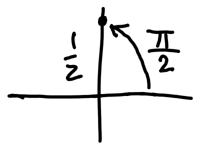
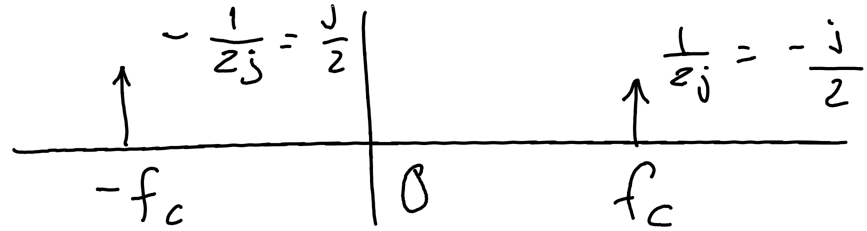
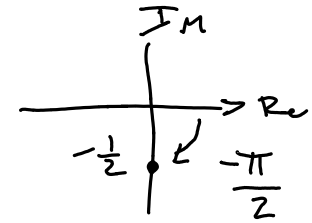
OFDM

Exercise 1: Are the magnitude and phase of this signal even or odd?

$$\mathcal{F}\{\sin(2\pi f_c t)\} = \frac{1}{2j}\left(\delta(f - f_c) - \delta(f + f_c)\right)$$







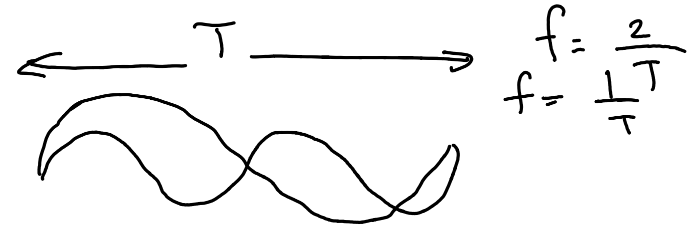
$|\cdot| = \frac{1}{2}$ $\quad$ $|\cdot| = \frac{1}{2}$

$\measuredangle = \frac{\pi}{2}$ $\quad$ $|f(f)| = |f(-f)|$: even $\quad$ $\measuredangle = -\frac{\pi}{2}$

$\measuredangle f(f) = -\measuredangle f(-f)$: odd

Exercise 2: Show this.

e.g.



$$\int_0^T \sin\left(2\pi \frac{m}{T} t\right) \sin\left(2\pi \frac{n}{T} t\right) dt$$

using: $2\cos\theta\cos\varphi = \cos(\theta - \varphi) + \cos(\theta + \varphi)$

$$= \frac{1}{2}\int_0^T \cos 2\pi \frac{t}{T}(m-n)\, dt - \frac{1}{2}\int_0^T \cos 2\pi \frac{t}{T}(m+n)\, dt$$

$$= k\left[\sin\left(2\pi \frac{t}{T}(m-n)\right)\right]_0^T - k\left[\sin\left(2\pi \frac{t}{T}(m+n)\right)\right]_0^T$$

$= 0 - 0 = 0$ $\quad$ (m & n are integers)

Exercise 3: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$. What is the subcarrier frequency spacing?

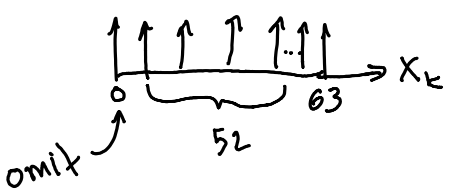
$T = 64 \cdot 50\text{ns} = 3.2\mu s.$

$T_s = 50\text{ns} = \frac{1}{f_s} = \frac{1}{20\text{ MHz}}$

subcarrier spacing: $\frac{1}{T} = \frac{1}{3.2 \times 10^{-6}} \approx 300$ kHz

Exercise 4: The 802.11g specification uses 52 of the $N = 64$ possible subcarriers and omits both the DC (zero frequency) and the highest-frequency subcarriers. What is the bandwidth of the signal?

52 subcarriers spaced $\frac{1}{3.2\mu s.}$



Bandwidth $= \frac{(52+1)}{3.2 \times 10^{-6}} \approx 16$ MHz

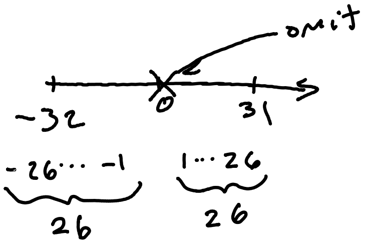
why +1?

Instead of using 52 contiguous subcarriers, the DC subcarrier corresponds to the frequency at the center of the RF channel. So instead of using subcarriers 1...52 we use −26...−1 and 1...26. This corresponds to subcarriers 1...26 and 38...63.

Why? because the spectrum of a sampled signal is periodic in frequency, $S(0) = S(\pm n f_s)$

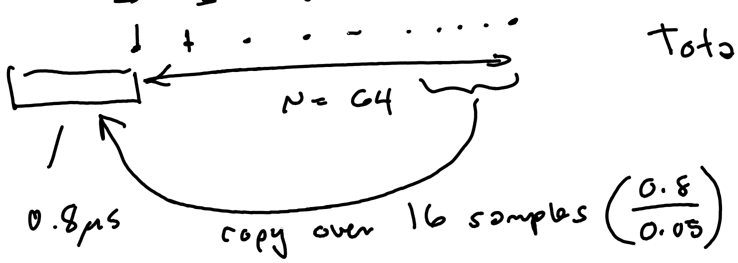
or $S(k) = S(k \pm nN)$ $\quad S(0) = S(64)$

$S(-32) = S(32)$



Exercise 5: The 802.11g WLAN standard uses OFDM with a sampling rate of 20 MHz, with $N = 64$ and guard interval of $0.8\mu s$. What is the total duration of each OFDM block, including the guard interval? How long is the guard time?

$T_s = \frac{1}{20\,MHz} = 50\,ns$



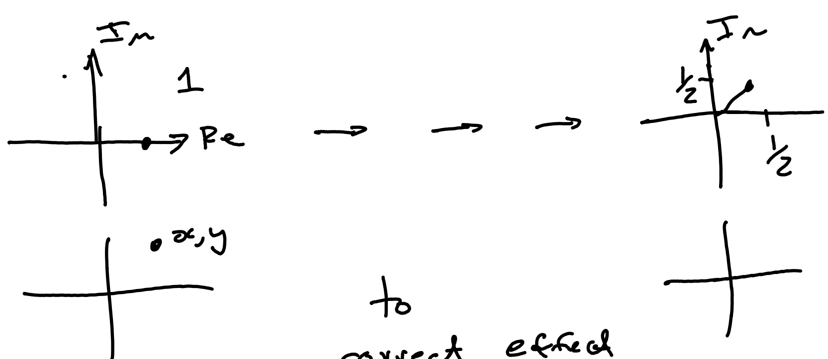
Total duration $= 3.2 + 0.8$
$= 4.0\mu s.$

Exercise 6: The 802.11g preamble contains a "short" followed by a "long" training symbol. The short symbol contains only every fourth subcarrier. What is the period of this symbol? The long training symbol contains fixed data on each of the data subcarriers. How would you use the long training symbol to correct the phase and amplitude of subsequently-received data subcarriers?

frequency spacing $= \frac{4}{T}$

$f = \frac{4}{T}, \frac{8}{T}, \frac{12}{T}, \ldots, \frac{60}{T}$

period $= \frac{T}{4}$ $\frac{1}{4}$ original symbol duration



$\frac{1}{2} + \frac{j}{2} = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}}$

$\sqrt{\frac{1}{2}^2 + \frac{1}{2}^2}$

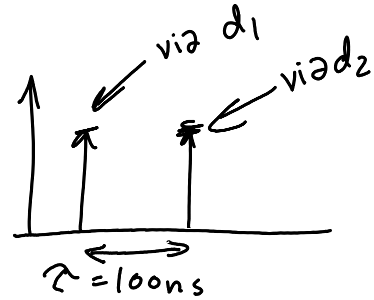
$= \frac{1}{\sqrt{2}}$

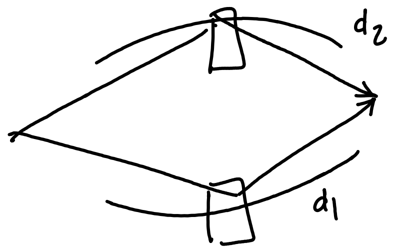
to correct effect of channel   divide by $\frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}}$

or multipy by $\sqrt{2} e^{j\frac{\pi}{4}}$

in general, if channel multiplies a pilot tone by X, we multiply the data on that subcarrier by X^{-1} (multiply by $\frac{1}{|X|} e^{-j\angle X}$).

Exercise 7: A channel's impulse response is two equal-level impulses separated by 100 ns. What difference in propagation path lengths (m) would result in such an impulse response? How far apart are the nulls of this channel? What OFDM signal bandwidth(s) would be required to provide frequency diversity?





$$|d_2 - d_1| = ?$$

$$V = 300\ m/\mu s$$

$$d = v \cdot \tau = 0.1 \cdot 300 = 30m.$$

The signals received along the two paths cancel at the null frequencies:

$$\sin(2\pi f t) + \sin\left(2\pi f (t+\tau)\right) = 0$$

for a phase shift that is an odd multiple of π:

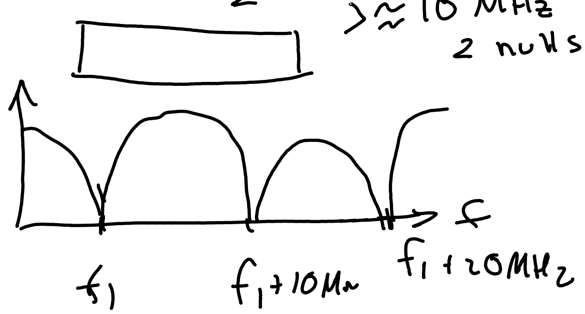
$$2\pi f \tau = (2n+1)\pi$$

$$f = \frac{2n+1}{2\tau} = \frac{1}{2\tau}, \frac{3}{2\tau}, \frac{5}{2\tau}$$

$$= 5, 15, 25\ MHz$$

$$\Delta f = 10MHz$$

OFDM signal bandwidth $\gtrsim$ 10 MHz to include 2 nulls



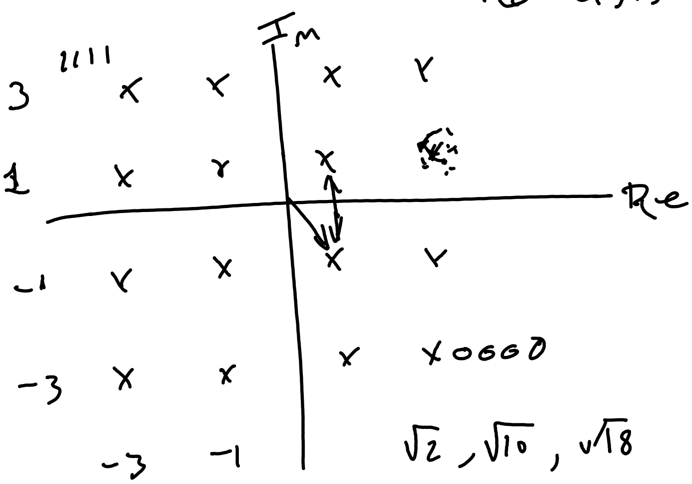
In general:

we would like the COFDM signal bandwidth $> \frac{1}{\text{channel delay spread}}$

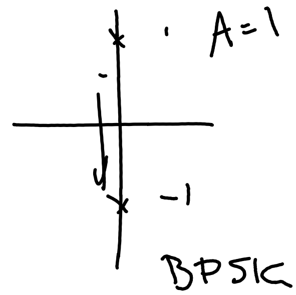
Exercise 8: How many bits per subcarrier are transmitted by an OFDM system using 16-QAM? Assuming equal noise powers, how much more power does this system need to achieve the same BER than a system using 4-QAM? Than a system using BPSK?

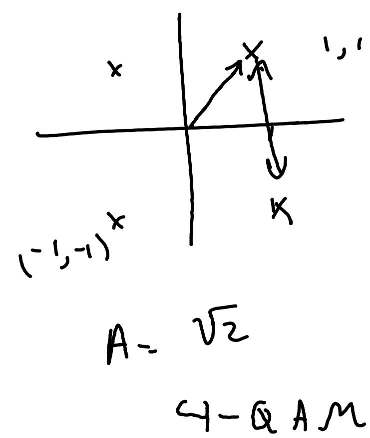
4-QAM: 4
BPSK: 1
} bits per subcarrier

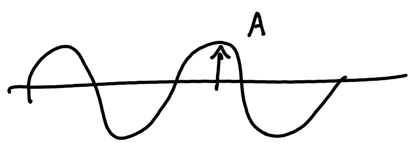
16-QAM



4 bits/sub carrier







$\frac{A^2}{\sqrt{2}}$

Note: this is the power of each subcarrier, not of the OFDM signal. The total power is the sum of the subcarrier powers (assuming independent modulation).