Minimum-Loss Network Reconfiguration: A Minimum Spanning Tree Problem

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Abstract

Topological reconfiguration of power distribution systems can result in operational savings by reducing the power losses in the network. In this paper, an efficient heuristic is proposed to find an initial solution for the minimum-loss reconfiguration problem with small optimality gap. Providing an initial solution for a mixed-integer programming (MIP) problem, known as "warm-start", allows for a significant speed up in the solution process. The network reconfiguration for loss reduction is mapped here into a problem of finding a minimum spanning tree (MST) in a graph, for which there are a number of efficient algorithms developed in the literature. The proposed method leads to very fast solutions (less than 1.4 s for systems up to 10476 nodes). For the test systems considered, the solution provided by the proposed method lies within a relative optimality gap of about 2.2% with respect to the optimal solution. The existing MST algorithms guarantee the scalability of the proposed routine for large-scale distribution systems. Sensitivity factors are also employed to refine the solution to a smaller optimality gap.

Keywords: Power distribution systems, topological reconfiguration, minimum spanning tree.

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1. Introduction

The tendency towards optimizing the utilization of the current infrastructure in power systems has been significantly increased in the past decade. The costs and technical difficulties associated with building new lines is a major motivation [1]. Also, reducing power losses has become a desirable objective for many distribution companies (DISCOs). Distribution systems (DS) are the last stage of the electricity network supplying energy to customers. Conventionally, DS's have been operated in radial configuration due to the simplicity of operating and protecting a radial system. Although not often, weakly-meshed configurations are sometimes encountered. Despite their extended use, radial structures are relatively vulnerable in that they have a single point of supply. Nonetheless, losing a single feeder within the whole of a DS is not potentially a threat for the entire power system operation and, therefore, the radial structure is still usually preferred.

One of the important operational problems in DS is the topological reconfiguration for loss reduction, load

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balancing, voltage profile improvement, etc. There are usually controllable switches in DS's using which the operator can alter system's topology by performing opening/closing actions. Since the number of switches are relatively high in practical systems, it is almost impossible for the operator to find the best topology without an optimization study. Many algorithms for DS reconfiguration have been developed, e.g., [2]-[4]. In these studies, the condition of radiality of the final network is normally imposed. This constraint, however, is not easy to represent as a mathematical formula, which is also recognized in [5] and [6].

The term "radial" refers to a configuration that includes all the nodes but has no loops. A literature review on this subject is provided in [5] and [6] which articulates the different approaches for imposing the radiality constraint in a configuration optimization problem. In heuristic methods, the radiality constraint is usually dealt with implicitly. Examples of heuristic methods used in the literature for network reconfiguration are Genetic Algorithms [7] and [8], Harmony Search [9], Simulated Annealing [10], Artificial Neural Networks [11], Plant Growth Simulation [12], Tabu Search [13], Particle Swarm Optimization [14], and Ant Colony [15]. In direct mathematical models, on the other hand, there needs to be a mathematical formulation for representing the radiality constraint. A few studies provide mathematical models for the radiality constraint, such as [16]-[21].

The standard constraints in an optimal DS reconfiguration problem are power flow equations, nodal voltage limits, feeders ampacity limits, and radiality. Formulating this problem using the traditional power flow equations leads to a mixed-integer nonlinear programming (MINLP) problem [22]. Solving large-scale MINLP problems is not practically possible due to the tremendous amount of computations required. Therefore, researchers have tried to tackle the problem using heuristic methods. The advantage of heuristic methods is their low computational complexity while being unreliable and/or suboptimal. A sensible combination of algorithms would be to use heuristics to find a good initial solution and then use this solution to initiate a deterministic mathematical optimization, e.g., [20].

There are a few deterministic mathematical formulation of the network reconfiguration problem that are in the form of a mixed-integer programming (MIP) problem. A mixed-integer conic programming formulation is proposed in [18] for network reconfiguration. It takes over 0.5 h for the algorithm in [18] to find an optimal/near-optimal solution for a network with 135 nodes. Network reconfiguration problem is formulated as a second-order cone programming in [19]. This method takes hours to provide optimal solution for a system with 880 nodes. A mixed-integer quadratically-constrained programming (MIQCP) formulation of the reconfiguration problem is proposed in [20] which is based on a linear power flow algorithm developed in [23]. The optimization problem in [20] takes about three minutes to solve the problem for a 135-node system. A mixed-integer linear programming formulation for the reconfiguration problem is developed in [24] which linearizes the current injections at each node as well as the quadratic terms in the objective function and constraints. Although all the aforementioned deterministic approaches provide many advantages in terms of flexibility of the formulation (e.g., for adding extra constraints, aiming at different objectives, knowledge about the optimality gap at every iteration, etc.), they do not take advantage of a good initial solution to start the MIP solver. Many commercial MIP solvers allow for a "warm- start", i.e. starting from a known solution, to speed up the search for the global optimum, e.g., CPLEX [25] and GUROBI [26].

In this paper, a fast method is sought which provides a suboptimal solution for the minimum-loss reconfiguration problem with a small optimality gap. This can replace the first stage of the MIP solution process that most of the commercial solvers have as a built-in routine. This routine searches for a feasible solution using some general-purpose heuristic methods. Those heuristics, however, are relatively slow and often find a solution with a large optimality gap. Providing a good initial solution to start with, the solution time can be significantly reduced [25].

The problem of finding a radial topology for a DS with minimum losses can be interpreted as finding a spanning tree in the network that also generates minimal losses. The minimum spanning tree (MST) algorithm has been adopted in [27] to find a radial topology that minimizes the energy-not-supplied. In order to assure radiality, different MST algorithms are utilized in [28] (Kruskal algorithm) and [29] (Prim algorithm) within a Genetic Algorithm-based search. However, the only use of the MST algorithms in the referred studies is to help finding a radial configuration, not to find the minimum losses.

In this paper, the problem of minimum-loss DS reconfiguration is mapped into a MST problem based on some assumptions. The main assumption here, based on engineering knowledge, is that the meshed network is a good solution (if not the best) for loss minimization when the radiality constraint is relaxed. The same assumption has been made in [21] with a slight difference. In [21], it is assumed that the meshed network generates the least possible losses, which may not be always true as shown in this paper. However, the difference between the losses in the meshed network and the best-possible network configuration is negligible, as is also shown later in this paper. The second assumption in the present paper is that reconfiguring the network in order to reduce the losses will implicitly improve the voltage profile. This assumption has been previously validated in [4], and is also validated in this paper on various test systems. The voltage profile can also be modified by adjusting capacitor banks, voltage regulators, and transformers tap positions, as is done in [20], outside of the reconfiguration process.

Based on the assumptions mentioned in the previous paragraph, the problem of DS reconfiguration for loss reduction is defined as "finding a spanning tree that imitates, as closely as possible, the same flow pattern as the meshed network". The idea of starting from a meshed network and opening switches sequentially has been proposed in [4], called DISTOP. There are two main differences between DISTOP and the proposed method in this paper. The first difference is that after every switching action, a new power flow solution is required in DISTOP, while only one power flow solution is sufficient for the proposed algorithm in this paper. The other difference is how the algorithms check whether a configuration is radial. It is stated in [4] that the switches are opened one after another until the network becomes radial. However, it is not explicitly explained how to check the radiality. In many cases, disconnecting a switch that carries the minimum current leads to a disconnected network. It is not trivial to check if the network remains connected after opening a particular switch. The proposed algorithm in this paper, on the other hand, is guaranteed to provide a radial configuration.

The proposed heuristic method here has the advantage of using a fast and robust algorithm that also finds a high-quality solution (with small optimality gap). The efficient MST algorithms developed in the literature such as Kruskal [30] and Prim [31] algorithms can be employed to solve the formulated problem. A refinement to the solution is done by sensitivity analysis around the neighborhood of the candidate tie switches. In order to do that, line outage distribution factors (LODF) are derived based on the linear power flow formulation in [23].

The rest of this paper is organized as follows. In Section 2, a brief background on graph theory and the MST problem is presented. In Section 3, the DS reconfiguration problem is converted into a MST problem. Section 4 presents the application of the proposed method to various test cases. The main findings of this study are summarized in Section 5.

2. Background: Minimum Spanning Tree

A spanning tree is a subgraph of an undirected graph that contains all the vertices (i.e. it is connected) and has no cycles (has no loops). The MST in a weighted undirected graph is the subgraph that is a spanning tree, and the sum of its weights is the minimum possible. This problem is well-addressed in the literature under the name "minimum spanning tree" (MST) and there are efficient algorithms such as Kruskal [30] and Prim [31] to solve the problem. Besides, there are parallel algorithms to solve the MST problem even faster, e.g., [32]. By negating the weights, the algorithms for MST will find the maximum spanning tree [33]. The time complexity (the amount of time taken by the algorithm to run as a function of the network size) of a version of Prim's algorithm which is suitable for sparse graphs is $\mathcal{O}(e \log v)$, which is similar to the time complexity of Kruskal's algorithm.

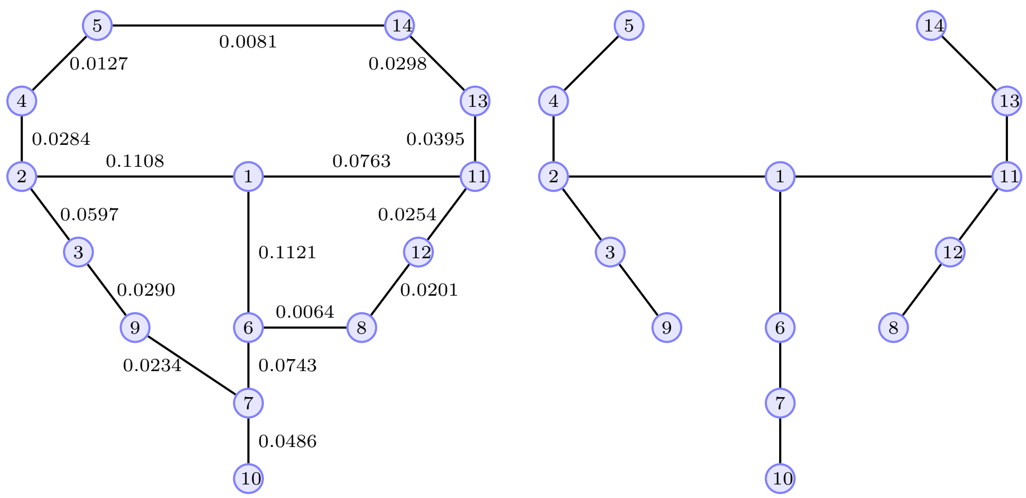
Prim's algorithm is used in this paper to find the MST. There is basically no advantage of using Prim's algorithm over Kruskal's, and one may choose the other. It is important to emphasize that these algorithms provide the optimal solution to the MST problem. The structure of the Prim's algorithm for a weighted undirected graph $G(V, E)$ with V vertices and E edges is summarized in the following steps.

1. Choose any vertex r in V to be the root node. Set $V_t = \{r\}$ and $E_t = \emptyset$.

2. Find an edge with the smallest weight such that one of its end points is in S and the other is in $V \backslash V_t$. Add this edge to E_t and its new vertex to S.

Table 1: MST Search For The 14-Node Graph Using Prim's Algorithm

Iteration	1	2	3	4	5	6	7
New Edge	1-6	1-2	1-11	6-7	2-3	7-10	11-13
New Vertex	6	2	11	7	3	10	13
Iteration	8	9	10	11	12	13	
New Edge	13-14	3-9	2-4	11-12	12-8	4-5	
New Vertex	14	9	4	12	8	5	



(a) Weighted 16-node graph

(b) MST of the 16-node graph

Figure 1: The 16-node graph and its MST.

3. If $V \backslash V_t = \emptyset$, then terminate. Otherwise, go back to Step 2.

Prim's algorithm is applied to the 14-node graph shown in Fig. 1(a). The final spanning tree obtained using the MST algorithm is shown in Fig. 1(b).

3. Distribution System Reconfiguration: A Minimum Spanning Tree Problem

In this section, the MST algorithm is applied, as a heuristic method, to the DS reconfiguration problem. This problem aims at finding a radial topology which admits the minimum resistive losses. It is stated in [21] that the minimum-loss network is achieved when all the switches are closed. This statement, however, does not always hold. In order to show this the problem is solved without imposing the radiality constraint. The results of these simulations obtained by the MIQCP method proposed in [20] are reported in Section 4. The following assumptions are derived based on the engineering knowledge of DS and are used throughout this paper. When all the tie-switches are closed, forming a weakly-meshed network, then

- the losses are close (if not exactly the same) to the losses of the best-possible configuration.
- the load is distributed between the feeders and a moderate balance (but not the best-possible) among all the feeders is achieved.
- the voltage profile of the network is close to the best-possible voltage profile.

The validity of these assumptions is demonstrated through examples in Section 4. Note that loss reduction, load balancing between feeders, and voltage profile improvement are three different objectives and may lead to different configurations. However, loss reduction implicitly improves the voltage profile, as was also previously acknowledged in [4], and maintain a moderate balance between feeders, as was also recognized in [34].

It is shown here, mathematically, that the loss minimization is in alignment with voltage profile improvement. The total losses in a DS, as derived in [20], are obtained as:

$$P_{\text{loss}} = \sum_{(i,j)\in S_L} G_{ij}\left[(V_i^{\text{re}} - V_j^{\text{re}})^2 + (V_i^{\text{im}} - V_j^{\text{im}})^2\right] \tag{1}$$

where G is the branch conductance; S_L is the set of all the branches; V^{re} and V^{im} are the real and imaginary pars of the nodal voltages. The voltage at the substation node is usually considered to be a known quantity. Normally, the following values are assumed for the substation voltage: $V_s^{\text{re}} = 1$ and $V_s^{\text{im}} = 0$. Recall the fact from [20] that the nodal voltage magnitudes are mainly governed by their real parts, i.e. V^{re}. Minimizing (1) can be translated into minimizing the differences between the real parts of the nodal voltages. Since one of the voltages, i.e. V_s^{re}, is already fixed at 1 p.u., the other nodal voltages are also pushed to stay close to 1 p.u. to minimize the objective in (1). This argument reveals the fact that minimizing losses implicitly improves the voltage profile of a distribution network.

Let us look at a meshed DS as a weighted undirected graph, where the **negative** of the current magnitudes are taken as the weights. The problem of finding a tree for the meshed network that closely imitates it can be converted into a MST problem. The MST algorithm tries to keep the branches with the largest current magnitudes (note the negative sign) and open the branches with the least currents to form a tree. For example, take the 14-node graph shown in Fig. 1(a), which is the 14-node system of [2], with the current magnitudes shown for each branch. Figure 1(b) shows that the branches with the least currents, i.e. 5-14, 6-8, and 7-9 are open in the MST solution.

Although the above discussed method appears reasonable, network constraints, such as nodal voltage limits and feeder ampacities have not been explicitly considered. However, it is shown in Section 4 that the system operational constraints are implicitly satisfied in the final results. The flowchart of the proposed algorithm is shown in Fig. 2. Recall that the MST algorithm proposed here is meant to quickly find a good initial solution to be used in a direct mathematical approach, such as the MIQCP problem in [20]. Most

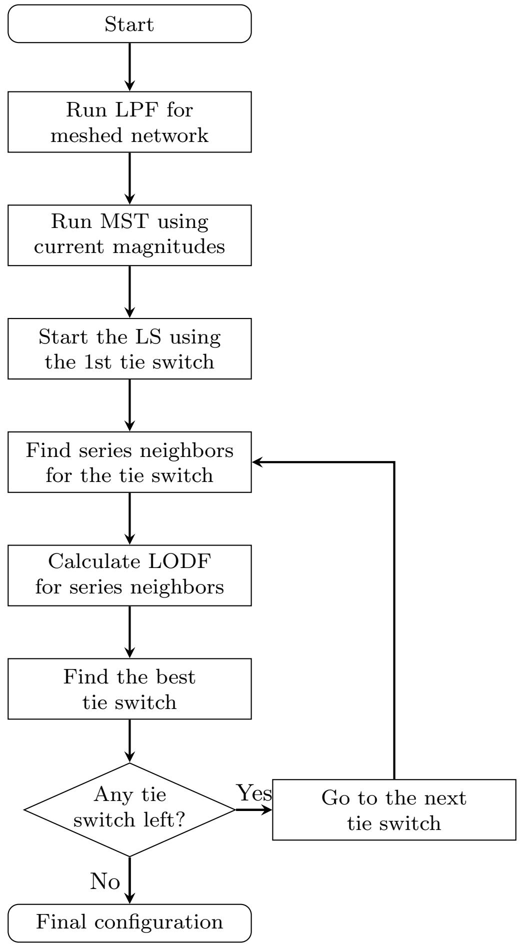


Figure 2: Flowchart of the MST+LS algorithm for network reconfiguration.

of the commercial solvers, e.g., CPLEX, have embedded heuristics to find an initial solution to start the branch-and-cut procedure. These heuristics need to provide a feasible solution in a short time. The proposed algorithm is to replace those general-purpose heuristics to quickly find a good initial solution.

3.1. Series Neighborhood Search

In order to further improve the optimality of the solution obtained by the MST algorithm, a local search (LS) is performed based on sensitivity factors derived using the linear power flow (LPF) algorithm of [23]. Each new tie switch obtained by the MST algorithm breaks a particular loop in the graph. The simulation results, discussed in the next section, show that the tie switches in the actual optimal solution are mostly in the series neighborhood of the tie switches obtained by the MST algorithm. Two branches are called "Series Neighbors" if their common node is only connecting those two branches. Mathematically, the degree of the common node must be two. Figure 3 shows examples of series neighbors (e.g., branches 3-4 and 4-5) and

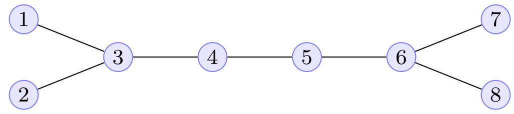


Figure 3: Visual examples of series and non-series neighbors.

non-series neighbors (e.g., branches 5-6 and 6-7). Note that one branch may have multiple series neighbors. For instance, branches 3-4, 4-5, and 5-6 are all in series. By closing an open switch and opening its series neighbor, the radiality of the solution is preserved, while the optimality of the solution may improve. Note that the branch exchange between two non-series neighbors may not necessarily lead to a tree and, hence, is not sought here.

In order to reevaluate the losses after each branch exchange, a new power flow solution is required. This is, however, computationally expensive to run a power flow after each branch exchange. The line outage distribution factors (LODF) are derived here to avoid running a new power flow solution. The LPF equations in complex form are given in [23] as:

$$\bar{Y}\,V = I \tag{2}$$

where V is the vector of complex nodal voltages; I is the vector of complex nodal current injections, and $\bar{Y}$ is the modified admittance matrix in which the diagonal elements contain the admittance part of the loads. Note that (2) is derived in [23] in rectangular coordinates, while here the complex form is used instead. In the network reconfiguration problem, the control variables are the switches which are usually three-phase operated units. Therefore, it suffices to use the balanced model of the network to formulate the power flow equations.

Suppose the voltages are obtained for an initial case using (2). Suppose also that a change occurs in the current injection at Node k. The aim here is to find the changes in the nodal voltages without solving (2) for the second time. Since (2) is linear, one can write

$$\Delta V = \bar{Z}\Delta I \tag{3}$$

where $\bar{Z}$ is the inverse of $\bar{Y}$, and is not the network impedance matrix. Note that $\bar{Y}$ is the admittance matrix of a network with non-zero branch impedances and current sources in parallel with some impedance connected at each node. Also, the substation is represented by a voltage source. From a circuit theory point of view, this network has a solution and, therefore, the inverse of $\bar{Y}$ exist. Now suppose it is desired to find the changes in the line currents. The current flowing through the line connecting Node i to Node j is calculated as

$$f_{ij} = (v_i - v_j)y_{ij} \tag{4}$$

Therefore, the changes in f_{ij} due to the changes in the current injection at Node k is calculated as

$$\Delta f_{ij} = (\Delta v_i - \Delta v_j) y_{ij} \tag{5}$$

Substituting the values of Δv_i and Δv_j from (3) into (5), one has

$$\Delta f_{ij} = (\bar{Z}_{ik} - \bar{Z}_{jk}) y_{ij} \Delta I_k \tag{6}$$

Now, assume a line is disconnected from the grid. In order to find the new power flow solution (2) has to be solved with a slightly modified $\bar{Y}$. To avoid the refactorization of $\bar{Y}$, sensitivity factors can be used. One way to model the outage of Line i-j (Fig. 4) is to inject an appropriate current source I_x into Node i and draw the same current from Node j such that the resulting current through Line i-j (f'_{ij}) is only circulating between the two added current sources, i.e. $I_x = f'_{ij}$. Under these conditions, the line can be considered as disconnected from the rest of the network. It can be verified by applying the Kirchhoff's Current Law (KCL) to Nodes i and j in Fig. 4. It is also important to note that by adding the new current sources to Nodes i and j, the flows in the rest of the network are also affected, which is equivalent to the disconnection of Line i-j. The new flow in Line i-j due to the added current sources can be calculated as

$$f'_{ij} = (v'_i - v'_j) y_{ij} = f_{ij} + (\Delta v_i - \Delta v_j) y_{ij} \tag{7}$$

Substituting the values of voltages from (3), the new flow becomes

$$f'_{ij} = f_{ij} + y_{ij}(\bar{Z}_{ii} + \bar{Z}_{jj} - 2\bar{Z}_{ij}) I_x \tag{8}$$

Remember that the aim is to have $f'_{ij} = I_x$. Therefore, the appropriate current injection at Nodes i and j would be

$$I_x = \frac{1}{1 - y_{ij}(\bar{Z}_{ii} + \bar{Z}_{jj} - 2\bar{Z}_{ij})} f_{ij} \tag{9}$$

By injecting the calculated I_x at both ends of Line i-j, this line can be considered as isolated from the rest of the network. The changes in the voltage of Node k due to this current injection is calculated using (3) as

$$\Delta v_k = \frac{\bar{Z}_{ki} - \bar{Z}_{kj}}{1 - y_{ij}(\bar{Z}_{ii} + \bar{Z}_{jj} - 2\bar{Z}_{ij})} f_{ij} \tag{10}$$

Finally, the changes in the current flowing through the other lines l-m is calculated using (5) as

$$\Delta f_{lm} = M_{ij,lm} f_{ij} \tag{11}$$

where $M_{ij,lm}$ is the LODF and is calculated as

$$M_{ij,lm} = \frac{(\bar{Z}_{li} - \bar{Z}_{lj}) - (\bar{Z}_{mi} - \bar{Z}_{mj})}{1 - y_{ij}(\bar{Z}_{ii} + \bar{Z}_{jj} - 2\bar{Z}_{ij})} y_{lm} \tag{12}$$

The new flows in the remaining lines after a line outage are calculated by adding the changes obtained in (11) to the initial line currents. For a network with d lines, all the $M_{ij,lm}$ form a $d \times d$ matrix with -1 on

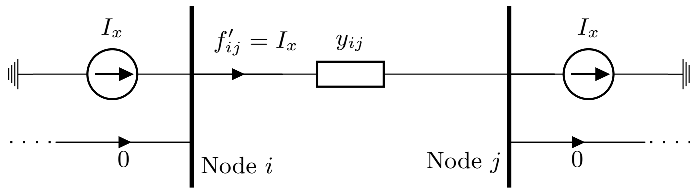


Figure 4: Line outage modeling using nodal current injections.

the diagonal elements. If disconnecting a branch creates an isolated part in the network, the LODF's will still admit a solution. This solution is equivalent to disconnecting all the nodes and branches fed by that particular branch. Note that the conventional methods, such as Newton-Raphson power flow, fail to find a solution if the network has disconnected parts.

Let S^{TS} represent the set of tie switches obtained by the MST algorithm with N^{TS} members. Assume that each tie switch k in S^{TS} has n_k^{TS} series neighbors. Thus, $\sum_{k=1}^{N^{\mathrm{TS}}} n_k^{\mathrm{TS}}$ power flow solutions are required to search for an improved solution. Instead, for every tie switch, the LODF's are calculated for its series neighbors assuming this particular switch is closed and all other tie switches are open. For instance, if 3-4 in Fig. 3 is a tie switch in the MST solution, both 4-5 and 5-6 are candidates for a branch exchange. The LODF for the outage of these two branches (i.e. 4-5 and 5-6), assuming all other tie switches are open, are calculated. The new current flow in Line l-m is then computed as:

$$f_{lm}^{\mathrm{new}} = f_{lm}^0 + f_{ij}^0 M_{ij,lm} \tag{13}$$

where f^0 and f^{new} are the vectors of initial and new current flows, respectively. Using the new currents, the new losses can be calculated as:

$$P_{\mathrm{loss}}^{\mathrm{new}} = \sum_{(l,m)\in S_L} R_{lm} |f_{lm}^{\mathrm{new}}|^2 \tag{14}$$

where S_L is the set of all the lines. The LODF calculation routine needs to be called only N^{TS} times, whereas in the normal case the power flow routine needs to be called $\sum_{k=1}^{N^{\mathrm{TS}}} n_k^{\mathrm{TS}}$ times. Also, each LODF evaluation needs less computation than running a normal power flow. The size of the LODF calculated for each tie switch depends on the number of series neighbors for that tie switch, which is obviously quite smaller than the total number of branches in the network. The flowchart of the proposed algorithm is shown in Fig. 2.

4. Simulation Results

In this section, the differences between losses for the meshed network and the best-known configuration are shown through several test cases to demonstrate the assumptions made in Section 3. Then, the proposed MST and LS algorithms are applied to sample distribution test systems to show their effectiveness.

4.1. Meshed Network Versus Best-Known Configuration

The MIQCP formulation of [20] is used here to obtain the optimal configuration by dropping the radiality constraint. The linear power flow equations used in [20] are derived in [23] based on the following assumptions: 1) loads are voltage-dependent elements; 2) voltage angles are small in DS's. The LPF algorithm is also capable of modeling constant-power load models by choosing appropriate parameters for the load model proposed in [23]. Several test systems are used and the problem is solved using CPLEX. Common test systems used in the literature are the 14-node [2], 33-node [3], 70-node [35], 84-node [36], 119-node [37], 135-node [38], 415-node [18], 873-node [39], and 10476-node [39] systems. The system dimensions, total loads, and initial losses are given in Table 2. Simulation results for the meshed network and the optimal solutions are provided in Table 3. As can be seen in Table 3, the difference between the losses in the meshed network (P_1) and the optimal solution (P_2) is negligible, which partially confirms the assumption in [21]. These results support the assumptions stated in Section 3.

4.2. Radial Configurations and Active Losses

The algorithm depicted in Fig. 2 is applied here to the test systems described in Table 2. The results of the analysis are reported in Table 4. The optimum solution for each case is obtained using the MIQCP method in [20]. In Table 4, ϵ_1 is the relative optimality gap for MST solution, defined as

$$\epsilon_1 = \frac{|P_{\text{opt}} - P_{\text{MST}}|}{P_{\text{opt}}} \times 100 \tag{15}$$

And ϵ_2 and ϵ_3 are defined similarly for the LS and DISTOP solutions. As can be seen, the optimality gaps for the MST solutions are mostly less than 3%. For the 10476-node system, the optimal solution is not known to the authors. Nonetheless, it is possible to provide a lower bound on the optimal solution using the losses in the meshed network, which is 7838.7 kW. This gives a gap of 4%, which guarantees that the solution provided by the MST routine is within an optimality gap of less than 4%. For the 10476-node system, the proposed algorithm provides 47.4% loss reduction, and the same value is also reported in [40] for loss reduction. However, this reduction in losses is achieved in [40] in 472s. The computation time of the proposed method is discussed later in this section.

The column denoted by ϵ_2 in Table 4 shows the relative gap for the local search (LS) results. In all the cases, LS has led to an improved solution as compared to the MST results. The rightmost column of Table 4 shows the optimality gap for the DISTOP solution. Since there is no explanation on how to check the connectivity of the network at each iteration, it was not possible for the authors to fully implement the DISTOP algorithm to compare CPU times. It was also not possible to check the connectivity constraint manually for the 10476-node system due to its dimensionality. Despite more computations required by this algorithm, the optimality of its solution is, in most of the cases, worse than the proposed MST+LS method in this paper.

Table 2: Dimensions of the Test Systems

Test Case	Branches	Feeders	Load(MVA)	Losses (kW)
14-node	16	3	$28.70 + j17.30$	514
33-node	37	1	$3.7 + j2.3$	202.7
70-node	79	4	$4.47 + j3.06$	227.5
84-node	96	11	$28.3 + j20.7$	532
119-node	132	3	$22.7 + j17.0$	1298.1
135-node	156	8	$18.31 + j7.93$	320.4
415-node	488	55	$141.75 + j103.5$	2660
873-node	900	7	$124.9 + j74.4$	3450.3
10476-node	10736	84	$1490.7 + j886.7$	15531.1

The tie switches in the MST solution and after applying the LS are listed in Table 5. The branches that are different from the optimal solution are shown in bold. It is easy to check that the branches in bold are electrically close to the ones in the optimal solution. For example, in the 33-node system, to get the optimal solution Branch 28 has to replaced by Branch 37, which have Node 29 in common, but are not in series. For the 70-node system, the following two replacements gives the optimal solution: $39 \rightarrow 70$ and $71 \rightarrow 79$. These pairs of branches have one node in common, but are not in series. Similar differences were observed for other cases.

The aforementioned results motivate the idea of focusing on the branches close to the MST+LS solution to find an improved solution in the branch-and-cut procedure. The initial solution is used as the incumbent solution which helps prune all subproblems for which the value of the objective function is no better than the incumbent. Moreover, in the branch-and-cut process, it is possible to issue priority orders for variables in the branching strategy. Variables with higher priorities will be branched on first. The knowledge obtained from the MST+LS algorithm about the variables that may provide an improved solution can be invoked here to assign priority orders. An appropriate variable ordering can result in speed up in the solution process by branching on more influential variables [25]. The above discussed idea will be implemented in a future work by the authors.

4.3. Voltage Profile

It is interesting to look into the voltage magnitudes in the initial configuration and the radial configuration found by the MST method. For illustrative purposes, the 135-node and 873-node systems are considered here. The voltage profiles for these test systems before and after reconfiguration are shown in Fig. 5. The minimum and average values for the voltage magnitudes are also shown in the figure. As can be seen, a

Table 3: Comparison of The Best Possible Configuration and Meshed Network

Test Case	P_1(kW)[1]	P_2(kW)[2]	ΔP(%)[3]	Open Lines
14-node	427.814	427.814	0	-
33-node	123.291	123.253	0.03	9
70-node	198.425	198.355	0.03	51, 75, 78
84-node	462.682	459.824	0.62	13, 63, 83, 84
119-node	819.359	818.342	0.12	23, 51, 75, 129, 130
135-node	271.846	270.83	0.37	84, 106, 135, 138
873-node	990.336	986.346	0.40	132, 241, 412, 451, 596, 639, 882, 888, 890, 900

[1] Losses for meshed network.

[2] Losses for best-known configuration. [3] Relative difference.

Table 4: Comparison of Network Losses Obtained by The Proposed Algorithms and Other References

#Nodes	Losses(kW)				ρ(%)[i]	ϵ_1[ii]	ϵ_2[iii]	ϵ_3[iv]
	MIQCP [20]	MST	LS	DISTOP [4]				
14	468.3	468.3	468.3	468.3	8.9	0	0	0
33	139.5	140.7	139.9	139.5	31.0	0.8	0.2	0
70	201.4	208.7	203.6	203.9	10.5	3.6	1.1	1.2
84	469.9	471.7	470.08	471.4	11.3	0.38	0.04	0.32
119	869.7	894.3	883.5	891.9	31.9	2.8	1.5	2.5
135	280.2	289.4	286.4	295.9	10.6	3.3	2.2	5.6
415	2352.4	2358.6	2355.3	2357.2	11.4	0.26	0.12	0.20
873	991.3	1001.9	1000.3	991.8	70.1	0.92	0.76	0.05
10476	N/A	8173.4	8158.1	-	47.5	N/A	N/A	N/A

[i] Loss reduction by MST+LS w.r.t the initial configuration.

[ii] Relative optimality gap for the MST solution (%). [iii] Relative optimality gap for the LS solution (%). [iv] Relative optimality gap for the DISTOP solution (%).

Table 5: Radial Configurations Obtained by The LS Algorithm

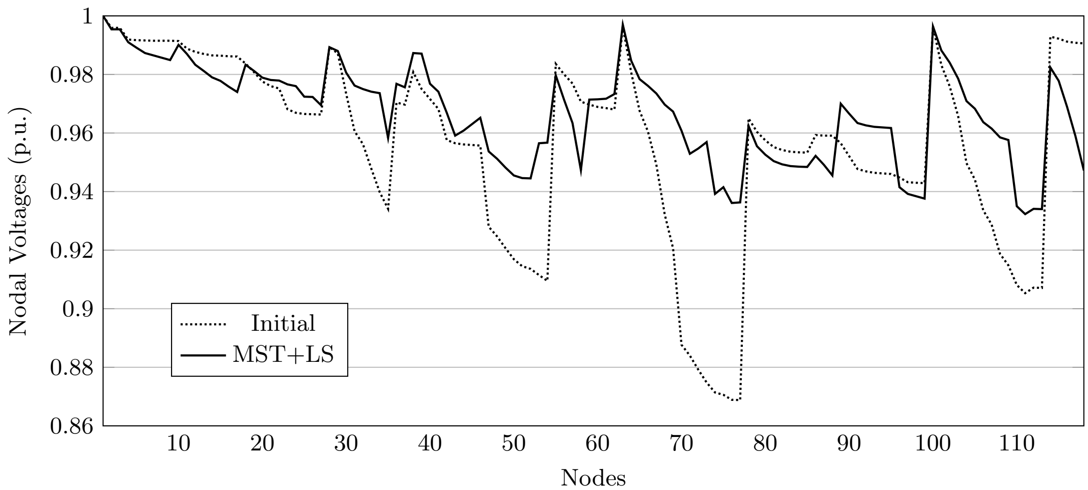
#Nodes	Tie Switches
14	7,8,16
33	7,9,14,**28**,32
70	14,30,**39**,45,51,66,**71**,75,76,77,78
84	7,34,39,42,**63**,72,83,**84**,86,89,90,92
119	23,26,34,39,42,**52**,58,**70**,**73**,**75**,95,109,122,129,130
135	**9**,35,51,**54**,90,96,106,126,135,**136**,138,141,**143**,144,145,146,147,148,150,151,155
415	7,**13**,**34**,39,42,63,72,**83**,84,86,89,90,92,103,**109**,**130**,135,138,159,168,**179**,180,182,185,186, 188,199,225,231,234,255,264,274,276,278,280,281,282,284,295,321,327,330,351,360,370,372, 374,376,377,378,380,391,417,423,426,447,456,466,468,470,472,473,474,476,481,482,483,484, 485,486,487,488
873	**84**,**130**,**144**,159,190,282,288,306,**330**,409,**412**,**434**,**451**,494,596,616,629,631,637,698,**818**, **843**,885,888,890,896,900

significant improvement in voltage profiles has been achieved. These observations support the idea that the radial network obtained by the MST routine provides improved voltage profiles.

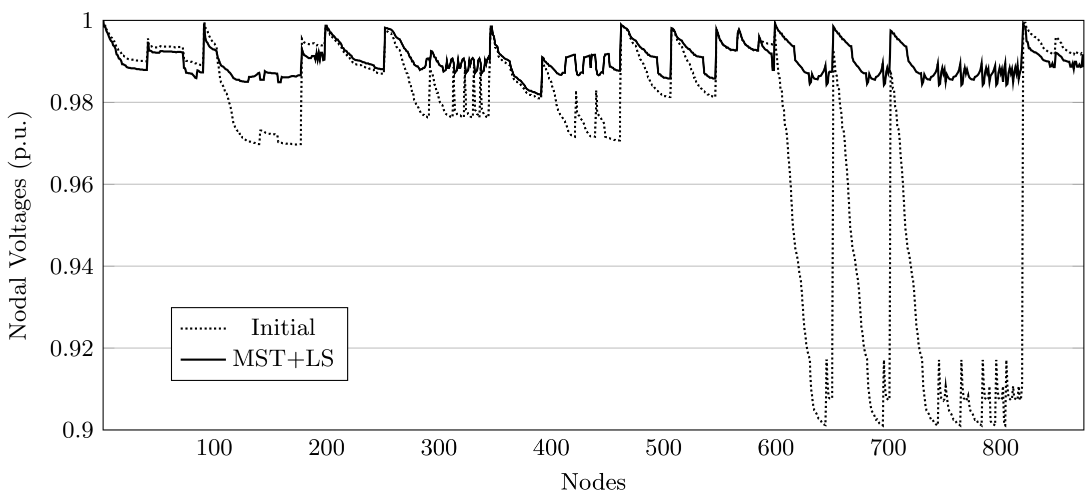
4.4. Feeders Loading

Although it is not its explicit objective, the minimum-loss network reconfiguration normally results in a moderate load balance among all the feeders [34]. However, there are cases in which a good load balance cannot be achieved due to the increment in the losses that would be imposed. Another case is when there is no connection between one cluster of feeders to the other cluster. In this case, every cluster can be optimized independently because they do not have any interdependencies. For instance, for the 84-node system in [36], Feeders F1, F7, and F8 are in one cluster and the rest are in another cluster. No branch exchange can be performed between these two clusters to reduce losses or improve the load balance. Therefore, maintaining a perfect load balance may not be achievable in some cases. For illustrative purposes, the feeder currents are shown in Fig. 6 for the 84-node and the 873-node systems, when the network is reconfigured for loss reduction using the MST+LS method. As can be seen, the MST+LS solution generates fairly close currents to the ones in the optimal solution.

It is of importance to notice that the main objective of the reconfiguration here is to minimize the losses, not to find the best-possible load balance among the feeders. In order to balance the load and minimize the losses simultaneously, a multi-objective optimization problem needs to be solved. The solution provided by the MST+LS routine is also a good initial solution to start such multi-objective optimizations and reduce



(a) The 119-node system



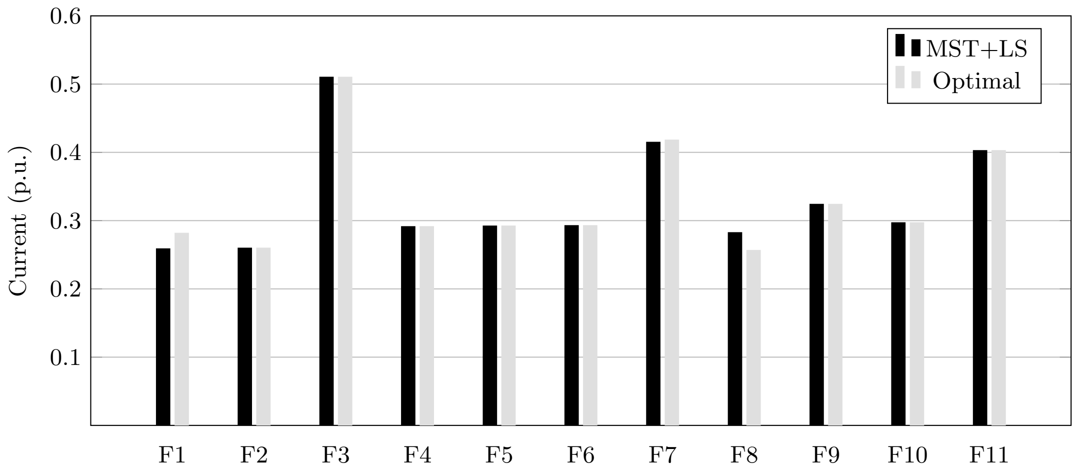
(b) The 873-node system

Figure 5: Voltage profiles for the 119-node and 873-node systems for the initial and obtained configurations.

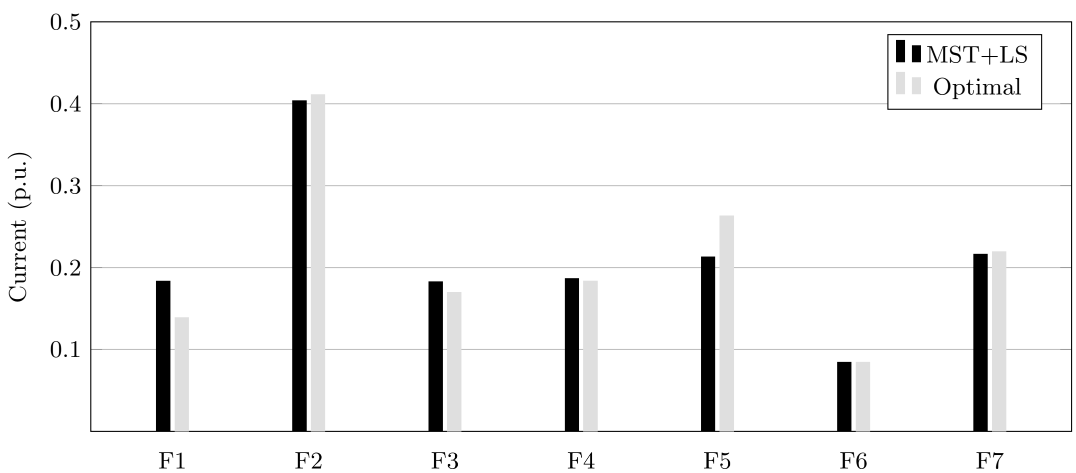
the computational burden of the branch-and-cut procedure.

4.5. Computational Complexity

As discussed in Section 2, the time complexity of Prim's algorithm to find the MST is $\mathcal{O}(e \log v)$. The MST algorithm and the local search (LS) algorithm were implemented in the MATLAB platform on an Intel Core i7-2600 CPU @ 3400 MHz with 8GB of RAM. The average CPU times for each test system is divided into the MST time and LS time, as shown in Table 6. The computation times of the proposed method are, by orders of magnitude, smaller than many previously proposed heuristics. Also, the scalability of the MST



(a) The 84-node system



(b) The 873-node system

Figure 6: Feeders currents for the MST+LS and the optimal solutions.

method is a significant advantage. There are only a few references in the literature that use systems with over 10,000 nodes to demonstrate the applicability of their method on large-scale systems. The CPU times in Table 6 shows that the proposed method is able to find the solution for a system with over 10,000 nodes in about a second, where to achieve the same optimality, the algorithm in [40] requires 472s.

The speed up in the DS reconfiguration problem is achieved by sacrificing some possible optimality of the solution. However, this is a fast and robust algorithm which provides a good initial solution to start the more sophisticated algorithms, such as the MIP algorithms in [18]-[20], [22], and [24], to find the optimal solution.

Table 6: Comparison of CPU Times for Different Methods

System	MST	LS	Reference 1	Reference 2
33-node	0.015	0.022	7.2 ([9])	0.3 ([29])
70-node	0.021	0.085	3 ([35])	2.4 ([41])
84-node	0.023	0.151	36.15 ([36])	0.3 ([29])
119-node	0.023	0.261	8.61 ([9])	9.04 ([37])
135-node	0.026	0.551	7.3 ([18])	0.4 ([29])
873-node	0.048	4.85	20.9 ([40])	874 ([19])
10476-node	1.360	37.11	472 ([40])	-

5. Conclusion

The problem of distribution system reconfiguration for loss reduction is mapped into a minimum spanning tree (MST) problem, for which efficient solution methods have been developed in the literature. A relative optimality gap of less than 2.2% was achieved for several test systems. The main highlights of the present framework are summarized here, as follows:

- The proposed algorithms guarantee the radiality of the final configuration.
- The MST algorithm is a low-cost heuristic that provides a solution with small optimality gap.
- The proposed local search method which is based on sensitivity factors is able to quickly polish the MST solution.
- The MST+LS solution can be readily used as an initial solution for a direct mathematical optimization such as MIP methods.

The application of the proposed heuristics to improve the branch-and-cut procedure is currently under development by the authors. By applying the MST+LS algorithm instead of the default heuristics applied in commercial solvers, it is expected to substantially reduce the processing time.

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