.

4.4. Branch Ampacity

The current flowing through each branch is calculated as:

$$I_{m,k}^2 = u_{m,k}[G_{m,k}^2 + B_{m,k}^2][(V_m^{\mathrm{re}} - V_k^{\mathrm{re}})^2 + (V_m^{\mathrm{im}} - V_k^{\mathrm{im}})^2] \tag{13}$$

The production of binary and continuous variables are dealt with as described in Appendix. The current ampacity limits are then imposed by the following constraint:

$$I_{m,k}^2 \leq |I_{m,k}^{\max}|^2 \tag{14}$$

where $I^{\max}$ is the maximum current allowed in a branch.

4.5. Nodal Voltage Limits

The nodal voltage magnitudes are bounded by the following constraints:

$$V_m^{\min} \leq \sqrt{|V_m^{\mathrm{re}}|^2 + |V_m^{\mathrm{im}}|^2} \leq V_m^{\max} \tag{15}$$

In a typical distribution system, voltage angles are negligible, as discussed in [25]. This assumption allows for eliminating $|V_m^{\mathrm{im}}|^2$ in (15), giving the following box constraint:

$$V_m^{\min} \leq V_m^{\mathrm{re}} \leq V_m^{\max} \tag{16}$$

4.6. Complete Formulation

The minimum-loss network reconfiguration problem has (7) as its objective, subject to (11)-(14), (16), the additional constraints resulting from replacing the binary-continuous products using (17a) and (17b), and the radiality constraint. The radiality constraint is represented by the most recent formulation proposed in [5] and the proposed formulation in this paper, and the results are compared in terms of CPU time.

5. Simulation Results

In this section, the problem of network reconfiguration for loss reduction is solved for several test systems using the proposed method. Common test systems used in the literature are the 14-node [1], 33-node [2], 70-node [26], 84-node [27], 119-node [28], and 135-node [29]. The system sizes and total loads are given in Table 2. The mixed-integer programming problem is written in AIMMS environment [30] and GUROBI is used to solve it.

Simulation results for the test systems obtained using the radiality constraint in (6) as well as the results obtained using the radiality constraint in (4) are given in Table 3. The power losses are calculated in the post-process using a constant-power load model, after the configuration is determined by the voltage-dependent load model formulation. This is done for the purpose of comparison with reported values in other literature, e.g., [5]. The optimality of the results is guaranteed by the solver. A comparison of the optimal losses and configurations with previous literature is done in [7] and is not reproduced here. It is important to notice that the amount of saving in CPU time increases as the size of the problem increases. For instance, 17%, 24%, and 30% reduction in CPU time are achieved for the 84-node, 119-node, and 135-node systems, respectively. This is due to tighter relaxations provided by the subproblems generated during the branch-and-cut algorithm. When a disconnected network is generated by (6), the corresponding quadratic programming relaxation would be infeasible since the power flow equations are impossible to satisfy in a disconnected network.

This fact does not affect the quality of the final solution. However, it causes the whole solution process to take longer to converge, as can be clearly seen in the reported CPU times.

The optimal configurations are shown in Table 4. Only the open switches are reported. There is no difference between the configurations obtained when (4) is used and the configurations obtained when (6) is used.

6. Conclusion

Distribution systems are modeled as planar graphs in this study. The purpose of this modeling is to facilitate the formulation of the radiality constraint using a proven mathematical procedure. The problem of DS reconfiguration for loss reduction is formulated as a mixed-integer quadratically constrained programming problem, which is then solved using a commercial software. It is shown that the CPU time required by the branch-and-cut procedure is decreased when the proposed formulation for radiality constraint is used. Whenever the connectivity of the network is left to be imposed only by the power flow equations, infeasible subproblems may be generated which slows down the whole solution process. The proposed formulation, on the other hand, guarantees a radial configuration and provides tighter quadratic programming relaxations at every iteration.

7. Appendix: Product of Binary-Continuous Variables

The production of a binary variable (z) and a bounded continuous variable (x) can be eliminated by introducing a new continuous variable (w) and the following four inequality constraints [31]:

$$x - (1-z)x_{\max} \leq w \leq x - (1-z)x_{\min} \tag{17a}$$

$$z\, x_{\min} \leq w \leq z\, x_{\max} \tag{17b}$$

If $z = 1$, then (17a) enforces $w = x$ and (17b) limits x within its bounds. If $z = 0$, then (17b) enforces $w = 0$ and (17a) is the bounds on x. Therefore, w is equivalent to $z \times x$. This technique is adopted here to eliminate the multiplication of binary-continuous variables in (7)-(13).

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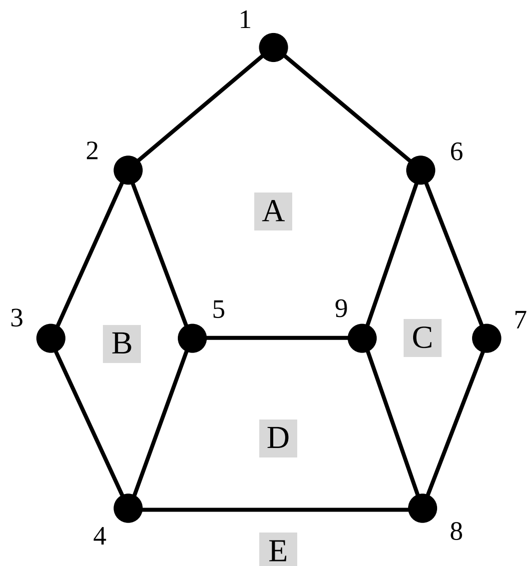
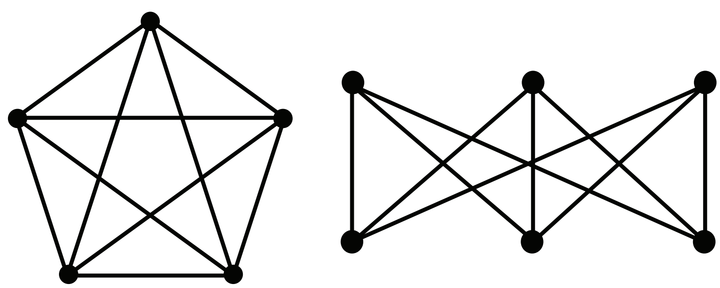


Figure 1: The 9-node network. The letters show the faces of the graph.



(a) K_5 (b) $K_{3,3}$

Figure 2: The Kuratowski's two graphs.

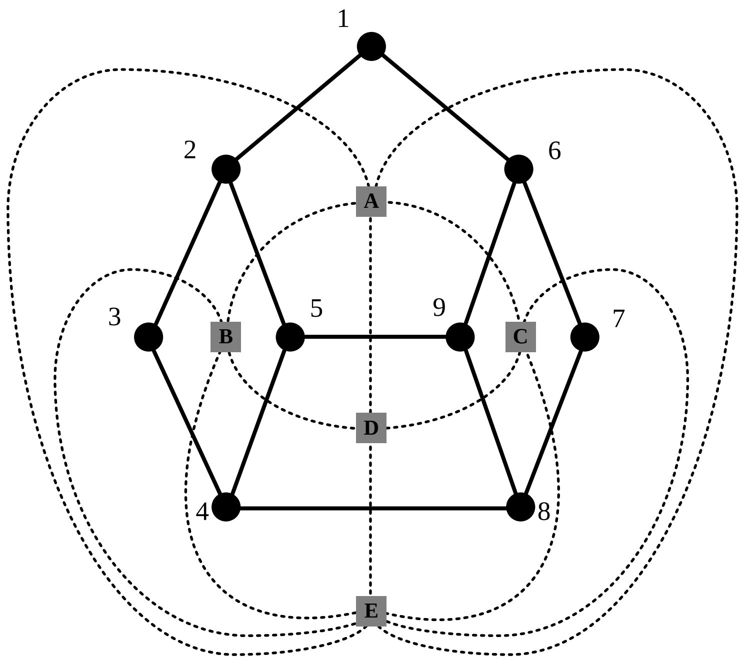


Figure 3: Dual graph (dotted lines) of the 9-node network.

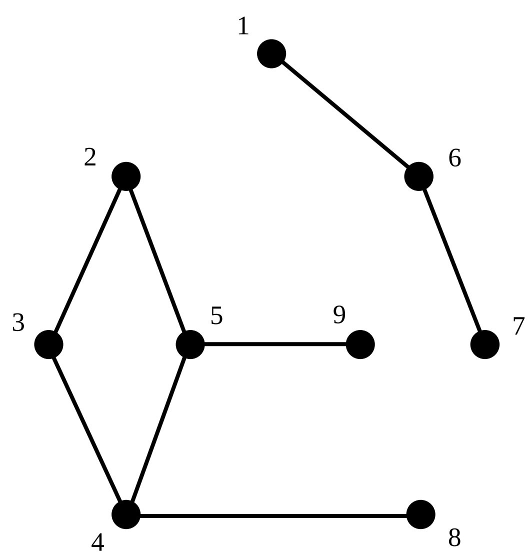


Figure 4: The network obtained by applying the conventional radiality constraints.

Table 1: Values for β_{ij} According to (6) for the Network in Fig. 4

Value	Variables							
1	β_{61}	β_{52}	β_{23}	β_{34}	β_{84}	β_{45}	β_{95}	β_{76}
0	β_{21}	β_{96}	β_{89}	β_{87}				

Table 2: Dimensions of the Test Systems

Test Case	Branches	Feeders	Load(MVA)
14-node	16	3	$28.70 + i17.30$
33-node	37	1	$3.7 + i2.3$
70-node	79	4	$4.47 + i3.06$
84-node	96	11	$28.3 + i20.7$
119-node	132	3	$22.7 + i17.0$
135-node	156	8	$18.31 + i7.93$

Table 3: Comparison of Network Losses Obtained by The Proposed Algorithm and Other References

Test Case	Losses(kW)		T_1(s)*	T_2(s)*
	Initial	Optimum		
14-node	514	468.3	0.14	0.16
33-node	202.7	139.6	3.2	3.0
70-node	227.5	201.4	5.7	4.2
84-node	532	469.9	9.4	7.8
119-node	1298.1	869.7	39.4	30.1
135-node	320.4	280.2	188.2	132.5

* T_1: CPU time when (6) is used. T_2: CPU time when (4) is used.

Table 4: Radial Configurations Obtained by The Proposed Algorithm

Test Case	Off-line Branches
14-node	6-8,7-9,5-14
33-node	7-8,9-10,14-15,32-33,25-29
70-node	14-15,9-38,15-67,49-50,39-59,38-43,9-15,21-27,28-29, 62-65,40-44
84-node	7-6,13-12,18-14,26-16,32-28,34-33,39-38,43-11,72-71, 83-82,55-5,41-42,63-62
119-node	23-24,26-27,35-36,41-42,44-45,51-65,53-54,61-62,74-75,77-78,86-113,95-100,101-102,89-110,114-115
135-node	6-7,10-32,57-61,78-125,20-130,137-138,59-145,139-154,141-154,155-156,154-204,211-212,138-217,125-219,141-220,222-223,144-145,43-46,63-64,130-131, 214-215