ELEX 7860 : Wireless System Design
2019 Winter Term

Assignment 1

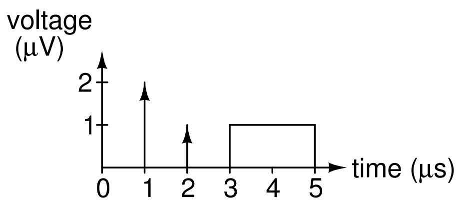
Due Thursday, February 21, 2019. Submit your assignment using the appropriate Assignment folder on the course web site. Assignments submitted after the solutions are made available will be given a mark of zero. **Show how you obtained your answers.**

Question 1

A Rayleigh-fading signal has a mean signal power of −90 dBm. What is the probability that the received signal strength will be less than −105 dBm?

Question 2

A channel has the following impulse response:

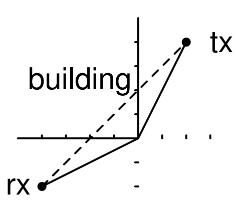


What is the mean *excess* delay? What is the RMS delay spread?

Hints: This distribution has both discrete and continuous components. Substitute the appropriate integrals for the sums used in the lecture notes. The vertical axis is in units of volts, not watts. The distribution is not normalized.

Question 3

Consider two FRS radios (unlicensed NBFM personal communication devices operating on frequencies around 465 MHz) communicating around the edge of a building as shown below:



where each tick marks a distance of 2 m. Assuming diffraction is the only significant propagation mechanism and the corner of the building can be considered a knife-edge, estimate the path loss, including both diffraction and distance effects.

Question 4

Consider a receiver with two diversity branches. The SNRs on the two branches are 10 dB and 6 dB.

(a) What SNR would result from selection diversity? From equal-gain combining (where both branches are weighted equally)? From maximal-ratio combining?

(b) What SNRs could result from switching diversity if the switching threshold was 8 dB? If it was 5 dB?

Question 5

A cellular radio system operating at 1.8 GHz requires a short-term average signal power of -105 dBm at the mobile for acceptable service. The base station transmit power is 20 dBm, the path loss at a distance of 100 m is equal to the free-space path loss, the distance-dependent path loss can be assumed to be well-approximated by a power law with an exponent of -2.7, and the signal is affected by log-normal shadowing with a standard deviation of 10 dB.

At what distance is there is a 95% probability of having the required mean signal level?

Question 6

Two random variables, X and Y represent two flips of a coin. The outcome of each toss can be H or T (heads or tails).

(a) Draw the joint pdf if the two coins are fair (unbiased) and the outcomes are independent.

(b) Draw the joint pdf if H is twice as likely as T but the outcomes are independent.

(c) Draw the joint pdf if the coins are fair but the outcome of the second toss depends on the first and is always the opposite.

(d) (i) For which of these conditions are X and Y identically distributed? (ii) For which are they independent r.v.? (iii) For which are they i.i.d.?

Question 7

A channel has three possible inputs (1, 2, 3) and four possible outputs (1, 2, 3 and ?). The '?' ("erasure") message is output when the channel is faded and the receiver can't make a decision.

The following channel transition probability matrix gives the probability of an output y for an input x:

		y			
		1	2	3	?
x	1	0.6	0.0	0.2	0.2
	2	0.2	0.7	0.0	0.1
	3	0.2	0.0	0.6	0.2

Find the capacity of this channel in bits per channel use.

In other words, find the maximum value of the mutual information, $I(X, Y)$, for the probability distribution of the input, $P(X)$, that maximizes $I(X, Y)$.

Some hints:

- A numerical solution is suggested. The following Matlab (or Octave) function returns the mutual information for a column vector of input probabilities, **px**, and a matrix of transition probabilities, **c**, organized as above, with the rows as the inputs and the columns as the outputs:

```
c=[.6 .0 .2 .2;.2 .7 .0 .1;.2 .0 .6 .2] ;

% ... your code here ...

% mutual information for channel input and transition
% probabilities

function in = mi(px,c)
   pxy=px.*c ;
   py=sum(pxy) ;
   in=sum((pxy.*log2(pxy./(px*py)))(pxy>0)) ;
end
```

- To find the capacity you need to find the input probabilities, $p(X) \equiv$ **px**, that maximize the mutual information, $I \equiv$ **mi()**.

- The search space is two-dimensional since knowing $p(x = 1)$ and $p(x = 2)$ determines $p(x = 3)$: **px(3)=1-(px(1)+px(2))**.

- The objective function, mutual information, is a convex function – it has one maximum.

- You may use any numerical software and optimization method you wish. For example, you could evaluate **mi()** over a range of values for **px**, print or plot the results, and select the value of **px** that results in the maximum.

  Or you could use an optimization function such as **fminsearch()** by passing it a handle to a function that returns the (negative) of the mutual information and an initial guess for **px**. Such a function would have to compute the third probability and return an appropriate value for invalid locations in the search space (where $p(x = 1) + p(x = 2) > 1$).

- *Show your work – simply giving an answer and saying you "guessed" is not sufficient.*