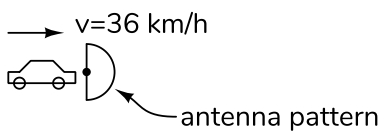
ELEX 7860 : Wireless System Design
2021 Winter Term

Solutions to Midterm Exam 1

Question 1

An antenna mounted on a vehicle receives only from directions within ±90 degrees of the direction of travel. If we make the same assumptions as Clarke's model for the distribution of scatterers, sketch the shape of the power spectrum (power spectral density versus frequency) that you would expect to see for an unmodulated carrier.

Label the frequency axis if the carrier frequency is 900 MHz and the vehicle speed is 36 km/h.



Answer

The assumptions in Clarke's model about the distribution of scatterers is *"a large number of signal paths arriving from directions that are uniformly distributed in a circle around a moving receiver."* The Doppler shift for each path is $f_m \cos(\theta)$ where θ is the angle relative to the direction of motion. In this case θ is uniformly distributed over $-\pi/2$ to $\pi/2$ and the shape of the spectrum will be the positive portion of the "bathtub" curve shown in the lecture notes.

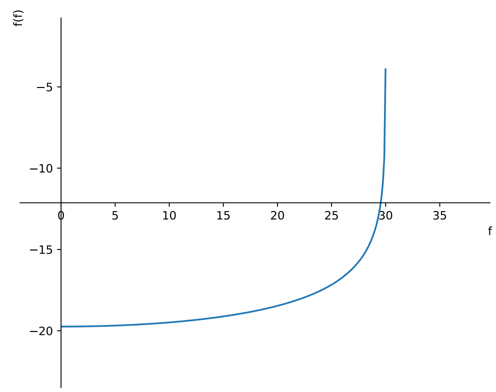
The maximum Doppler shift is $f_m = \frac{v}{c} \cdot f_c = (10)/(3 \times 10^8) \cdot 9 \times 10^8 \approx 30$ Hz.

The Doppler shift is given by $f_m \cos(\theta)$ where θ is the angle to the direction of motion. In this case the angle of arrival is uniformly distributed between $-\pi/2$ and $\pi/2$ and so the value of the pdf is $1/\pi$ and the frequency components increase from $-f_m$ to f_m.

If the angle of arrival were uniformly distributed over 0 to 2π the shape of the curve would be the Arcsine distribution which, in this application is given by:

$$S(f) = \frac{2}{\pi f_m \sqrt{1 - \left(\frac{f}{f_m}\right)^2}}$$

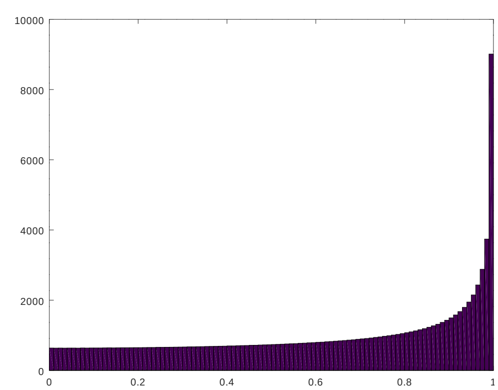
However, in this case the antenna only receives from directions in the direction of travel so only the positive frequency components will be received and the curve is only the positive half. In dB this looks as follows:



As a check, we can approximate the pdf by computing the histogram of a cosine over the range $-\pi/2$ to $\pi/2$:

```
hist(cos(-pi/2:pi/1e5:pi/2),100)
```

which gives this:



Question 2

What is the ratio of the level crossing rate (N_R) to the maximum Doppler rate (f_m) when the threshold level is equal to the mean signal level ($\rho = 1$)?

Answer

The level crossing rate is given by:

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

so solving for the ratio N_R/f_m when $\rho = 1$:

$$\frac{N_R}{f_m} = \frac{\sqrt{2\pi}}{e} \approx 0.92$$

which means the mean signal level is crossed at a rate approximately equal to the (maximum) Doppler rate.

Question 3

The loss due to propagation through walls in a building can be modelled as having a log-normal distribution. If the standard deviation of this loss is 10 dB, what fraction of locations will have a loss of more than 20 dB above the mean due to the log-normal fading?

Answer

The probability that a normal random variable, z, with mean m and standard deviation σ will exceed the value γ is:

$$\Pr[z > \gamma] = \frac{1}{2}\text{erfc}\left(\frac{\gamma - m}{\sqrt{2}\sigma}\right)$$

In this question $\sigma = 10$ and $\gamma - m$ is 20 so

$$= \frac{1}{2}\text{erfc}\left(\frac{20}{\sqrt{2} \cdot 10}\right) =$$

which can be computed using, for example, Matlab:

```
0.5*erfc(20/(sqrt(2)*10))
```

to be about 2.3%.

Question 4

(a) A cellular telephone system suffers from Rayleigh fading. The mean SNR is 20 dB, and the minimum SNR to detect the base station signal is 10 dB. What is the probability of not detecting the base station signal if diversity is not used?

(b) What is the probability of not detecting the base station signal when using three-branch space diversity with switched-diversity combining, assuming independent fading on each branch?

Answer

(a) From the Rayleigh CDF with $\rho = 10^{\frac{10-20}{20}} = \frac{1}{\sqrt{10}}$, the probability that the signal is not detected because it's below the threshold is: $1 - e^{-\rho^2} = 1 - e^{-0.1} \approx 0.095$.

(b) With three independently-fading branches and switched-diversity combining the probably of all three being faded is the product of the three probabilities or $(0.095)^3 \approx 0.9 \times 10^{-3}$.

Question 5

A garage door opener transmits a 10 mW signal at a frequency of 450 MHz. How much power is received at a distance of 10 m assuming the transmit and receive antenna gains are each −6 dB?

Answer

Using the Friis equation:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

with $P_T = 10$ mW, $\lambda = c/f = 300 \times 10^6/450 \times 10^6 = 2/3$ m, $G_T = G_R = 10^{-6/10} \approx 1/4$, and $d = 10$ m the received power is:

$$P_R = 10 \times 10^{-3} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \left(\frac{2/3}{4\pi \cdot 10}\right)^2 \approx 17.6 \text{ nW}$$

$$\approx -47.5 \text{ dBm}$$

Question 6

You would like to implement a space diversity system but can only receive on one antenna at a time. What type(s) of diversity combining can you use?

Answer

Of the three types of diversity combining studied in the course (combining, selection and switching) only switching diversity can be implemented with a single receiver.