Solutions to Midterm Exam

Part 1

Question 1

This question asks for the received signal power at a distance of $d = 1\,\text{km} = 1000$ m under two propagation conditions. In both cases the frequency is $f = 900$ MHz and so the wavelength is $\lambda = c/f = 3\times10^8/8\times10^8 = \frac{1}{3}$ m.

In one version of this question the transmit power is $P_T = 20\,\text{W}$ and the transmit antenna gain $G_T = 10\,\text{dBi} = 10$. In the other version $P_T = 10\,\text{W}$ and $G_T = 13\,\text{dBi} \approx 20$. The receive antenna gain is $G_R = 0\,\text{dBi} = 1$.

(a) Over a free-space path the Friis equation gives the received signal power as:

$$\begin{aligned} P_R &= P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \\ &= 20\cdot 10\cdot 1\left(\frac{1/3}{4\pi\cdot 1000}\right)^2 \\ &= 141\,\text{nW} = -68.5\,\text{dBW} = -38.5\,\text{dBm} \end{aligned}$$

for both versions of the question.

(b) For the NLOS case, the path loss at $d_0 = 100$ m is the difference between the transmitted signal level ($20\,\text{W} \approx 43\,\text{dBm}$) and the received signal level ($-40\,\text{dBm}$): $\text{PL}(d_0) = 43 - -40 = 83\,\text{dB}$.

Over a NLOS path the path loss in dB computed according to a power-law is:

$$\text{PL}(d)_{\text{dB}} = \text{PL}(d_0) + 10n\log_{10}\left(\frac{d}{d_0}\right)$$

with a path loss exponent of $n = 2.5$, the path loss at 1 km is:

$$\text{PL}(1\,\text{km}) = 83 + 2.5\cdot 10\cdot\log_{10}\left(\frac{1000}{100}\right) = 108\,\text{dB}$$

and the received power is the transmit power minus the path loss: $43\,\text{dBm} - 108\,\text{dB} = -65\,\text{dBm}$.

Question 2

(a) The Rayleigh CDF is:

$$P(r \le R) = \int_0^R p(r)\,dr = 1 - e^{-\rho^2}$$

where $\rho = R/R_{\text{rms}}$ is relative to the mean signal level (voltage) R_{rms}. In this case the mean is $R_{\text{rms}} = -80\,\text{dBm}$ and the question asks for $P(r \le R)$ for $R = -93$ dBm. Thus $\rho = -93 - -80 = -13\,\text{dB} = 10^{-13/20} \approx 0.225$ and $P(r \le R) = 1 - e^{-0.05} \approx 4.8\%$.

(b) For a uniform angle of arrival and a maximum Doppler rate of f_m, the rate at which the signal crosses a threshold ρ is given by:

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

In one version of the question the speed is $v = 100\,\text{km/hr} = 27.8\,\text{m/s}$ and the carrier frequency is $f_c = 2.4\,\text{GHz}$ which gives $f_m = f_c\cdot\frac{v}{c} = 2.4\times10^9\cdot\frac{27.8}{3\times10^8} \approx 222\,\text{Hz}$ and $N_R = \sqrt{2\pi}\cdot 222\cdot 0.225\cdot e^{-(0.225^2)} \approx 120\,\text{Hz}$.

In the other version $v = 50\text{km/hr} = 13.9\,\text{m/s}$ and $f_c = 5.2\,\text{GHz}$ which results in $f_m = f_c\cdot\frac{v}{c} = 5.2\times10^9\cdot\frac{13.9}{3\times10^8} \approx 241\,\text{Hz}$ and $N_R = \sqrt{2\pi}\cdot 241\cdot 0.225\cdot e^{-(0.225^2)} \approx 129\,\text{Hz}$.

Part 2

Question 1

There were two versions of this question about a controller generating messages with the following probabilities:

	p		$\log_2(p)$
F	0.50	1/2	−1
L	0.25	1/4	−2
R	0.25	1/4	−2

	p		$\log_2(p)$
F	0.250	2/8	−2
L	0.375	3/8	−1.42
R	0.375	3/8	−1.42

(a) The entropy of a source is given by:

$$H = \sum_i (-\log_2(P_i) \times P_i)$$

For the first version of the question the entropy, in bits per message, is:

$$H = 1 \cdot 0.5 + 2 \cdot 0.25 + 2 \cdot 0.25 = 1.5\,\mathrm{b}$$

and for the second version:

$$H = 2 \cdot 0.25 + 1.42 \cdot 0.375 + 1.42 \cdot 0.375 = 1.56\,\mathrm{b}$$

(b) The Hamming distances between the three codewords, 000000, 010101 and 111111 are shown in the table below:

	000000	010101	111111
000000		3	6
010101			3
111111			

(i) the smallest value in the table is the minimum distance, $d_{\text{min}} = 3$.

(ii) A block FEC code can correct $\lfloor \frac{d_{\text{min}}-1}{2} \rfloor = 1$ errors and

(iii) detect $d_{\text{min}} - 1 = 2$ errors.

(c) For minimum-distance decoding the receiver should decide that the valid codeword with smallest distance to the received codeword was transmitted. The table below shows the distances of the valid codewords to the received codeword for the two versions of this problem:

	110000	000011
000000	2	2
010101	3	3
111111	4	4

And in both cases the receiver should decide that the codeword 000000 was transmitted.

Question 2

There were two versions of this question with generator matrices for an $(n = 4, k = 2)$ block code with:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \text{ from which } P = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

or

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ from which } P = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

(a) As given in the question, there are $k = 2$ information bits and $n - k = 2$ parity bits in each codeword.

(b) The valid codewords will correspond to the 2^k possible combinations of information bits and the corresponding parity bits for each set of information bits. This can be computed by doing the matrix multiplication in GF(2) or by evaluating the two parity equations that are defined by P.

Label the information bits as $d_0 \dots d_1$ and the parity bits $p_0 \dots p_1$. For the first version of the question, the parity bits are $p_0 = d_1$ and $p_1 = d_0 \oplus d_1$. For the second version the parity bits are $p_0 = d_0 \oplus d_1$ and $p_1 = d_0$.

Thus the valid codewords are the four combinations of four bits (d_0, d_1, p_0, p_1) in the following table:

		version 1		version 2	
d_0	d_1	p_0	p_1	p_0	p_1
0	0	0	0	0	0
0	1	1	1	1	0
1	0	0	1	1	1
1	1	1	0	0	1

(c) The parity check matrix H is an $(n-k) \times n$ matrix that results in a zero when multiplied by a correct codeword. Each row includes a 1 in the column corresponding to the bits included in the each parity check equations and in the corresponding parity check bit. It can can be derived from the generator matrix (as $H = [P^T \,|\, I]$) or from the parity check equations.

For the first version it is:

$$H = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

and for the second version of the question:

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$