PN Sequences and Spread-Spectrum

Introduction

Pseudo-Random Bit Sequence (PRBS) signals have many important applications in communications systems. In this chapter we will study the properties of a type of PRBS called a maximal-length (ML) sequence, learn how to generate these sequences and look at one of their applications – spread-spectrum systems.

Properties of a ML PRBS

ML PRBS sequences, sometimes called m-sequences, have a number of useful properties including:

- a long period: the sequence is called maximum-length because the sequence has a period of (repeats after) $2^m - 1$ bits where m is the number of bits of state in the generator. This is one less than the maximum number of states of an m-bit counter.

- approximately equal number of 1's and 0's: there are 2^{m-1} ones and $2^{m-1} - 1$ zeros.

- the expected distribution of run lengths for independent bits: one-half of the runs[1] have length 1, one-quarter have length 2, etc. (except that there is one run of length m ones and one of length m-1 zeros)

- adding any (circular) shift of the sequence to itself is also an m-sequence

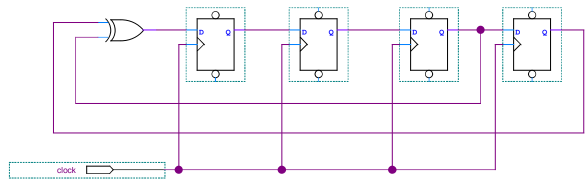
- low autocorrelation: the dot product of two time-shifted versions of the sequence is small except when the shift is zero.

Exercise 1: How many bits are there in an m-sequence for $m = 6$? How many are 1's? How many are 0's?

[1]A run is a sequence of bits with the same value.

Generating a ML PRBS

A ML PRBS can be implemented using a shift register whose input is the modulo-2 sum of certain other taps.



Exercise 2: If the initial value of each flip-flop is 1, what are the values of the next 4 bits output by the right-most flip-flop?

This is known as a linear-feedback shift register (LFSR) generator. There are published tables showing the LFSR tap connections that result in a ML PRBS generator.

If the contents of the shift register ever become all zero then all future values will be zero. This is why the generator has only $2^m - 1$ states – the state corresponding to all zeros is not allowed.

Exercise 3: How many flip-flops would be required to generate a ML PRBS of period 8191? How many ones would the sequence have? What is the longest sequence of 0's?

Walsh-Hadamard Sequences

A Hadamard matrix is a square matrix with entries of ± 1 whose rows (or columns) are mutually orthogonal. This makes the rows useful as spreading codes and they are used in WCMDA downlink (forward) channels to multiplex various synchronization, control and data streams (called "channels") onto the same carriers.

An easy way to construct a Hadamard matrix of order $n = 2^k$ is by replicating a Hadamard matrix of 2^{k-1} in the following structure:

$$\begin{bmatrix} H & H \\ H & -H \end{bmatrix}$$

starting with $H_1 = [1]$.

Exercise 4: Derive H_2 and H_4. Show that the first two rows and last two columns of each matrix are orthogonal.

This construction technique includes a replicated version of the lower-rate code in each higher-rate code. This allows the creation of a family of spreading codes with different lengths that are orthogonal when transmitted at different rates. In WCDMA this is called OVSF (Orthogonal Variable Spreading Factor) codes. For example, one stream could use the spreading code H_1 (no spreading) while another could use one of the codes from H_4. These two streams would be orthogonal and could be separated at the receiver.

Exercise 5: Spread the value a with H_{4_0} (equal to H_1) and the value b with H_{4_3}. Add the spread sequences and then de-spread them. Do you get back the original values?

Although the spreading sequences obtained from Hadamard matrices are orthogonal to each other, they do not have the minimum-autocorrelation properties of m-sequences. Thus these are useful for multiplexing but not for time synchronization or for situations where the spreading codes might be offset in time (e.g. for separating different users on the uplink).

Hadamard matrices have other useful applications. For example, the rows of a Hadamard matrix can also be used as the codewords of an error-correcting code and the Walsh transform uses the rows of a Hadamard matrix as the orthogonal basis functions for the Walsh transform.

Applications of Spread-Spectrum

Spread-spectrum systems transmit signals that occupy much more bandwidth than is necessary. This may appear wasteful of spectrum, but there are advantages for certain applications.

Military and Covert Spreading a signal reduces its power spectral density, making the signals more difficult for conventional narrow-band receivers to detect and for narrow-band transmitters to jam (cause intentional interference). This is useful for military and covert applications where low probability of intercept (LPI) and resistance to jamming are important.

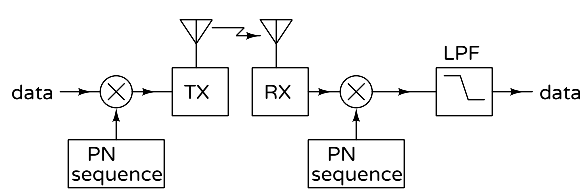
Resistance to Interference Spread-spectrum systems can help with sharing of unlicensed spectrum such as the ISM bands where there are no requirements for devices to limit interference with each other. In this case interference is unintentional and unavoidable. Spread spectrum allows system performance to degrade gradually (e.g. through reduced data rates and range) rather than catastrophically.

Multipath Propagation Multipath propagation causes fading when different delayed paths add up destructively. A receiver with high time resolution (which requires high bandwidth) can separate the different multipath components and combine them coherently rather than allow them to cancel out (fade).

Multiple Access By assigning different spreading codes to different users, they can share the same spectrum at the same time. This is called Code Division Multiple Access (CDMA). In principle this is an elegant approach and provides certain advantages such as gradual degradation with increasing number of users and allowing "soft" handover between base stations. However, the reliance on spreading for separation of users adds complexity in other ways (such as the need for accurate and low-latency power control feedback) and makes CDMA less robust than other multiple-access approaches.

Ranging Accuracy of time measurement is inversely proportional to the signal bandwidth. Although not a communications application, DSSS systems are widely used when it is necessary to accurately measure delays. Application examples include satellite-based navigation systems (e.g. GPS) and LIDAR (Light Detection and Ranging).

Direct-Sequence Spread Spectrum



In Direct-Sequence Spread Spectrum (DSSS) the data-bearing signal, which is conventionally modulated (e.g. with m-ary QAM), is multiplied by a pseudo-random spreading sequence, typically having values ± 1, at the transmitter. The spreading signal's "chip" rate is many (e.g. 100 to 1000) times the symbol rate.

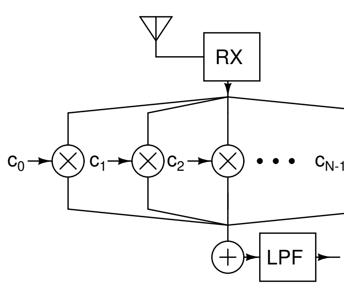
The receiver multiplies the received signal by a synchronized replica of the spreading sequence. This "de-spreads" the received signal and recovers the data signal. A low-pass filter passes frequencies required to recover the data. However, signals that are not synchronized to the (de-)spreading code will remain spread in spectrum and only the portion of these signals that falls within the low-pass filter bandwidth will cause interference to the desired signal.

The ratio of chip rate to symbol rate is the spreading factor and is also (approximately) the ratio of the spread to de-spread bandwidth. It is also the factor by which signals uncorrelated with the spreading signal are reduced in power.

Exercise 6: Consider a 30 kHz signal. What is the SIR if a jammer is transmitting on the same frequency with equal received power? If the jammer is on a different frequency? What is the SIR if DSSS with a spreading factor of 100 is used? Does the SIR depend on the jammer's frequency?

Rake Receiver

A rake receiver (so-called because the block diagram looks like a garden rake) uses multiple correlators to de-spread signals arriving with different delays. Each "finger" generates the same PN sequence but with a different time offset. These time offsets correspond to the delays of the multipath components. The receiver usually has a correlator that continuously scans for new multipath components resulting from changes to the channel impulse response (e.g. due to movement).



Spreading Sequences

Spreading codes tend to have either low autocorrelation for non-zero lags (e.g. m-sequences) or low cross-correlation between codes (e.g. OVSF codes).

- PN sequences with good autocorrelation properties, such as m-sequences, are used when it is necessary to separate different delays of the same signal. An example application is the uplink of an IS-95 CDMA system. Different users' signals will experience different multipath and the base station must identify and coherently combine each multipath component.

- PN sequences with good cross-correlation properties, such as Walsh codes, are used when it is necessary to separate different signals with the same delay. An example application is the downlink of a WCDMA system.

Exercise 7: Why do all the downlink codes have the same delay? Why do different user's uplink signals have different delays?

Near-Far Problem and Power Control

A problem with DSSS CDMA is that the amount of interference caused by a co-channel user is proportional to its received power. This is because the only separation between unsynchronized users (e.g. on the uplink) is due to de-spreading. However, because of the power-law relationship between distance and path loss, a nearby user can be received at a much higher signal strength than a far user. This can overwhelm the ability of the receiver to separate out users based on their spreading codes.

More importantly, the capacity of a DSSS CDMA system (number of users or aggregate bit rate) is inversely proportional to the SIR. A co-channel signal being received at high power can greatly degrade the SIR and thus capacity of system. For a CDMA system to maintain reasonable capacity it must ensure that the received signal levels of all co-channel users be kept close to the same level. This requires an accurate and low-latency power-control feedback channel.

Example: 2G CDMA Cellular

DSSS was used in Qualcomm's "IS-95" second-generation cellular standard. DSSS was used to provide both CDMA and to deal with multipath propagation. The chip rate was 1228800 MHz. The PN sequence was composed of both a 64-element Walsh code and an m-sequence of length $2^{15}-1$. The Walsh code was used to separate logical channels (pilot, paging and traffic) while the m-sequence was used by a rake receiver to coherently combine multiple propagation paths.

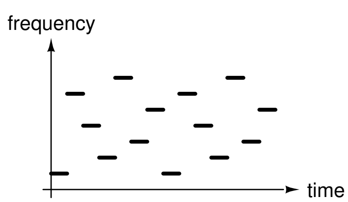
Multi-User Detection

System performance can be improved by estimating and subtracting out interference. Normally this is difficult because interference is weaker than the desired signal. However, with DSSS it's possible to recover two co-channel signals at the same time, regenerate the interfering signal and subtract it from the received signal. This is called Multi-User Detection (MUD).

In theory, this process can be repeated until all the interference is eliminated. In practice there are limitations resulting from imperfect estimates of the channel and errors in demodulating the interfering signals.

Frequency Hopping Spread Spectrum

Another way to increase the bandwidth is to change ("hop") the transmit frequency over a frequency range much wider than the bandwidth of the signal.



The transmit frequency is determined by pseudo-random number generator that controls a frequency synthesizer. For two devices to communicate they must hop in synchronized manner to the same sequence of frequencies. For example, slave devices can synchronize to timing and hop patterns controlled by a master station.

Collisions with narrow-band signals or other unsynchronized frequency-hopping users will only last the duration of one hop. We can identify two types of FHSS systems:

- In slow frequency hopping (SFH) multiple symbols are transmitted at each frequency. This means multiple symbols could be lost during one hop.
- In fast frequency hopping (FFH) multiple hops are taken for each symbol. This will average out the interference present on multiple frequencies.

The effect of multipath propagation will also depend on the relationship between the channel's delay spread (or coherence bandwidth) and the hop duration. If the hop is much shorter than the delay spread then only one (presumably the strongest) path will be seen. If the hop is much longer than the delay spread then the performance will be the same as for a non-spread spectrum case.

FHSS has the advantage that hop frequencies that are undesirable can be omitted from the hop sequence. This would be the case for frequencies that are detected to have narrow-band interference, that are faded or that are known to cause interference to other services or devices. FHSS is not subject to the near-far problem but also cannot be used for MUD.

Example: Bluetooth

BT is a wireless personal area network (WPAN) operating in the 2.4 GHz ISM band. It hops between 79 channels spaced 1 MHz apart (40×2 MHz channels for BLE) at a rate of 1600 Hz. Adaptive frequency hopping is used to avoid interference to other services (e.g. a WiFi transceiver on the same device).

Exercise 8: Is BT FFH or SFH?