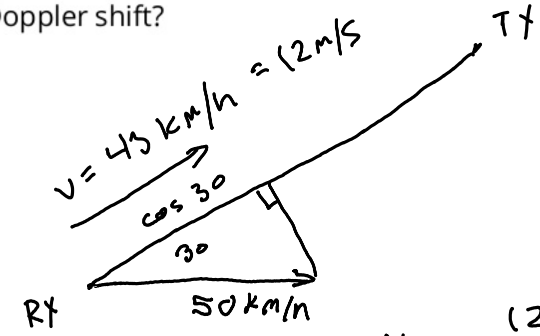
Multipath Propagation

Exercise 1: A receiver in a car receives a 1.8 GHz signal while travelling on a road at 50 km/h. The road is at an angle of 30 degrees relative to the direction of arrival of the signal. What is the velocity relative to the direction of arrival of the signal? By how much does the path length change each second (in meters)? In wavelengths? What is the Doppler shift?



$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{1.8\times 10^9}$$

$$\approx 0.17\ m/\lambda$$

$$v = \frac{12}{0.17} \simeq 70 \text{ wavelength/s}.$$

$$f_D = \frac{v}{c}\cdot f_c = \frac{12}{3\times 10^8}\cdot 1.8\times 10^9 = 72\ Hz$$

$$\text{units: } \frac{m/s}{m/s}\cdot Hz = Hz$$

Exercise 2: A channel has three multipath components with delays of 1, 2 and 3 μs and amplitudes of 10, 6 and 0 dBm respectively. What are the excess delays, the power delay profile, the normalized power delay profile, the mean excess delay and the RMS delay spread?

$$10 = 10\log 10$$

$$\log_{10} 10 = 1$$

delay = 1, 2, 3

excess delays = 0, 1, 2

powers are 10 mW, 4 mW, 1 mW

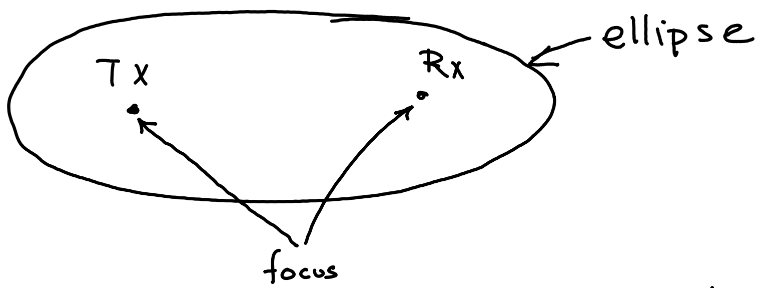
$$p(\tau) = \frac{10}{15},\ \frac{4}{15},\ \frac{1}{15}$$

$$\bar{\tau} = 0\cdot\frac{10}{15} + 1\cdot\frac{4}{15} + 2\cdot\frac{1}{15} = \frac{6}{15}\ \mu s.$$

$$\sigma = \sqrt{\frac{10}{15}\left(0-\frac{6}{15}\right)^2 + \frac{4}{15}\left(1-\frac{6}{15}\right)^2 + \frac{1}{15}\left(2-\frac{6}{15}\right)^2} = 0.611$$

Exercise 3: Imagine a receiver traveling in a straight line towards a transmitter but with no LOS path. How could you arrange reflecting objects such that there was no time dispersion (flat fading)? What arrangement would result in no time-varying fading? Neither?

no time dispersion ⇒ equal delays



more practically, all scatterers located close to each other (e.g. near one antenna)

no time-varying fading ⇒ signal constant

⇒ no multipath ⇒ all scatterers in one place (one point).

neither: one scatterer (also)

Exercise 4: What fraction of the time is a Rayleigh-distributed signal 10dB below the mean? 20dB? 30dB? This is a useful result to remember.

$$P(r < \overset{\text{threshold}}{R}) = 1 - e^{-\rho^2} \qquad \rho^2 = \left(\frac{R}{R_{rms}}\right)^2$$

R_{rms}: root mean square (voltage)

if R is 10dB below mean:

$$\rho^2 = 0.1 = \underline{\text{power}} \text{ ratio} \quad : \quad \left(\frac{R}{R_{rms}}\right)^2 = \frac{1}{10}$$

$$P(10\ dB) = 1 - e^{-0.1} \approx 0.1$$

$$P(20\ dB) = 1 - e^{-0.01} \approx 0.01$$

$$P(30\ dB) = 1 - e^{-0.001} \approx 1 \times 10^{-3}.$$

$$1 - e^{-0.001} = 0.0009995$$

(corrected Feb. 20) → power ratio

Exercise 5: How often will the signal drop 10dB below the mean if the carrier frequency is 1.8 GHz and the velocity is 100 km/h? On average, how long will each of these fades last?

$$\rho^2 = 10^{\frac{-10}{10}} = 0.1$$

$$f_m = f_c \cdot \frac{v}{c}$$

$$v = \frac{100 \times 10^3 \text{ m/s}}{3600 \text{ s/hr}} \approx 27.8 \text{ m/s}$$

$$f_m = 1.8 \times 10^9 \cdot \frac{27.8}{3 \times 10^8} = 167 \text{ Hz}$$

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

$\sqrt{2\pi} \times 167 \times \sqrt{.1} \times e^{-0.}$

119.7779724

$\approx$ 120 Hz

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} = \frac{e^{.1} - 1}{\sqrt{.1} \times 167 \times \sqrt{2\pi}} =$$

0.000794491

= 0.8 ms