OFDM

Modulation

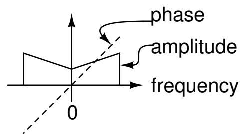
The purpose of modulation is to shift the spectrum of a baseband signal to a higher frequency so it can be transmitted through a bandpass channel.

For example, in AM broadcasting a baseband signal (0-10 kHz) is shifted to a carrier frequency between 0.6 and 1.6 MHz, so it can propagate over free space.

This frequency shift is accomplished by multiplying the baseband signal with a sinusoidal signal at the carrier frequency, f_c.

Complex Modulation

From Fourier analysis we know that real signals have a spectrum with even symmetry in amplitude and odd symmetry in phase:



Modern transmitters and receivers often process signals in complex form. These complex signals do not need to be symmetrical. For example, a complex sinusoidal carrier $e^{j2\pi f_c t}$ has a Fourier transform $\delta(f - f_c)$. By multiplying a complex baseband signal by a complex sinusoid we can obtain a passband signal that does not need to be symmetric about the carrier.

Digital vs. Analog Modulation

Digital modulation is used by most modern communication systems because of its advantages:

- better power efficiency due to use of digital modulation and channel coding (FEC)
- better spectral efficiency due to use of source coding (compression)
- more flexibility because we can transmit any information including digitized speech and images
- enhanced security through encryption, authentication and integrity checks

Analog modulation today is restricted to legacy systems that would be expensive to replace (e.g. aircraft still use AM for air traffic control) or very low-cost systems (e.g. baby monitors).

Note that implementation technology and modulation technique are independent. Either analog or digital circuits can be used to implement either analog or digital modulation. For example an FSK signal can be generated with an analog VCO (voltage controlled oscillator) or a digital NCO (numerically controlled oscillator).

ISI

If the delay spread of the channel is a significant fraction of the symbol period then the receiver will see a signal that is composed of overlapping copies of the transmitted symbols. This Inter-Symbol Interference (ISI) may degrade the performance of the receiver.

Wireless communication systems need to deal with ISI caused by multipath propagation.

There are many ways to correct for ISI. These include:

- linear equalizers - the received signal is passed through an FIR filter that attempts to remove the effect of the channel on the received signal
- decision-feedback equalization - previously demodulated data is passed through a filter that has the same impulse response as the channel and the estimated ISI on subtracted from the subsequently received signal, and
- sequence estimation - we estimate the most likely sequence that was transmitted based on the received signal and a knowledge of the channel impulse response

However, these techniques require that either the transmitter or receiver estimate the impulse response of the channel. This is difficult when the channel is rapidly changing such as when a device is moving.

OFDM

The trend for modern communication systems is to avoid the need to estimate the channel impulse response by using a technique called Orthogonal Frequency Division Multiplexing (OFDM).

An OFDM transmitter collects N groups of bits ("subsymbols") at a time and uses them to modulate N "subcarriers." These subcarriers are transmitted in parallel over approximately the same time duration that would have been required to transmit the N subsymbols serially. The net effect is to reduce the symbol rate by a factor N but with no significant impact on the overall bit rate.

OFDM is used by many systems that must cope with ISI including voiceband telephone modems, digital broadcasting, ADSL, WLAN and cellular radio systems.

On the other hand, communication systems operating over LOS links such as with satellites and over co-axial cables do not experience significant amounts of ISI and can use serial modulation such as conventional PSK and QAM instead of OFDM.

OFDM subcarriers are orthogonal because their frequencies are multiples of the inverse of the symbol duration. For example, if the symbol duration is $T_s = 250\ \mu$s the the subcarriers would be at frequencies that are multiples of 4 kHz (i.e. 4, 8, 12, ... kHz).

Exercise 1: Show this by finding the integral over a duration T_s of any two subcarriers with arbitrary phase and amplitude.

Exercise 2: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$. What is the subcarrier frequency spacing?

This orthogonality makes it possible to extract an individual subcarrier by multiplying the composite signal by the desired subcarrier and integrating over the period of one symbol. Only the desired subcarrier will contribute to the result of the integral.

Implementation

OFDM is implemented using an N-point Discrete Fourier Transform (DFT). The Fast Fourier Transform (FFT) is an efficient (O($n \log n$) algorithm for computing the DFT. The existence of the FFT algorithm makes OFDM practical for many applications.

Exercise 3: How much more computation is required to compute a DFT ($O(N^2)$) versus an FFT ($O(N \log_2(N))$) for $N = 64$? For $N = 1024$?

The DFT is defined as:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{j2\pi}{N}kn}$$

and the inverse DFT as:

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{j2\pi}{N}kn}$$

Where X_k is the k'th (complex) subcarrier and x_n is the n'th time-domain sample. Note that the only difference between the two equations is the scaling and the sign of the complex exponential.

The value of N is typically a power of 2 to allow efficient implementation using the Fast Fourier Transform (FFT) algorithm. Not all of the subcarriers need to be used. Those that are not needed are set to zero at the transmitter and ignored at the receiver. This makes it easy to accurately control the occupied bandwidth.

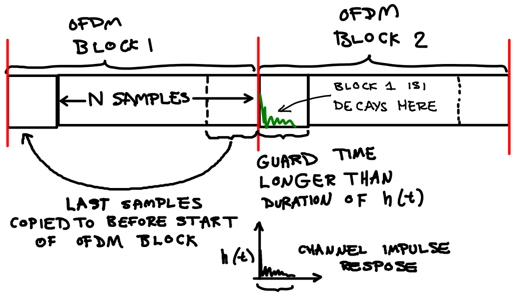
Exercise 4: The 802.11g specification uses 52 of the $N = 64$ possible subcarriers and omits both the DC (zero frequency) and the highest-frequency subcarriers. What is the bandwidth of the signal?

Any signalling constellation (e.g BPSK or 64-QAM) can be applied to the subcarriers. Frequently, different modulation and different powers are used on different subcarriers to adjust to different channel gains and to interference that might be different at different frequencies.

Cyclic Extension

The use of OFDM by itself reduces the impact of ISI but does not eliminate it. We can eliminate ISI completely by leaving a "guard time" (or "guard interval") before each symbol. If this time is longer than the duration of the channel impulse response then the ISI from the previous symbol will decay before the start of the symbol. We can avoid ISI by sampling the received signal starting after the ISI from the previous symbol has ended

For practical reasons it's common to extend each symbol into the guard interval rather than not transmitting anything at all during the guard interval. This is called a "cyclic" or "periodic" prefix. Since the symbol is periodic with period equal to the symbol duration, we can sample any N-sample portion of this extended symbol without affecting the orthogonality or performance of the system.



Since no (new) data is transmitted during the guard time, this reduces the average data rate. However, the guard time is typically a small fraction of the OFDM symbol duration and so the impact on the overall throughput is relatively small.

Exercise 5: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

Synchronization

Synchronization of OFDM signals involves two steps. The first is time synchronization: selecting a set of N samples that does not suffer from ISI. The second is estimating the amplitude and phase shift of the channel at each subcarrier frequency so they can be reversed (sometimes called a "single tap" equalizer).

Synchronization for frame-based systems such as 802.11 typically uses a preamble and pilot subcarriers. Synchronization for continously-transmitting systems such as LTE uses periodically-transmitted synchronization symbols as well as pilot subcarriers.

Since multiplication in frequency is the same as convolution in time, the DFT can be used to efficiently detect a known (short) pilot symbol by correlating the input signal with the known pilot symbol.

Exercise 6: The 802.11g preamble contains a "short" followed by a "long" training symbol. The short symbol contains only every fourth subcarrier. What is the period of this symbol? The long training symbol contains fixed data on each of the data subcarriers. How would you use the long training symbol to correct the phase and amplitude of subsequently-received data subcarriers?

Interpolation between regularly-spaced pilot tones can be used to estimate the amplitude and phase of the channel. These pilots often "walk" between subcarriers in a predefined sequence to allow more accurate estimation of the channel.

Coded OFDM

The frequency response of a channel is the Fourier transform of its impulse response. If the channel's impulse response is not an impulse then it will be frequency-selective.

Multipath channels where the different paths have different delays (called "dispersive" channels) are therefore frequency-selective channels.

This means the gain of the channel can be different for different subcarriers of an OFDM signal. This will result in different errors rates.

FEC is used to address this issue. Typically, all of the bits transmitted in each OFDM block, regardless of which subcarrier they were transmitted on, are interleaved and FEC-coded. This is called bit-interleaved coded modulation. The FEC code corrects errors introduced on faded subcarriers and thus provides a type of frequency diversity.

Exercise 7: A channel's impulse response is two equal-level impulses separated by 100 ns. What difference in propagation path lengths would result in such an impulse response? How far apart are the nulls of this channel? What OFDM signal bandwidth(s) would be required to provide frequency diversity?

Subcarrier Modulation

Each subcarrier is modulated using M-ary QAM (this means using a constellation with M points to transmit $\log_2(M)$ bits per subcarrier. As with any other use of QAM, the constellations are Gray-coded to reduced the BER which improves the performance of bit-interleaved modulation.

Exercise 8: How many bits per subcarrier are transmitted by an OFDM system using 16-QAM? Assuming equal noise powers, how much more power does this system need to achieve the same BER than a system using 4-QAM? Than a system using BPSK?

On channels where the SNR is different for different subcarriers (e.g. due to frequency-selective fading resulting from multipath), each carrier can use a different value of M. This is commonly used with ADSL where the channel multipath does not change over time (it is due to taps on twisted-pair transmission lines).

OFDMA

It's possible to allocate different subcarriers to different users. This allows a single high-rate channel to be divided up among users while still retaining the frequency-diversity advantages of using a wide-band signal.

On the uplink, different users can be allocated different subcarriers. This that all user transmissions be synchronized.

Advantages

Avoids ISI As explained above, one advantage of OFDM is that the reduced symbol rate makes it practical to leave inter-symbol guard times that avoid ISI.

Beamforming and MIMO Another advantage is that each subcarrier can be considered to have a single amplitude and phase. This makes it possible to coherently combine multiple OFDM signals received from multiple antennas. This enables implementation of space diversity, beamforming and spatial multiplexing (MIMO).

OFDMA Another advantage is that it's possible to divide up a channel among multiple users by allocating different sets of subcarriers to different users. This allows very fine control over bandwidth allocation and is known as OFDMA.

Flexible Bit and Power Allocation Another advantage is that it's possible to transmit different amounts of power and use different modulation on different subcarriers. This can be used to make some subcarriers more reliable (e.g. for pilots) or to compensate for frequency-varying attenuation, noise or interference. This also makes it simpler to meet transmit spectral mask requirements.

Narrow Power Spectrum The spectrum of each subcarrier is proportional to the inverse of the symbol duration which is N time narrower than the spectrum of a serial modulation. This makes it much simpler to produce a signal with a power spectrum that is confined to the desired channel bandwidth. This is done by selecting the number of subcarriers that are transmitted

The shape of each subcarrier's spectrum is controlled by windowing each symbol in time (e.g. with a Hamming window). This is done by gradually decreasing the level of one symbol and increasing the level of the next one during the cyclic extension.

In addition, the DC subcarrier is often not used to avoid implementation issues (the presence of carrier due to a DC offset in mixer inputs).

Disadvantages

High PAPR The main disadvantage of OFDM is the high peak-to-average power ratio (PAPR). This is due to adding up a large number of sinusoids (subcarriers) each of which has a random phase and amplitude due to the subcarrier modulation. The Central Limit Theorem implies that, as with Rayleigh multipath fading, the pdf of the amplitude will be Rayleigh with a high maximum amplitude.

Practical systems will limit the peaks of the envelope resulting in interference between subcarriers and to adjacent channels. To limit this, amplifiers need to be operated an an average power that is significantly less than their peak power. This typically results in lower power efficiency compared to systems that operate with constant envelope amplitudes.

Effects of Time-Varying Fading Another disadvantage is that if the channel gain (amplitude or phase) changes during the transmission of a symbol the subcarriers will no longer be sinusoids and will no longer be orthogonal. This will result in interference between the subcarriers.