## Solutions to Final Exam

## Question 1

(a) The probability of each message is the fraction of messages of each type. In one version of the exam these are $\frac{30}{60}=0.5, \frac{18}{60}=0.3, \frac{6}{60}=0.1$. In the other they are $\frac{30}{60}=0.5, \frac{20}{60}=0.333, \frac{5}{60}=0.083$.
(b) The entropy of a source is given by:

$$
H=\sum_{i}\left(-\log _{2}\left(P_{i}\right) \times P_{i}\right) \quad \text { bits } / \text { message }
$$

Where the probabilities are as given above. The result in both cases is $\approx 1.6 \mathrm{bit} /$ smessage.
(c) The information rate is the amount of information transmitted per second. In this case since one message is generated per second the information rate is also $\approx 1.6$ bits/second.

## Question 2

The bits are transmitted in little-endian order. There were two versions of the question. One was for the character ' $\mathfrak{s}$ ' $(0 x 73$, binary 01110011$)$ and one for the character 'L' (0x4c, binary 0100 1100).

The differential voltage is the difference between the two voltages (in this case labelled $\mathrm{V}+$ and $\mathrm{V}-$ ). These must have opposite values for differential signalling. If the two voltages are 0 and 3 V then the differential voltages must be $0-3=-3 \mathrm{~V}$ and $3-0=3 \mathrm{~V}$. Similarly for 0 and 5 V levels. The duration of each symbol is the bit period, $\frac{1}{10 \times 10^{3}}=100 \mu \mathrm{~s}$.

The waveforms are:



## Question 3

(a) If the propagation delay through a 100 m section of co-ax cable is found to be $1 \mu \mathrm{~s}$ then the velocity factor is:

$$
V F=\frac{v}{c}=\frac{\frac{100}{1 \times 10^{-6}}}{3 \times 10^{8}}=\frac{1}{3}
$$

from the velocity factor we can obtain the dielectric constant:

$$
\varepsilon_{r}=\frac{1}{V F^{2}}=9
$$

(b) We also know the relationship between characteristic impedance and the product of distributed inductance and capacitance, $L C$, $\left(Z_{0}=\sqrt{\frac{L}{C}}\right)$ and between $L C$ and velocity of propagation ( $v=$ $\frac{1}{\sqrt{L C}}$ ) from which we can derive:

$$
C=\frac{1}{v Z_{0}}=200 \mathrm{pF}
$$

for $Z_{0}=50 \Omega$ and 133 pF for $Z_{0}=75 \Omega$.

## Question 4

To achieve error-free communication over an AWGN channel requires that we meet the Shannon capacity limit. This is:

$$
C=B \log _{2}\left(1+\frac{S}{N}\right)
$$

We are given the SNR ( -10 dB or 0.1 ) and the bit rate which must be less than the capacity. Thus we can compute the required bandwidth, $B$ :

$$
B=\frac{C}{\log _{2}\left(1+\frac{S}{N}\right)}
$$

and for a capacity (bit rate) of $1 \mathrm{~kb} / \mathrm{s}$ this is $\approx 7.3 \mathrm{kHz}$ and for a bit rate of $2 \mathrm{~kb} / \mathrm{s}$ it is $\approx 14.6 \mathrm{kHz}$.

## Question 5

HDLC framing requires adding a start flag, an end flag, bit-stuffing within the frame after 5 consecutive 1's.

For the bit sequence $\begin{array}{lllllllllll}1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1\end{array}$ the transmitted bits would be:

011111101101111100101111110
and for $0 \begin{array}{lllllllll}0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1\end{array}$ it would be:
011111100011111010101111110

## Question 6

To compute the IP checksum we add up the bytes arranged as 16 bit words in big-endian byte order. Then add the MS 16 bits to the LS 16 bits and invert the bits of the result.

Both versions of the question give the same result: the sum is $0 \times 13$ c7e and the IP checksum is $0 \times 3880$.

## Question 7

For the routing table:

| Destination | Netmask | Interface |
| :---: | :---: | :---: |
| 127.0 .0 .0 | 255.255 .0 .0 | lo |
| 10.0.3.0 | 255.255 .255 .0 | eth0 |
| 0.0.0.0 | 0.0.0.0 | eth1 |

We can determine the interface by AND'ing the destination IP address and the Netmask column and comparing the result to the Destination column. The first match is used to select the interface. The last row matches all addresses and is thus the default route.

There were two versions of the question with the same IP addresses arranged in different orders. The results are:

| IP address | selected interface |
| :---: | :---: |
| 74.125 .25 .104 | eth1 |
| 10.0 .3 .255 | eth0 |
| 127.0 .0 .1 | lo |
| 10.0 .5 .1 | eth1 |

