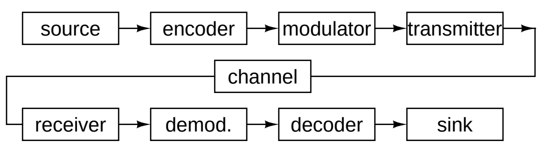
Modulation - Part 1

After this lecture you should be able to:

- *compute Shannon capacity for the AWGN channel*
- *compute modulation index and bandwidth for FM signals*
- *determine if a filter meets Nyquist criteria for no ISI*
- *compute transfer function and impulse response of RRC and Gaussian filters*
- *compute output of above filters to binary signals*
- *compute bandwidth for a given PSD and bandwidth definition*
- *compute output SNR for FM discriminator given the modulation index and input SNR*
- *compute output SIR for DS SS receiver in AWGN*
- *compute BER for FH SS receiver*

Communication System



- we will concentrate on modulation suitable for wireless systems
- for an AWGN channel the maximum error-free data rate is

$$C = B\log_2(1+\frac{S}{N})$$

- this is the Shannon capacity bound

Exercise: What is the capacity of the telephone channel assuming it is an AWGN channel, has a bandwidth of 4 kHz and an SNR of 255 (24 dB)? If you could either double the bandwidth or double the SNR, which would provide the greater increase in capacity?

Analog: AM, FM, SSB, PM, VSB

- older systems are analog
- AM used for aeronautical mobile and MF, HF broadcast
- NBFM used for most land-mobile systems
- WBFM for VHF audio broadcast
- VSB used for VHF video broadcast
- SSB used for HF

FM

- most widely used analog modulation for wireless communication
- due to:
- immunity to noise that affects amplitude only
- capture effect eliminates interference at high-enough SIR
- demodulator output (audio) SNR can be higher than input (IF) SNR
- transmitters can use efficient, non-linear (class C) RF power amplifiers
- a sine-modulated FM signal of amplitude A, carrier frequency f_c and peak frequency deviation of Δ_f:

$$s(t) = Acos\left[2\pi f_c t + \frac{\Delta_f}{f_m} sin(2\pi f_m t)\right]$$

- modulation index (β) for sinusoidal modulation of FM:

$$\beta = \Delta_f / f_m$$

- for non-sinusoidal modulation use the maximum signal frequency for f_m
- Carson's rule: 98% of the power is contained in a bandwidth:

$$B = 2(\beta_+ 1) f_m$$

Exercise: Commercial FM broadcasting uses Δ_f of about 80 kHz and the audio signals have f_m of about 20 kHz. What is the modulation index? What is the bandwidth?

- for WBFM $\beta \gg 1$, for NBFM $\beta \ll 1$

Exercise: What is β when $B = 2 f_m$?

FM Demodulators

- for an AM demodulator the output SNR is the same as the input SNR
- for an FM discriminator the output SNR can be higher than the input SNR (if the input SNR is above a threshold)
- SNR improvement and the threshold depend on β, for sinusoidal modulation, and input SNR above threshold,

$$SNR_{out} = 3(\beta^3 + \beta^2) SNR_{in}$$

- at SNRs below threshold, get "click" (impulse) noise instead of Gaussian noise

Digital Modulation

- more flexible: can be used for voice, data and video
- source coding to reduce data/bandwidth
- FEC coding to improve performance
- encryption is possible

Choice of Modulation

- power efficiency (BER versus E_b/N_0)
- spectral efficiency (b/s/Hz)
- error floor on fading channel
- complex decision - need to consider many factors:
- sensitivity to fading (e.g. phase synchronization requirements)
- performance in multipath and interference
- implementation costs (e.g. need for linear amplifiers)
- complexity of transmitter and receiver
- ability to use RAKE, time diversity, soft handoff, discontinuous transmission, etc.
- e.g. often it's spectral efficiency in b/s/Hz/km that matters

Bandwidth

- many definitions
- based on the power spectral density (PSD) of the signal
- spectrum can have continuous and discrete (periodic) components
- "absolute bandwidth" - total bandwidth where PSD is non-zero
- null-to-null bandwidth - for periodic modulation
- half-power (3dB) bandwidth - where PSD drops -3 dB from peak
- FCC definitions (e.g. CFR 47 90.209):
- authorized bandwidth - what the transmitter license says (e.g. 20 kHz)
- occupied bandwidth - contains 99% of power plus spurious
- necessary bandwidth - usually same as occupied

- spectral mask
- limits to PSD or spurious
- often quoted in regulations or standards to specify allowed adjacent-channel interference levels

  Exercise: The PSD of a signal is 1 mW/Hz at 1MHz and decays linearly until it is zero at 900 kHz and 1.1 MHz. What are the absolute, half-power, and 99%-power ("occupied") bandwidths?

Nyquist Criteria

- limited bandwidth ($\equiv$ impulse response, $h(t)$, is not an impulse) causes inter-symbol interference (ISI)
- to ensure no ISI at sampling times $h(t)$ must have zeros at multiples of the symbol period
- $h(t)$ includes effect of transmit filters, channel bandwidth limits and receive filters
- Nyquist no-ISI criteria: the overall channel transfer function must be symmetrical about $\frac{1}{2T}$
- use transmit and receive filters plus channel equalizers so complete channel meets Nyquist criteria
- transmit and receive filters typically have transfer function $\sqrt{H(f)}$
- parameter α is "excess bandwidth"
- $\alpha = 0$ is a "brick wall" filter
- $\alpha = 1$ means twice the minimum required bandwidth is used
- most common example is the "root raised-cosine" filter
- no-ISI symbol rate is twice the (1-sided) baseband bandwidth
- RF bandwidth is twice the baseband bandwidth
- can use other filters than raised-cosine
- e.g. Gaussian filter: has smoother $h(t)$ and smoother $H(f)$, but does not meet Nyquist requirement for no ISI

Signal Space Description

- information to be transmitted is encoded into a sequence of symbols
- assume have M different symbols
- we can represent each of the possible symbols as a weighted sum of orthonormal basis functions
- might require up to M basis functions
- e.g. basis functions might be $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ for $0 \le t < T_s$
- plotting the symbols along axis corresponding to basis functions gives the "constellation"
- we can derive symbol error rate (for a receiver using filters matched to basis functions) in terms of the Markum's $Q(\cdot)$ function

Spread Spectrum

- is a multiple-access technique, not a modulation method
- will cover in detail later
- two types: direct sequence (DS) and frequency-hopping (FH) SS
- both use pseudo-random sequences to the spread signal over a much wider bandwidth than necessary (orders of magnitude more than the symbol rates)
- use law of large numbers (distribution of a sum of large number of RV is Gaussian) to average out interference for all users
- number of practical advantages:
- robust to narrow-band interference
- low probability of intercept
- multiple paths can be demodulated and combined with RAKE receiver instead of causing ISI (equivalent of frequency diversity)
- practical disadvantages for DS SS:

- correlators not as effective as IF filters in cancelling undesired signals (requires accurate power control)
- requires linear(ized) PA
- more complex signal processing and system planning

PN sequences

- repeatable binary sequence
- low cross-correlation, auto-correlation
- maximal-length sequence repeats with period $2^N - 1$

DS SS

- transmitter multiplies the baseband signal with a PN code
- (convolves the signal in frequency, thus spreading it)
- each PN symbol is called chip, chip rate
- at receiver a synchronized PN generator multiplies by multiplicative inverse of the transmitting PN code
- result is the transmitted baseband signal which is demodulated normally
- the post-despreading IF filter has bandwidth approximately equal to symbol rate
- signals that are uncorrelated with PN signal are not despread, only a fraction of this signal gets through the post-despreading filter
- processing gain (rejection of uncorrelated interference) is same as ratio of pre- to post-decorrelator bandwidths
- SNR is the same as for a narrowband system
- SIR can be more equally distributed between users than in a narrow-band system

FH SS

- the PN sequence is used to select from a large set of possible carrier frequencies (channels)
- transmitter frequency "hops" from channel to channel
- can have slow (multiple symbols/hop) or fast (multiple hops/symbol) FH
- the receiver's synchronized PN sequence is used to tune the receiver
- performance determined by probability that two users will hop to the same frequency at the same time

Performance of SS

- for DS SS:
- performance in fading and multipath difficult to predict since results sensitive to assumptions about propagation
- can derive simple results in absence of fading or multipath and assuming others users generate uncorrelated interference
- for FH SS:
- results will vary with propagation assumptions
- if no collision, then BER is unchanged
- if collision, BER is about 0.5
- probability of collision
- use FEC to correct for inevitable collisions