OFDM

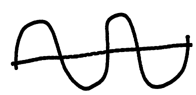
Exercise 1: Show this by finding the integral over a duration T_s of any two subcarriers with arbitrary phase and amplitude.

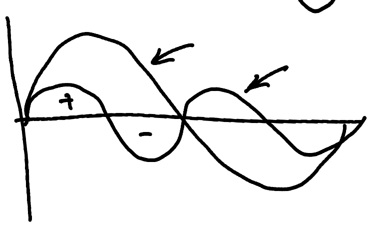
$$\int_0^{T_s} A\cos\left(2\pi \frac{i}{T_s}t+\phi\right)\cdot B\cos\left(2\pi\frac{j}{T_s}t+\theta\right)\,dt \overset{?}{=} \begin{cases} 0 & i\neq j \\ \neq 0 & i=j \end{cases}$$

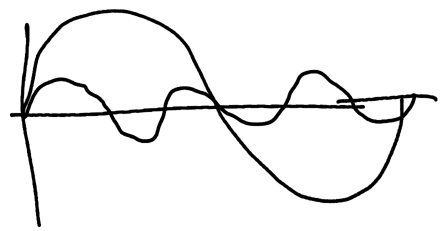
$$AB\int_0^{T_s} \cos\left(2\pi\frac{(i\pm j)}{T_s}t+\alpha\right)dt + AB\int_0^{T_s} \sin\left(2\frac{(i\pm j)}{T_s}t+\beta\right)dt$$

$$\text{for}\begin{cases} i=j & AB\int_0^{T_s}\cos(\alpha)\,dt + AB\int_0^{T_s}\sin(\beta)\,dt \neq 0 \\ i\neq j & AB\int_0^{T_s}\cos(k\,2\pi\,t)\,dt = 0 \end{cases}$$

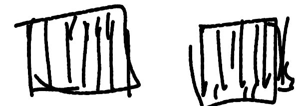
$K = i\pm j,\ i\neq j$

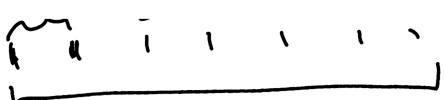






Exercise 2: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$. What is the subcarrier frequency spacing?





$$T_s = 64\cdot\frac{1}{20\,MHz} = 3.2\,\mu s.$$

subcarrier spacing $= \dfrac{1}{3.2\mu s} = 312.5$ kHz

$O(N^2)$ – DFT

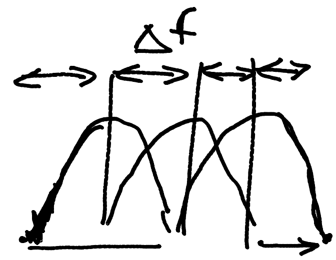
$O(N \log_2 N)$ – FFT

Exercise 3: How much more computation is required to compute a DFT ($O(N^2)$) versus an FFT ($O(N\,log_2(N))$) for $N = 64$? For $N = 1024$?

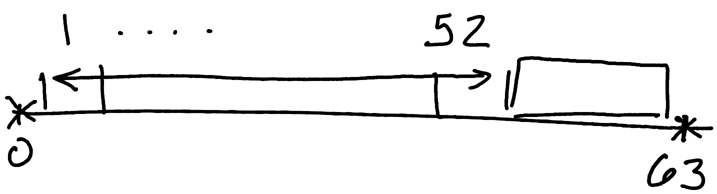
$$\frac{N^2}{N \log_2 N} = \frac{N}{\log_2 N}$$

$N=64$ $\quad \frac{64}{6} \approx 10$ ✓

$N=1024$ $\quad \frac{1024}{10} \approx 100$ ✓



Exercise 4: The 802.11g specification uses 52 of the $N = 64$ possible subcarriers and omits both the DC (zero frequency) and the highest-frequency subcarriers. What is the bandwidth of the signal?

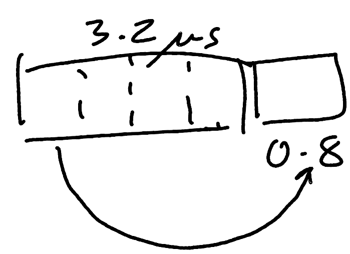


bandwidth = $52 \cdot 312.5$ kHz

(actually $53 \cdot 312.5$ kHz if considering null–null bandwidth).

Exercise 5: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

$T_s = \frac{1}{20\text{ MHz}}$ $\quad T_{symbol} = N T_s = \frac{64}{20\text{ MHz}} = 3.2\mu s$.

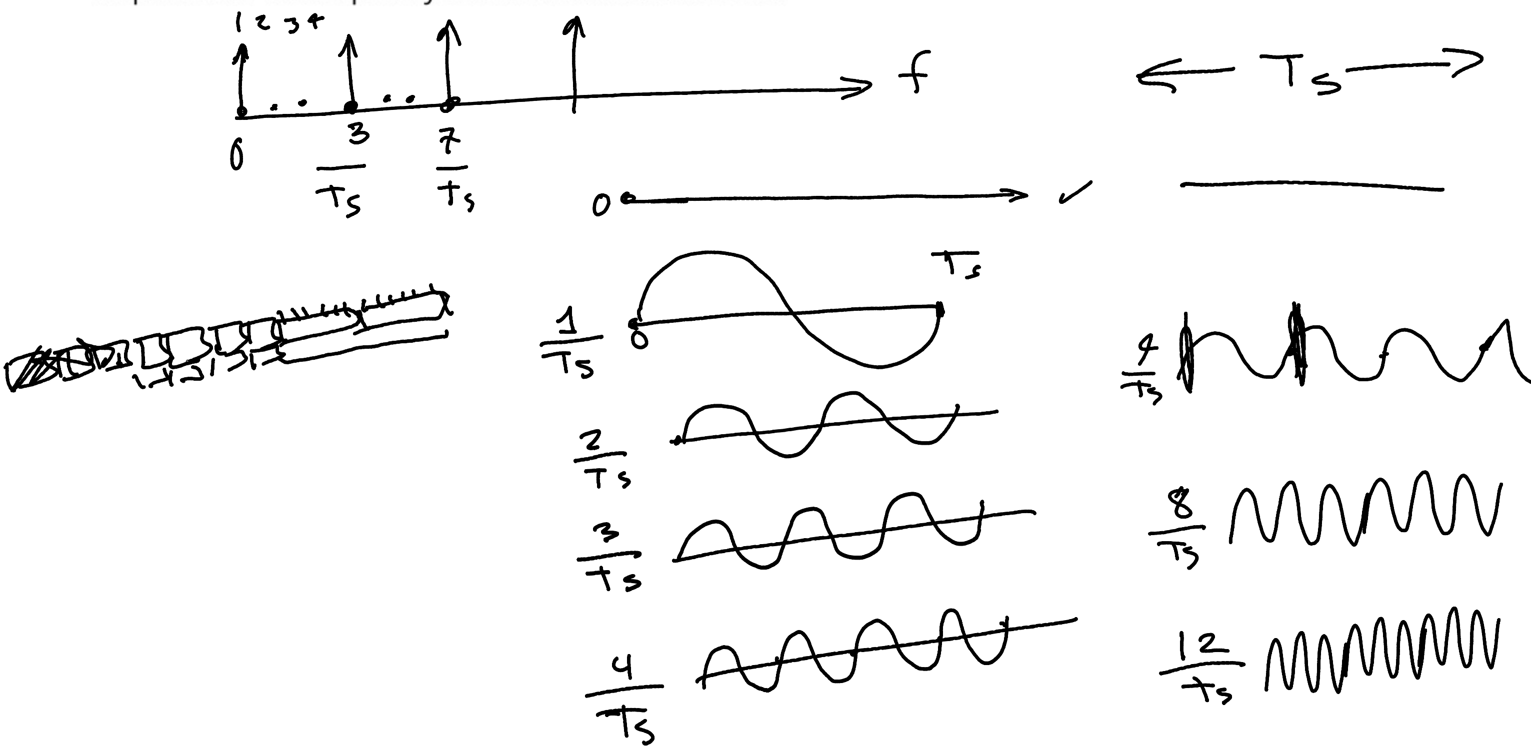


duration = $3.2 + 0.8 = 4.0\mu s$

Guard time in samples =

$$\frac{0.8\mu s}{T_s} = 16 \text{ samples.}$$

Exercise 6: The 802.11g preamble contains a "short" followed by a "long" training symbol. The short symbol contains only every fourth subcarrier. What is the period of this symbol? The long training symbol contains fixed data on each of the data subcarriers. How would you use the long training symbol to correct the phase and amplitude of subsequently-received data subcarriers?



period of pilot = $T_s/4$

short

period = T_s

long

if transmitted $1 + 0j = 1 \angle 0$

received $|r| \angle \theta$

then to correct received data multiply by $\frac{1}{|r|} \angle -\theta$

or $\frac{1}{r^*}$ ("single tap") equalization