OFDM

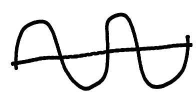
Exercise 1: Show this by finding the integral over a duration T_s of any two subcarriers with arbitrary phase and amplitude.

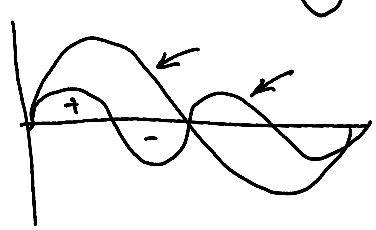
$$\int_0^{T_s} A\cos\left(2\pi \frac{i}{T_s}t+\phi\right)\cdot B\cos\left(2\pi\frac{j}{T_s}t+\theta\right)dt \overset{?}{=} \begin{cases}0 & i\neq j\\ \neq 0 & i=j\end{cases}$$

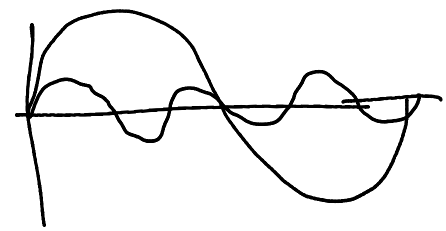
$$AB\int_0^{T_s}\cos\left(2\pi\frac{(i\pm j)}{T_s}t+\alpha\right)dt + AB\int_0^{T_s}\sin\left(2\frac{(i\pm j)}{T_s}t+\beta\right)dt$$

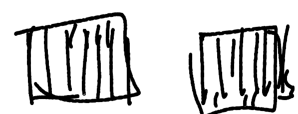
$$\text{for}\begin{cases} i=j & AB\int_0^{T_s}\cos(\alpha)\,dt + AB\int_0^{T_s}\sin(\beta)\,dt \neq 0\\ i\neq j & AB\int_0^{T_s}\cos(k\,2\pi t)\,dt = 0\end{cases}$$

$$K = i\pm j,\ i\neq j$$



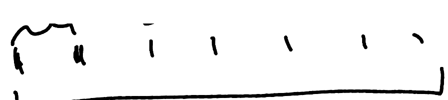






50ns

Exercise 2: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$. What is the subcarrier frequency spacing?



64

$$T_s = 64\cdot\frac{1}{20\,MHz} = 3.2\,\mu s.$$

$$\text{subcarrier spacing} = \frac{1}{3.2\,\mu s} = 312.5\ kHz$$

Exercise 3: How much more computation is required to compute a DFT ($O(N^2)$) versus an FFT ($O(N\, log_2(N))$) for $N = 64$? For $N = 1024$?

$O(N^2)$ – DFT

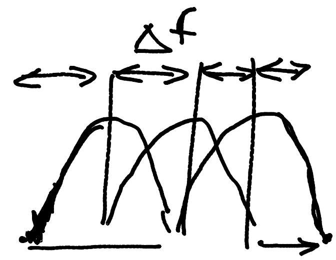
$O(N \log_2 N)$ – FFT

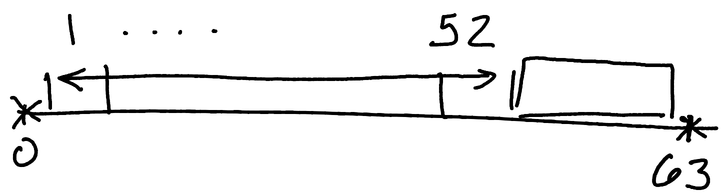
$$\frac{N^2}{N \log_2 N} = \frac{N}{\log_2 N}$$

$N=64 \qquad \frac{64}{6} \approx 10$ ✓

$N=1024 \qquad \frac{1024}{10} \approx 100$ ✓

Exercise 4: The 802.11g specification uses 52 of the $N = 64$ possible subcarriers and omits both the DC (zero frequency) and the highest-frequency subcarriers. What is the bandwidth of the signal?





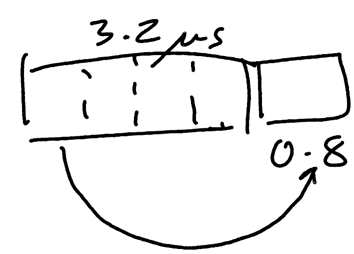
bandwidth = $52 \cdot 312.5$ kHz

(actually $53 \cdot 312.5$ kHz if considering null–null bandwidth).

Exercise 5: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

$T_s = \frac{1}{20\text{ MHz}}$

$T_{symbol} = N T_s = \frac{64}{20\text{ MHz}} = 3.2\,\mu s.$

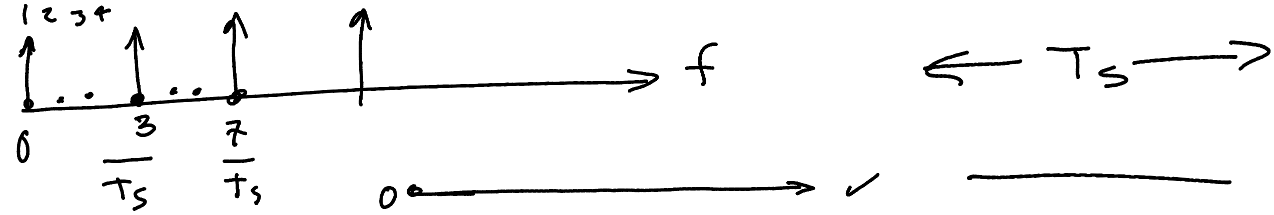


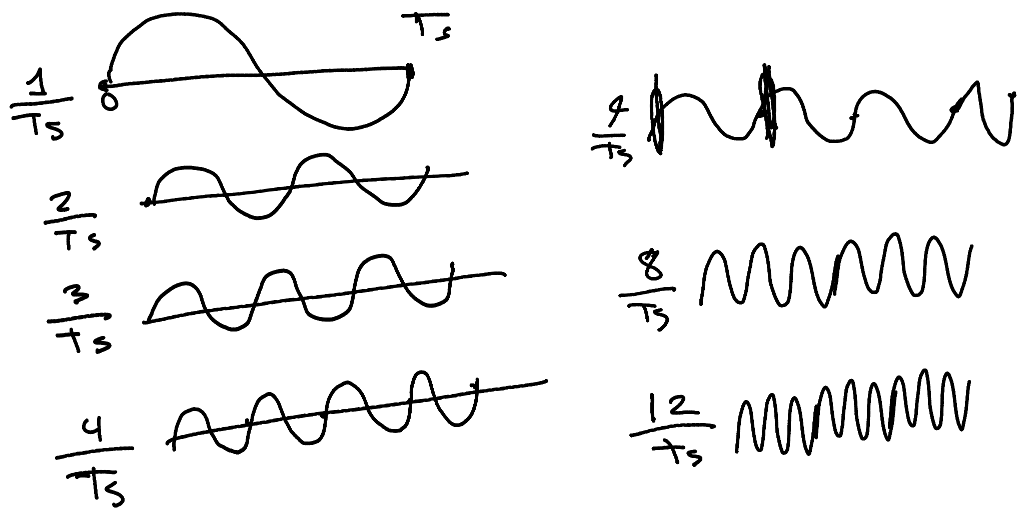
duration = $3.2 + 0.8 = 4.0\,\mu s$

Guard time in samples =

$$\frac{0.8\mu s}{T_s} = 16 \text{ samples.}$$

Exercise 6: The 802.11g preamble contains a "short" followed by a "long" training symbol. The short symbol contains only every fourth subcarrier. What is the period of this symbol? The long training symbol contains fixed data on each of the data subcarriers. How would you use the long training symbol to correct the phase and amplitude of subsequently-received data subcarriers?



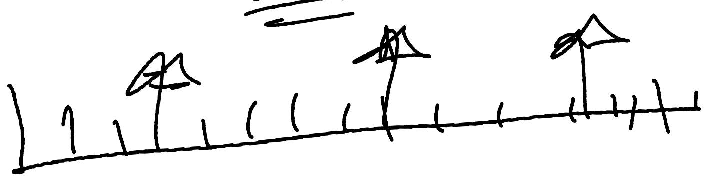


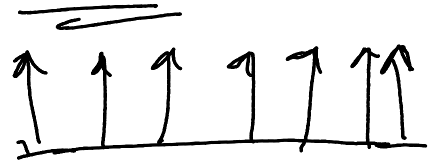
period of pilot = $T_s/4$

<u>short</u>

period = T_s

<u>long</u>





if transmitted $1+0j = 1\angle 0$

received $|r| \angle \theta$

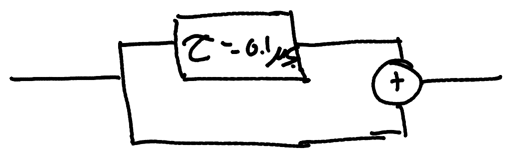
then to correct received data. multiply by $\underline{\underline{\frac{1}{|r|}}}\ \underline{\underline{\angle -\theta}}$

or $\frac{1}{r^*}$ ("single tap") equalization

$1\,ns = 1\,ft.$ ←

$300\,m/\mu s$

Exercise 7: A channel's impulse response is two equal-level impulses separated by 100 ns. What difference in propagation path lengths would result in such an impulse response? How far apart are the nulls of this channel? What OFDM signal bandwidth(s) would be required to provide frequency diversity?



path length diffence $\simeq$ 30 m ($\simeq 100'$).

phase shift = $\Delta\theta = 2\pi f \tau$

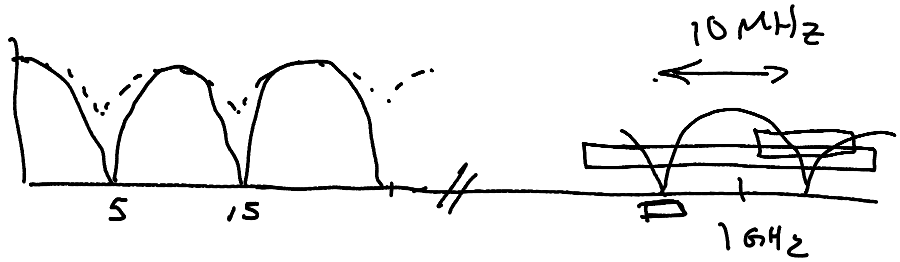
the two paths cancel when

$\Delta\theta = \pm\pi,\ \pm(2n+1)\pi$ $\quad \pm\pi, \pm 3\pi, \pm 5\pi$

$$\cancel{2\pi} f \cancel{\tau} = \frac{(2n+1)\cancel{\pi}}{2} = \frac{1}{\tau}\left(n + \frac{1}{2}\right)$$

for $\tau = 0.1\,\mu s$ $\quad \frac{1}{\tau} = 10\,MHz$

$f = 5 + 10n$ MHz

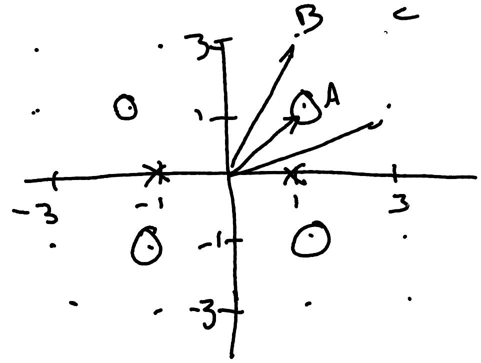


OFDM b/w $\gg \frac{1}{\tau}$

in this case $\gg 10\,MHz$

Exercise 8: How many bits per subcarrier are transmitted by an OFDM system using 16-QAM? Assuming equal noise powers, how much more power does this system need to achieve the same BER than a system using 4-QAM? Than a system using BPSK?

$\log_2 16 = 4$ bits / subcarrier.

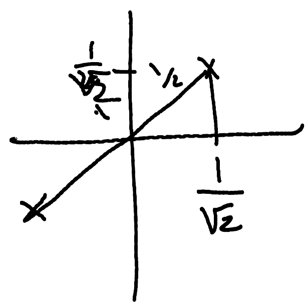


4-QAM power $= \left(\sqrt{1^2+1^2}\right)^2 = (\sqrt{2})^2$
$= 2$

average 16-QAM power

$$= \frac{(1 \cdot \overset{A}{2}) + (2 \cdot \overset{B}{10}) + (1 \cdot \overset{C}{18})}{4}$$

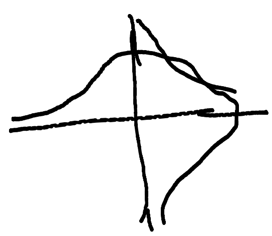
$$= \frac{1 + 20 + 18}{4} \approx 10$$

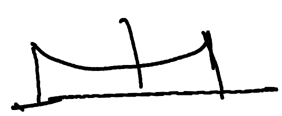


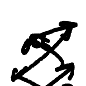
BPSK power = 1

<u>BUT</u> we're transmitting many subcarriers

→ Rayleigh distributed amplitude







peak power limitation.