Analysis of Feasible Synchronverter Pole-placement Region to Facilitate Parameter Tuning
Shuan Dong, Member, IEEE, and Yu Christine Chen, Member, IEEE

Abstract—This paper derives in analytical closed form the feasible pole-placement region of the synchronverter active-power loop (APL) so as to eliminate all trial-and-error effort in parameter tuning. We consider the well-established setting of a reduced third-order APL model with two controller parameters that can be tuned freely. Thus, only two of the three APL poles can be specified independently in hopes of achieving desired system dynamics. Central to the presented derivation is the realization that the two specified poles must represent the dominant mode of the system. Otherwise the actual system dynamics may be dictated by the third unspecified pole, leading to unexpected or undesired dynamic behaviour. Numerical simulations involving the full-order synchronverter dynamical model and a modified New England 39-bus test system validate the analysis and the resulting region within which poles must be placed for actual system dynamics to match desired ones.

Index Terms—Damping correction loop, grid-forming converter, parameter tuning, pole placement, synchronverter, virtual synchronous generator.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( D_f, D_p )</td>
<td>Damping and frequency droop coefficients.</td>
</tr>
<tr>
<td>( e_g, E_g )</td>
<td>Synchronverter inner voltage and its line-to-line RMS value.</td>
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<tr>
<td>( i_y )</td>
<td>Synchronverter output current.</td>
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<tr>
<td>( J_y )</td>
<td>Inertia constant.</td>
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<tr>
<td>( M_p, t_s )</td>
<td>APL overshoot and settling time.</td>
</tr>
<tr>
<td>( P_s, Q_l )</td>
<td>Active- and reactive-power outputs.</td>
</tr>
<tr>
<td>( R_c + jX_c )</td>
<td>Transmission line impedance.</td>
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<tr>
<td>( R_a + jX_a )</td>
<td>L-type filter impedance.</td>
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<tr>
<td>( R_v + jX_v )</td>
<td>Virtual impedance.</td>
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<td>( s_1, s_2, s_3 )</td>
<td>Poles of synchronverter APL.</td>
</tr>
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<td>( T_e, T_{ef} )</td>
<td>Electromagnetic torque and its filtered value.</td>
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<tr>
<td>( T_m )</td>
<td>Synchronverter input torque.</td>
</tr>
<tr>
<td>( u_{dc} )</td>
<td>De-bus voltage.</td>
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<tr>
<td>( u_t, U_t )</td>
<td>Voltage at point of common coupling and its line-to-line RMS value.</td>
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<td>( u_{\infty}, U_{\infty} )</td>
<td>Grid-side voltage and its line-to-line RMS value.</td>
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<tr>
<td>( X_t )</td>
<td>Total system reactance.</td>
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<td>( \zeta^* )</td>
<td>Desired APL damping ratio.</td>
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<tr>
<td>( \theta_g )</td>
<td>Virtual rotor angle.</td>
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<td>( \theta_{g\infty} )</td>
<td>Phase-angle difference between synchronverter inner voltage and grid-side voltage.</td>
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<td>( \tau_f )</td>
<td>Low-pass filter (LPF) time constant.</td>
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<td>( \psi_f, \psi_{ff} )</td>
<td>Excitation flux and its filtered value.</td>
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<td>( \omega_g )</td>
<td>Virtual rotor rotating speed.</td>
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<tr>
<td>( \omega_n )</td>
<td>APL natural frequency.</td>
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<tr>
<td>( \omega_N )</td>
<td>Rated rotating speed.</td>
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<tr>
<td>( \omega_{\infty} )</td>
<td>Grid-side voltage angular frequency.</td>
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<tr>
<td>( \Omega_f )</td>
<td>Feasible APL pole-placement region.</td>
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Superscript

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<tr>
<td>*</td>
<td>Reference value.</td>
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<td>o</td>
<td>Equilibrium value.</td>
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<td>ss</td>
<td>Steady-state value.</td>
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I. INTRODUCTION

Given greater integration of power electronic-interfaced renewable energy systems with little to no inherent inertia, e.g., wind and solar, the traditional high-inertia synchronous generator-dominated power system is transitioning into a lower-inertia one augmented by power electronics [1]. This transformation brings forth numerous technical challenges to maintain power availability and quality. In order to address these problems, the concept of the virtual synchronous generator (VSG) has been proposed to leverage power-electronic converters to provide virtual inertia to the grid [2]–[16]. The core idea of the VSG is to design the voltage-source converter (VSC) controller so that it mimics the dynamics of a synchronous generator. In addition to contributing virtual inertia to the grid, the VSG is also able to adjust its active- and reactive-power outputs and provide timely frequency and voltage regulation.

Among the various instantiations of VSGs, the synchronverter is a representative design with concise controller structure [8]–[11]. Our previous work in [9] augments the original synchronverter design from [8] with the so-called damping correction loop so that its response speed can be adjusted freely without violating the steady-state frequency droop characteristic. Then, aimed at a simple procedure to tune the synchronverter parameters, [10] develops a method that directly computes parameter values that satisfy desired transient and steady-state behaviours [10]. Specifically, the tuning method in [10] leverages a reduced third-order model of the synchronverter active-power loop (APL) that captures pertinent system dynamics. However, since there are only two controller parameters that can be tuned freely, only two of the three poles can be specified independently, while the third depends on the placement of the first two. Consequently, the parameter tuning method in [10] achieves desired time-domain dynamic behaviour only if the two specified poles indeed represent the APL dominant mode. Otherwise, the synchronverter would display unexpected or undesired dynamics governed by the third pole. For this reason, as shown in Fig. 1, the method...

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in [10] requires repetitive trial-and-error effort to specify the two desired poles until they represent the dominant mode. In this paper, we improve the method in [10] by eliminating the potential trial-and-error effort involved with ill-advised choices of desired poles. Particularly, we provide analysis and develop precise conditions to determine the feasible pole-placement region, within which the two placed poles represent the APL dominant mode.\textsuperscript{1}

In addition to the method developed in [10], lack of understanding of the feasible pole-placement region also complicates other VSG parameter tuning methods with cumbersome trial-and-error effort. For example, methods in [3]–[5] involve repeated specification of the natural frequency and the damping ratio for desired dominant poles to tune the VSG parameters with small-signal analysis before achieving desired dynamics. In view of this problem, numerous tuning methods endeavour to reduce or remove the required trial-and-error effort and facilitate the tuning procedure. For example, [6] leverages the virtual impedance to expand the feasible pole-placement region, and in so doing, reduces the trial-and-error effort of seeking feasible dominant pole locations. However, the adoption of virtual impedance may instead limit the feasible pole-placement region in some cases. Alternatively, the tuning methods in [11]–[16] do not pursue precise pole placement, and thus avoid some trial-and-error effort in the tuning procedure. The aforementioned tuning methods (at best) reduce the inherent trial-and-error effort involved instead of eliminating it altogether. However, the analytical characterization of the feasible pole-placement region a priori presented in this paper obviates all trial-and-error effort and significantly simplifies the VSG tuning procedure.

Given the literature reviewed above, this paper’s contributions are as follows. First, we develop an analytical condition that helps to directly compute the range of the achievable synchronverter APL natural frequency with given APL damping ratio. Within the range predicted by this condition, we can indeed choose the desired APL natural frequency freely and achieve desired dynamics by computing parameters with the method in [10]. In this way, we completely eliminate all trial-and-error effort from the parameter tuning method.

\textsuperscript{1}For ease of exposition, in the remainder of the paper, we refer to the pole(s) that represent the APL dominant mode interchangeably as dominant pole(s).
Also let \( \omega \) be the angular frequency of the grid-side voltage \( u \). The APL dynamic model is as follows:

\[
\frac{d\theta}{dt} = \omega_g - \omega_{\infty},
\]

\[
J_d \frac{d\omega_g}{dt} = T_m - T_{ef} - D_p(\omega_g - \omega^*) - D_f \frac{dT_{ef}}{dt} \left( \frac{T_{ef}}{\psi_{ff}} \right),
\]

\[
\tau_f \frac{dT_{ef}}{dt} = -T_{ef} + T_e,
\]

where \( J_d \) denotes the tuneable inertia constant, \( T_m = \frac{P_t^*}{\omega_N} \) represents the input torque with \( P_t^* \) being the active-power reference and \( \omega_N \) the rated rotating speed, \( \omega^* \) is the reference rotating speed, and \( \tau_f \) denotes the time constant of the low-pass filter (LPF). In (2), the term \( D_p(\omega_g - \omega^*) \) represents a simplified governor and achieves frequency-droop control, where \( D_p \) denotes the frequency droop coefficient. The damping correction loop \( D_f \frac{dT_{ef}}{dt} \) provides freely adjustable damping, since this term is in fact proportional to \( \omega_g - \omega_{\infty} \), where \( D_f \) and \( \omega_{\infty} \), respectively, denote the damping coefficient and the angular frequency of the grid-side voltage \( u_{\infty} \) [9]. We also note that the damping correction loop does not affect the steady-state frequency droop characteristics, since its output is zero at steady state.

2) Grid Interface: As shown in Fig. 2(c), the synchronverter is connected to the grid via an L-type filter \( R_s + jX_s \) and a transmission line with impedance \( R_e + jX_e \). Since the grid condition is assumed to be predominantly inductive, the total reactance \( X_t := X_s + X_e \gg R_s + R_e \). Let \( \psi_f \) denote the excitation flux from the RPL, then the synchronverter inner voltage reference \( e_g \) is evaluated as follows:

\[
e_g = \omega_g \psi_f \left[ \sin \theta_g \sin \left( \theta - \frac{2\pi}{3} \right) \sin \left( \theta + \frac{2\pi}{3} \right) \right]^T,
\]

and its line-to-line RMS value is \( E_g = \sqrt{3/2} \omega_g \psi_f \). With \( e_g \) in place, the VSC is operated with pulse width modulation (PWM) technique. Let \( U_{\infty} \) denote the line-to-line RMS value of \( u_{\infty} \). Then the electromagnetic torque is given by

\[
T_e = \frac{P_t}{\omega_N} \approx \sqrt{3} \frac{\psi_f U_{\infty} \sin \theta_{g,\infty}}{2 X_t},
\]

where \( P_t \) denotes the active-power output [9].

Remark 1 (On Cascaded Voltage and Current Loops and Virtual Impedance Branch). More detailed VSG models [21], which include the LCL filter, the cascaded voltage and current loops, and virtual impedance branch, can be reduced to the one in Fig. 2 via two considerations. First, the cascaded voltage and current loops (including the converter-side impedance \( R_1 + jX_1 \) of the LCL filter) can be approximated as unity gain blocks when tuning parameters or analyzing dynamics related to the output power [22]. The justification for this approximation is that the typical timescales of operation for the cascaded loops, in the range of 1–10 ms, are much smaller than those of the APL and RPL with 0.1–1 s [23]. Next, the virtual impedance branch with impedance \( R_e + jX_e \) can be equivalently connected in series to the grid-side impedance \( R_2 + jX_2 \) of the LCL filter. Thus, by setting \( R_s = R_1 + R_2 \) and \( X_s = X_1 + X_2 \), the synchronverter in Fig. 2 and other LCL-filter-based VSG designs share similar output-power dynamics. We also note that assigning a positive virtual resistance \( R_e \) effectively suppresses potential VSG synchronous resonance, which is caused by small \( R/X \) ratio or sufficiently fast output-power dynamics [7], [24]. Since the required virtual resistance value \( R_e \) is typically much smaller than the total system reactance \( X_t \), the predominantly inductive grid assumption in our paper still holds.

B. Parameter Tuning by Direct Computation [10]

As demonstrated in [10] and summarized above, the APL dynamics can be accurately captured by a third-order model, the block diagram of which is shown in Fig. 3. Furthermore, the third-order model has the following characteristic equation:

\[
s^3 + bs^2 + Ks + d = 0,
\]

where

\[
b = \frac{J_d + \tau_f D_p}{\tau_f J_g},
\]

\[
K = \frac{1}{\tau_f J_g} \left( D_p + D_f \sqrt{3 U_{\infty} \cos \theta_{g,\infty}} \right),
\]
Among them, parameters that need to be specified: its reference value LPF noise rejection ability. The remaining two parameters, based on the specified state variation in the input torque by obtain (6) in Appendix B. The three roots of (6), denoted $g_2, g_3$, represent the APL poles. Let $\omega_n^* \text{ and } \zeta^*$ denote the desired APL natural frequency and damping ratio, respectively. Then, as shown in Fig. 4, the desired locations for two of APL poles, $s_2$ and $s_3$, are specified as

\[
s_2 = s_2^\circ, \quad s_3 = s_3^\circ, \quad s_2^\circ = \omega_n^* e^{j(\pi - \phi^*)}, \quad s_3^\circ = \omega_n^* e^{j(\pi + \phi^*)}, \quad \phi^* = \arccos \zeta^* \in [0, \pi/2] \text{ rad}.
\]

respectively, where $\phi^* = \arccos \zeta^* \in [0, \pi/2] \text{ rad}$. According to the model outlined in Section II-A, the APL has four control parameters that need to be specified: $D_p$, $\tau_f$, $J_g$, and $D_f$. Among them, $D_p$ is determined based on the local grid code as

\[
D_p = \frac{\Delta T_{ns}}{\Delta \omega_{g}},
\]

where $\Delta \omega_{g}$ represents the steady-state deviation of $\omega_g$ from its reference value $\omega_g^*$, and $\Delta T_{ns}$ is the corresponding steady-state variation in the input torque $T_{ns}$ required by the grid code [8]. The time constant $\tau_f$ is chosen based on the required LPF noise rejection ability. The remaining two parameters, $J_g$ and $D_f$, are conventionally tuned iteratively by repeatedly checking system poles under different parameter settings via small-signal analysis [9]. However, the iterative tuning method requires repeated evaluation of system poles and significant trial-and-error effort. To overcome the shortcomings of iterative methods, the method in [10] computes $J_g$ and $D_f$ directly based on the specified $\omega_n^*$ and $\zeta^*$ as follows:

\[
J_g = \frac{\sqrt{2} \psi_j U_{\infty} \cos \theta^*_{g_{\infty}} - \tau_f D_p X_1 \omega_n^*}{\omega_n^* D_p X_1 (1 - 2\tau_f \omega_n^* \zeta^*)},
\]

\[
D_f = \frac{2 \psi_j \zeta^*}{\omega_n^*} + \frac{\tau_f \psi_j}{1 - 2\tau_f \omega_n^* \zeta^*}.
\]

With $J_g$ and $D_f$ computed from (13) and (14), as depicted in Fig. 5, we aim to achieve APL time-domain dynamics with desired maximum relative overshoot $M_p$ and settling time $t_s$ given by, respectively, [25]

\[
M_p = e^{-\pi \cos \phi^*}, \quad t_s = \frac{4}{\zeta^* \omega_n^*}.
\]

Via a numerical example 1 below, we find that for certain pairs of $\omega_n^*$ and $\zeta^*$, we cannot achieve the desired APL dynamics with transient-response specifications according to (15).

Example 1 (Impact of Choice of $\omega_n^*$ and $\zeta^*$ on APL Dynamics). In this example, we examine the impact of various settings for $\omega_n^*$ and $\zeta^*$ on actual APL dynamics. We use (13) and (14) to compute $J_g$ and $D_f$ based on specified $\omega_n^*$ and $\zeta^*$. Then using the PSCAD/EMTDC simulation software, we simulate the full-order system in Fig. 2 with the resulting values of $J_g$ and $D_f$ to obtain the actual APL step response to a change in $P_i^*$. Note that the synchronverter simulation model fully considers the APL and RPL dynamics as well as the PWM switching sequence control. Assume that $D_p = 120 \text{ N} \cdot \text{m} / \text{s} / \text{rad}$ according to local grid code, and all other parameter settings are reported in Appendix C. We consider two cases with $\omega_n^*$ set to be 48 (Case I) and 100 rad/s (Case II). In both cases, we choose $\zeta^* = 0.707$, which is the optimal damping ratio value that enables the APL to achieve fastest response with minimum overshoot. The actual APL step response (together with the desired APL step response with transient performance specifications in (15)) in Cases I and II are plotted in Fig. 6. As shown in
We derive the feasible pole-placement region that satisfies (16) in analytical form. We first derive the feasible range of \( \omega_n^* \) for a fixed \( \zeta^* \) (or equivalently \( \varphi^* \)). The key observation is that depending on the value of \( D_p \), there are three possible cases to consider. Then, by varying \( \zeta^* \) from 1 to 0 (or equivalently, varying \( \varphi^* \) from 0 to \( \pi/2 \) rad), we visualize the entire feasible pole-placement region in the \( s \)-plane, which also consists of three cases depending the value of \( D_p \). Finally, based on the pole-placement region analysis, we improve the direct computation method in [10] by eliminating the trial-and-error effort needed to specify \( \omega_n^* \) and \( \zeta^* \).

A. Analytical Characterization

Here, we specify a particular value for \( \zeta^* \) and study the range of values that \( \omega_n^* \) can take to ensure \( s_2 \) and \( s_3 \) satisfy (16) and represent the APL dominant mode. To begin our analysis, we express \( s_1 \) as a function of \( \omega_n^* \) and \( \zeta^* \). Application of Vieta’s formulas for (6) yields \( s_1 \) expressed as [26]

\[
 s_1 = -\frac{d}{s_2 s_3} = -\frac{d}{\omega_n^*}.
\]

(17)

By substituting (13) into (9), and further substituting the resultant into (17), we express \( s_1 \) as the following function of \( \omega_n^* \) and \( \zeta^* \):

\[
 s_1(\omega_n^*, \zeta^*) = -\frac{M^2 (2\tau_f \zeta^* \omega_n^* - 1)}{\tau_f (\omega_n^* + M)(\omega_n^* - M)},
\]

(18)

where

\[
 M = \sqrt{\frac{3 \psi U_{\infty} \cos \theta_{\infty}}{2 D_p \tau_f X_t}},
\]

(19)

with the values of \( \tau_f \) and \( D_p \) already set based on other design considerations mentioned in Section II-B. We note that according to (18), the value of \( s_1 \) is parametrized by the value taken by \( M \). Also, since \( M \) is inversely proportional to \( \sqrt{D_p} \), the range for \( \omega_n^* \) satisfying (16) similarly depends on the value of \( D_p \). In order to delineate the feasible range of \( \omega_n^* \), we define an auxiliary variable \( \mu \geq 0 \), as follows:

\[
 \mu := \frac{1}{2\tau_f M} = \sqrt{\frac{D_p}{N}},
\]

(20)

where

\[
 N = \frac{2\sqrt{\psi U_{\infty} \cos \theta_{\infty}}}{X_t}.
\]

(21)

Next, we separately consider cases where \( \mu = 0 \) and \( \mu > 0 \) and characterize the range of \( \omega_n^* \) that satisfies (16) in each.

1) \( \mu = 0 \) (\( D_p = 0 \)): The synchronverter is not required by the grid code to provide primary frequency regulation. With \( D_p = 0 \), the expression for \( s_1 \) in (18) simplifies as

\[
 s_1 = 2\omega_n^* \zeta^* - \frac{1}{\tau_f}.
\]

(22)

By substituting (22) into (16), and bear in mind that \( \omega_n^* > 0 \), we get the range of \( \omega_n^* \), which ensures that \( s_2 \) and \( s_3 \) are APL dominant poles, as follows:

\[
 \omega_n^* \in \left( 0, \frac{1}{3\tau_f \zeta^*} \right).
\]

(23)
2) $\mu > 0$ ($D_p > 0$): The synchronverter is required by the local grid code to provide frequency regulation with $D_p > 0$. By substituting (18) into (16), we get

$$F(\omega^*_n) - 1 + \frac{1}{(\omega^*_n + M)(\omega^*_n - M)} > 0,$$

(24)

where

$$F(\omega^*_n) = -\frac{\tau_f \zeta \omega^*_n^3}{M^2} + 3\tau_f \zeta \omega^*_n.$$  

(25)

Recognizing that $\omega^*_n > 0$ and $M > 0$ by definition, the conditions in (24) can be equivalently expressed as

$$0 < \omega^*_n < M \quad \text{and} \quad F(\omega^*_n) < 1,$$

(26)

or

$$\omega^*_n > M \quad \text{and} \quad F(\omega^*_n) > 1.$$  

(27)

We leverage a graphical approach to solve the range of $\omega^*_n$ from (26) and (27). Specifically, we plot $F(\omega^*_n)$ in Fig. 8, and note that when $\omega^*_n = M$, $F(\omega^*_n)$ takes its maximum value as

$$F_{\max} = F(M) = 2\tau_f M \zeta^* = \frac{\zeta^*}{\mu}.$$

(28)

The relationship in (28) implies that the ratio between $\zeta^*$ and $\mu$ determines the relationship between $F_{\max}$ and 1 and influences the feasible range of $\omega^*_n$ solved from (26) and (27). Next, we characterize the feasible range of $\omega^*_n$ with $0 < \mu < \zeta^*$ and $\mu \geq \zeta^*$.

(i) $0 < \mu < \zeta^*$ ($0 < D_p < N\zeta^2$): According to (28), $F_{\max} > 1$. As illustrated in Fig. 8(a), setting $F(\omega^*_n) = 1$ yields two positive-valued roots, which we denote by $\omega^*_n_1$ and $\omega^*_n_2$ (with $0 < \omega^*_n_1 < \omega^*_n_2$). Note that the solutions depend on $\zeta^*$ and the roots can be solved from $F(\omega^*_n) = 1$ via numerical methods or Cardano’s formula [26]. Here, leveraging the intuition from Fig. 8(a), the solution of (26) and (27) is given by

$$\omega^*_n \in (0, \omega^*_n_1) \cup (M, \omega^*_n_2).$$

(29)

(ii) $\mu \geq \zeta^*$ ($D_p \geq N\zeta^2$): In this case, (28) implies that $F_{\max} \leq 1$, so $F(\omega^*_n) \leq 1$ for all $\omega^*_n > 0$, and (27) is never satisfied. Thus, as shown in Fig. 8(b), the solution of (26) is given by

$$\omega^*_n \in (0, M).$$

(30)

To sum up, for a particular $\zeta^* = \cos \varphi^*$, the range of $\omega^*_n$ ensuring that $s_2$ and $s_3$ represent the APL dominant mode is given by

$$\omega^*_n \in \begin{cases} 
(0, \frac{1}{3\tau_f \zeta^*}), & \text{if } \mu = 0, \\
(0, \omega^*_n_1) \cup (M, \omega^*_n_2), & \text{if } 0 < \mu < \zeta^*, \\
(0, M), & \text{if } \mu \geq \zeta^*.
\end{cases}$$

(31)

We refer to these conclusions on $\mu$ summarized in (31) as the $\mu$-condition. Based on (15) and (31), we can indeed predict that the settling time of achievable APL dynamics is within the following range

$$t_s \in \begin{cases} 
(12\tau_f, +\infty), & \text{if } \mu = 0, \\
\left(\frac{4}{\zeta^* \omega^*_n_2}, \frac{4}{\zeta^* M}\right), & \text{if } 0 < \mu < \zeta^*, \\
\left(\frac{4}{\zeta^* M}, +\infty\right), & \text{if } \mu \geq \zeta^*.
\end{cases}$$

(32)

Example 2 (Revisiting Example 1 with the $\mu$-Condition). In this example, we use the $\mu$-condition to justify the impact of choice of $\omega^*_n$ and $\zeta^*$ on the pole-placement results observed in Example 1. Recall that $D_p = 120 \text{ N}\cdot\text{m/s}$ and $\zeta^* = 0.707$, so that $\mu = 0.84 > \zeta^*$ according to (20). Also, $M$ is computed to be 59.35 from (19). Thus, according to (31), setting $\omega^*_n \in (0, 59.35) \text{ rad/s}$ would ensure that $s_2$ and $s_3$ represent the APL dominant mode. In time-domain quantities, the achievable APL settling time $t_s$ is within the range $(0.095, +\infty)$ s. In Case I, we set $\omega^*_n$ to be $48 \in (0, 59.35) \text{ rad/s}$, and as shown in Fig. 7(a), (16) is satisfied and $s_2$ and $s_3$ are indeed the APL dominant poles as desired. However, in Case II, $\omega^*_n = 100 \notin (0, 59.35) \text{ rad/s}$, and thus as depicted in Fig. 7(b), (16) does not hold and instead $s_1$ represents the APL dominant mode.

Remark 2 (Range of $\omega^*_n$ to Ensure that $s_1 < m\text{Re}(s_2)$, $m > 1$). In order to achieve desired APL dynamics with greater precision, we may further limit the impact of $s_1$ by revising the condition in (16) to be $s_1 < m\text{Re}(s_2)$, where $m > 1$. To this end, when specifying $\omega^*_n$ before computing $J_g$ and $D_f$, the feasible range of values that $\omega^*_n$...
can take is of the following form:

\[
\omega_n^* \in \begin{cases} 
0, & \mu = 0, \\
\frac{1}{(m+2)\tau_f \zeta^*}, & 0 < \mu < \sqrt{\frac{(m+2)^3}{27m}} \zeta^*, \\
(0, M), & \mu \geq \sqrt{\frac{(m+2)^3}{27m}} \zeta^*,
\end{cases}
\]

where \( \omega_{n1}^* \) and \( \omega_{n2}^* (\omega_{n1}^* < \omega_{n2}^*) \) are the two positive-valued roots of

\[
-m\tau_f \zeta^* \omega_n^3 \frac{1}{M^2} + (m+2)\tau_f \zeta^* \omega_n^* = 1.
\]

For brevity, we omit the derivation of (33) as it is similar to that of (31).

**B. Visual Representation**

Based on the feasible range of \( \omega_n^* \) for specific \( \zeta^* = \cos \phi^* \) in (31), we vary \( \zeta^* \) from 1 to 0, or equivalently vary \( \phi^* \) from 0 to \( \pi/2 \), and visualize the entire feasible pole-placement region for the APL dominant poles \( s_2 \) and \( s_3 \) in the \( s \)-plane. There are three possibilities for the feasible pole-placement region of \( s_2 \) and \( s_3 \) depending on the value of \( \mu \), as shown in Fig. 9.

1) **Pattern (a) with \( \mu = 0 (D_p = 0) \):** Since (23) holds for all \( \phi^* \in [0, \pi/2) \), the real parts of \( s_2 \) and \( s_3 \) satisfy

\[
\text{Re}(s_2) = \text{Re}(s_3) = -\omega_n^* \zeta^* \left( -\frac{1}{3\tau_f}, 0 \right).
\]

Thus, as marked by the green colour in Fig. 9(a), the region in which we can place \( s_2 \) and \( s_3 \) as APL dominant poles is

\[
\Omega_f = \left\{ z \in \mathbb{C} \mid -\frac{1}{3\tau_f} < \text{Re}(z) < 0 \right\}.
\]

2) **Pattern (b) with \( 0 < \mu < 1 (0 < D_p < N) \):** The feasible pole-placement region \( \Omega_f \) for the APL dominant poles is given by the union of regions \( \Omega_{f1} \) and \( \Omega_{f2} \), i.e., \( \Omega_f = \Omega_{f1} \cup \Omega_{f2} \), which we describe separately below.

(i) **Region \( \Omega_{f1} \):** With \( \phi^* \) varying from 0 to \( \arccos \mu \), we have \( 0 < \mu < \cos \phi^* = \zeta^* \). This corresponds to the second case summarized in (31), and thus as marked by the sea-green colour in Fig. 9(b), \( \Omega_{f1} \) is given by

\[
\Omega_{f1} = \left\{ z = \omega_n^* e^{j(\pi \pm \phi^*)} \mid 0 \leq \phi^* < \arccos \mu, 0 < \omega_n^* < \omega_n^*(\phi^*) \right\}.
\]

where \( \omega_n^*(\phi^*) \) and \( \omega_n^*(\phi^*) \) \( (0 < \omega_n^*(\phi^*) < \omega_n^*(\phi^*)) \) are the two positive-valued roots of \( F(\omega_n^*) = 1 \). Note that \( \omega_n^1 \) and \( \omega_n^* \) are functions of \( \phi^* \), since the function \( F(\omega_n^*) \) in (25) parameterized by the value taken by \( \zeta^* \) and the equivalent of \( \phi^* \).

(ii) **Region \( \Omega_{f2} \):** With \( \phi^* \) varying from \( \arccos \mu \) to \( \pi/2 \), we have \( \mu \geq \cos \phi^* = \zeta^* \). This corresponds to the third case in (31). Also, based on (19), we note that \( M \) is independent of \( \phi^* \). Thus, as shown by the two circle sectors shaded in green colour in Fig. 9(b), \( \Omega_{f2} \) is given by

\[
\Omega_{f2} = \left\{ z = \omega_n^* e^{j(\pi \pm \phi^*)} \left| \arccos \mu \leq \phi^* < \frac{\pi}{2} \right. \right\}.
\]

3) **Pattern (c) with \( \mu \geq 1 (D_p \geq N) \):** For \( \phi^* \in [0, \pi/2) \), we have \( \mu \geq 1 \geq \cos \phi^* = \zeta^* \). This corresponds to the third case in (31). Accordingly, as marked by green colour in Fig. 9(c), the feasible pole-placement region for the APL dominant poles is given by

\[
\Omega_f = \left\{ z = \omega_n^* e^{j(\pi \pm \phi^*)} \left| 0 \leq \phi^* < \frac{\pi}{2}, 0 < \omega_n^* < M \right. \right\}.
\]

**C. Updated Parameter Tuning Method**

Based on the analysis of the feasible pole-placement region above, we improve the tuning method for the APL parameters \( D_p, \tau_f, J_g, \) and \( D_f \) in Section II-B as follows. As before, we first compute \( D_p \) with (12) according to local grid code and further determine \( \tau_f \) based on the LPF noise rejection requirements. With these, we solve the system equilibrium point and compute \( \mu \) using (20). Then, we choose the desired damping ratio \( \zeta^* \) for the APL dominant mode. After that, we determine the desired natural frequency \( \omega_n^* \) in the range (31) or (33) if \( s_1 < m\text{Re}(s_2), m > 1, \) is required) with due consideration for the desired APL response speed. Finally, we compute \( J_g \) and \( D_f \) according to (13) and (14). The revised method described above is depicted graphically in Fig. 10. Unlike the method in [10] portrayed in Fig. 1, we do not need to repetitively specify \( \omega_n^* \), and compute \( J_g \) and \( D_f \) until the APL poles \( s_2 \) and \( s_3 \) indeed represent the APL dominant
mode. In the next section, we verify the analytical development for the feasible pole-placement region via simulation studies.

**Remark 3** (RPL Parameter Tuning Method). Although the analysis and tuning procedure presented thus far focuses on the APL, a similar line of reasoning can be applied to parameter tuning for the RPL. Interested readers may refer to Appendix A for more details on the analysis and ensuing parameter tuning for the RPL.

### IV. CASE STUDIES

In this section, via time-domain simulations of the system in Fig. 2, we verify that by choosing $\omega_n^*$ within the range specified by the derived $\mu$-condition in (31), for a particular $\zeta^*$, we indeed satisfy (16) and ensure that $s_2$ and $s_3$ represent the APL dominant mode. We also validate the feasible pole-placement region derived in Section III-B based on the $\mu$-condition and visualized in the $s$-plane. Moreover, we demonstrate that the improved parameter tuning method effectively enables the synchronverter to achieve desired APL dynamics when it is connected to a benchmark test power system. The system under study in Sections IV-A and III-B is the synchronverter-connected system in Fig. 2 and values for all system parameters except $D_p$ are reported in Appendix C. The system simulated in Section III-C is modified from the New England 39-bus test system reported in [27] and [28].

#### A. Verification of $\mu$-Condition for Specific Choice of $\zeta^*$

In order to verify the proposed $\mu$-condition as summarized in (31), we choose APL damping ratio $\zeta^* = 0.707$ and consider three cases: I) $D_p = 0 \text{ N} \cdot \text{m} \cdot \text{s}/\text{rad}$, II) $D_p = 75 \text{ N} \cdot \text{m} \cdot \text{s}/\text{rad}$, and III) $D_p = 120 \text{ N} \cdot \text{m} \cdot \text{s}/\text{rad}$. According to (20), we get $\mu = 0$ in Case I, $0 < \mu < \zeta^*$ in Case II, and $\mu > \zeta^*$ in Case III, which correspond to the three possible options in (31). With these parameter settings, we compute the range of $\omega_n^*$ satisfying (16) based on the $\mu$-condition in (31) and shade the resulting regions in green colour in Figs. 11(a)–(c). To verify the range of feasible $\omega_n^*$ obtained from (31), we vary $\omega_n^*$ from 0 to 140 rad/s, compute $J_p$ and $D_f$ using (13) and (14), substitute the resultant values into the APL characteristic equation (6), and solve for the actual APL poles $s_1$, $s_2$, and $s_3$ as the roots of (6). We plot $s_1$ and $\text{Re}(s_2)$ with respect to $\omega_n^*$ as the blue and red traces, respectively, in Figs. 11(a)–(c) for Cases I–III, respectively. Visual inspection of the red and blue traces in Figs. 11(a)–(c) reveals that indeed (16) is satisfied only when $\omega_n^*$ lies within the green region, as predicted by (31). Thus, by specifying $\omega_n^*$ within the range predicted by the $\mu$-condition, we ensure that $s_2$ and $s_3$ represent the APL dominant mode and subsequently achieve desired APL dynamic performance. Furthermore, the latter is also evident by checking the actual APL step response with different choices of $\omega_n^*$ in Cases I–III against desired response satisfying (15) as shown in Figs. 11(d)–(f). Although we verify the $\mu$-condition in (31) only for $\zeta^* = 0.707$ above, we note that it holds for all possible $\zeta^* \in (0, 1]$.

#### B. Verification of Feasible Pole-placement Region

In this study, we further verify that by placing $s_2$ and $s_3$ within the feasible pole-placement region, we automatically satisfy (16) and ensure that $s_2$ and $s_3$ are the APL dominant poles. As shown in Fig. 9, the feasible pole-placement region consists of three cases (Cases I–III) depending on the value of $\mu$. We verify the three cases discussed in Section III-B by setting $D_p$ to be 0, 90, and 200 N · m · s/rad, which result in $\mu = 0$, $0 < \mu < 0.730 < 1$, and $\mu = 1.088 > 1$ in Cases I, II, and III, respectively. We compute the feasible pole-placement regions in Cases I–III with (36)–(39) and shade them with green or sea-green colours in Fig. 12. Then, we validate the pole-placement regions predicted in (36)–(39) by placing the APL poles $s_2$ and $s_3$ within and outside these regions and checking the actual resulting poles. In both Cases I and III,
Fig. 12. Verification of feasible pole-placement region developed based on the \(\mu\)-condition when (a) \(\mu = 0\) \((D_p = 0)\), (b) \(0 < \mu = 0.730 < 1\) \((0 < D_p = 90 < N)\), and (c) \(\mu = 1.088 > 1\) \((D_p = 200 > N)\). We shade the feasible pole-placement region with sea-green and green colours. By specifying the desired APL poles \(s^\star_2\) and \(s^\star_3\) within the feasible pole-placement region, (16) is satisfied and \(s_2\) and \(s_3\) indeed represent the APL dominant mode.

![Chart](image)

The poles \(s_2\) and \(s_3\) marked with \(\times\) within the green region satisfy (16), while it is not true for those marked with \(\Box\) outside the green region. Similarly, in Case II, we have (16) for the APL poles marked with \(\times\) and \(\triangle\) within the feasible pole-placement region, while not for those marked with \(\Box\) and \(\bigcirc\) outside the highlighted region. Based on these observations, we conclude that (16) is satisfied only if \(s^\star_2\) and \(s^\star_3\) are chosen to be within the feasible pole-placement region computed from (36)–(39). Thus, we validate the feasible pole-placement region visualized in the \(s\)-plane in Section III-B.

C. Verification in New England 39-Bus Test System

To further validate the feasible pole-placement region analysis as well as the updated tuning method in Section III, we implement it to tune the parameters for a synchronverter connected to a 39-bus test system, the one-line diagram of which is shown in Fig. 13. The test system is modified from the New England 39-bus system reported in [27] and [28], and as shown in Fig. 13, we replace the synchronous generator at Bus 30 with a synchronverter-controlled VSC with rated capacity \(S_N = 100\) MVA, rated AC voltage \(U_N = 22.0\) kV, and switching frequency \(f_{sw} = 5\) kHz. The synchronverter is connected to Bus 30 via an \(L\)-type filter with impedance \(R_s + jX_s = 0.080 + j0.754\) \(\Omega\). Note that all generators at Buses 31-39 are equipped with exciters, steam turbines, and governors, which help to stabilize system voltages and achieve primary-frequency regulation.

Next, we tune the APL parameters \(J_g\) and \(D_f\) following the updated procedure described in Section III-C. First, we set the desired APL damping ratio \(\zeta^\star = 0.707\) and compute the system equilibrium. With \(D_p = 0\), we have that \(\mu = 0 < \zeta^\star\). Based on the \(\mu\)-condition in (31), the APL natural frequency \(\omega^\star_n \in (0, 47.1)\) rad/s, and correspondingly, the achievable APL settling time \(t_s \in (0.12, +\infty)\) s. In accordance with its feasible range, we set the desired APL natural frequency \(\omega^\star_n = 22.6\) rad/s, which corresponds to \(t_s = 0.25\) s. Then using (13) and (14), we compute the APL parameters and get \(J_g = 4.63 \times 10^3\) kg \(\cdot\) m\(^2\) and \(D_f = 4.07\) V \(\cdot\) s/\text{rad}.

With the computed synchronverter parameters in place, we model and simulate the synchronverter-connected 39-bus test system in Fig. 13 in the PSCAD/EMTDC simulation software. In this study, we increase the active-power reference \(P^\star_t\) from 0 to...
to 60 MW at $t = 0.5$ s. As shown in Fig. 15, the actual APL step response obtained from the simulation (trace (i)) indeed nearly matches the desired one with $\zeta^* = 0.707$ and $\omega_n^* = 22.6$ rad/s (trace (ii)). Thus, we demonstrate that the updated parameter tuning method, which considers the feasible pole-placement region analysis, tunes the synchronverter without any trial-and-error effort.

**Remark 4** (Verification of RPL Dynamics). Making use of the 39-bus test system, we verify the analysis for RPL dynamics in Appendix A. Assume that the synchronverter is not required to perform voltage-droop control, i.e., $D_g = 0$. Next, recall that LPF time constant $\tau_f = 0.01$ s, so the settling time $t_s'$ of the RPL step response is freely adjustable within the range $(0.08, +\infty)$ s. Bearing this in mind, we choose the desired RPL dominant pole to be $s_1^* = -5 \in \Omega_f^{/2}$, which corresponds to $t_s' = 4/|s_1^*| = 0.8$ s. Then according to (47), we get $K_g = 2.49 \times 10^6 \text{Var} \cdot \text{rad}/\text{V}$. Finally, we simulate the system in PSCAD/EMTDC by increasing the reactive-power reference $Q_t^*$ from 0 to 40 MVar at $t = 5.5$ s. As shown in Fig. 16, the actual RPL step response (trace (i)) matches well with the desired one with $t_s' = 0.8$ s (trace (ii)).

**V. Concluding Remarks**

In this paper, we derive the $\mu$-condition to compute the range of the APL natural frequency $\omega_n^*$ for chosen APL damping ratio $\zeta^*$. Within the range of $\omega_n^*$ predicted by the $\mu$-condition, we are able to freely place the APL dominant poles and achieve desired APL dynamic performance. Thus, by incorporating the $\mu$-condition into the tuning procedure of the synchronverter parameters, we eliminate the trial-and-error effort of repeatedly specifying $\omega_n^*$ for the APL dominant mode, computing the APL parameters, and checking that $s_2$ and $s_3$ indeed represent the APL dominant mode. Based on the $\mu$-condition, we visualize the feasible pole-placement region in the $s$-plane. Within this region, we are able to place the APL dominant poles freely and achieve desired APL dynamic performance. A similar line of reasoning is applied to determine the feasible range of achievable RPL dynamics and the corresponding parameters that lead to desired RPL dynamics. Compelling directions for future work include exploring the impact of the LPF filter on the feasible pole-placement region, computing the feasible pole-placement region of other VSG designs, and developing a systematic method of computing the feasible pole-placement region for other higher-order controller systems.

**APPENDIX**

**A. Modelling, Analysis, and Tuning of RPL Dynamics**

**Modelling.** According to Figs. 2(b) and (c), we can describe the RPL dynamics with the small-signal RPL model in Fig. 17, in which [9]

$$Q_t = \frac{X_t}{X_t^2} E_g^2 - \frac{X_t}{X_t^2} t_{\infty}^2 + \frac{X_s - X_t}{X_t^2} E_g U_{\infty} \cos \theta_{\infty}, \quad (40)$$

$$U_t = \sqrt{\frac{X_t^2}{X_t^2} E_g^2 + \frac{X_s - X_t}{X_t^2} U_{\infty}^2 + \frac{2X_s X_t}{X_t^2} E_g U_{\infty} \cos \theta_{\infty}}. \quad (41)$$

Then, leveraging Mason’s gain formula [30], we get the RPL transfer function model as follows

$$\Delta Q_t = G_3(s) \Delta Q_t^* + G_4(s) \Delta U_t^* \quad (42)$$

where $G_3(s)$ and $G_4(s)$, respectively, describe the dynamics in $\Delta Q_t$ with respect to $\Delta Q_t^*$ and $\Delta U_t^*$. In (42), we note that $G_3(s)$ and $G_4(s)$ share the following second-order characteristic equation

$$1 + K s \left( s + \frac{1}{\tau_f} \right) = 0, \quad (43)$$

where

$$K := \frac{\partial Q_t}{\partial \psi_{f}} \bigg|_{x^o} + D_g \sqrt{\frac{2}{\partial U_t}} \bigg|_{x^o} \propto K_g. \quad (44)$$

**Analysis.** Let $s_4$ and $s_5$, where $\text{Re}(s_5) \leq \text{Re}(s_4)$, denote the two roots of the RPL characteristic equation (43). By varying $K$ from 0 to $+\infty$, we plot the root loci patterns of (43) in Fig. 18. According to Fig. 18, the feasible RPL...
pole-placement region $\Omega'_f$ is the union of two regions, i.e., $\Omega'_f = \Omega'_{f1} \cup \Omega'_{f2}$, which we describe separately below.

1) $K \in (0, 4\tau_f^2]$: As shown in Fig. 18, increasing $K$ from $0$ to $4\tau_f^2$ makes $s_4$ and $s_5$ move toward each other along the line $\Re(s_i) = -1/(2\tau_f)$ until they meet at the point $B$, i.e., $(-1/(2\tau_f), 0)$, on the real axis. Thus, the first part of the feasible RPL pole-placement region, in which we can place the RPL dominant poles $s_4$ and $s_5$ freely, is along the line described by

$$\Omega'_{f1} = \left\{ z \in \mathbb{C} \mid \Re(z) = -\frac{1}{2\tau_f} \right\}. \quad (45)$$

In the time domain, the settling time $t'_s$ of the RPL step response is fixed at $t'_s = 4/|\Re(s_4)| = 8\tau_f$ while the overshoot remains adjustable.

2) $K \in (4\tau_f^2, \infty)$: As shown in Fig. 18, increasing $K$ from $4\tau_f^2$ to $\infty$ makes $s_4$ and $s_5$ move away from each other along the real axis until they, respectively, arrive at points $A$ and $O$. In this case, we have $s_5 < s_4$ and so $s_4$ dominates the RPL dynamics. Thus, the second part of the feasible RPL pole-placement region, in which we can place the RPL dominant pole $s_4$ freely, is the segment $BO$, i.e.,

$$\Omega'_{f2} = \left\{ z \in \mathbb{R} \mid -\frac{1}{2\tau_f} < z < 0 \right\}. \quad (46)$$

In the time domain, the settling time $t'_s$ can be freely adjusted within $(8\tau_f, \infty)$ and the RPL dynamics are overdamped.

**Tuning.** Based on the analysis above, we recommend tuning the synchronverter RPL by placing $s_4$ within the region $\Omega'_{f2}$ delineated in (46). In this way, the RPL response is freely adjustable and also well damped. Let $s_4^*$ denote the desired location of $s_4$ in the $s$-plane, substitute $s_4 = s_4^*$ and (44) into (43), and we get

$$K_g = -\frac{\partial Q_e}{\partial s} \bigg|_{s^*} + D_q \sqrt{3} \frac{\partial U}{\partial s} \bigg|_{s^*} s_4^* \left(\tau_f s_4^* + 1\right). \quad (47)$$

Recall that the voltage-droop constant $D_q$ is determined based on the grid code, we can directly compute the remaining parameter $K_g$ according to (47). In this way, we achieve RPL time-domain dynamics with $t'_s = 4/|s_4^*|$ [25].

**B. Small-signal APL Model**

The APL dynamics are sufficiently described by (1)–(3) and (5). Let $\Delta(\cdot)$ denote small-signal perturbations in variable $(\cdot)$. Note that $\omega_g^*$ is a reference value for $\omega_g$ and remains fixed, i.e., $\Delta \omega_g^* = 0$. Thereafter, by linearizing (1)–(3) and (5) around the system equilibrium point, taking the Laplace transformation of the resultant, we can obtain the expression for $\Delta \theta_{g\infty}$ (see Section III in [10]). Finally, substitution of the expression for $\Delta \theta_{g\infty}$ into the linearized version of (5) yields the following small-signal third-order APL model [10]:

$$\Delta P_t = \frac{d \cdot (\tau_f s + 1) \cdot (\Delta P_t^* - \omega_N (J_p s + D_p) \Delta \omega_{\infty})}{s^3 + bs^2 + KS + d} = G_1(s) \Delta P_t^* + G_2(s) \Delta \omega_{\infty}, \quad (48)$$

where $b, K$, and $d$ are given by (7)–(9), respectively. Note that the third-order APL model in (48) considers the LPF dynamics in (3), and thus captures the APL dynamics with high accuracy, as verified in [10].

**C. Parameters of Synchronverter-connected System** In Fig. 2

$$R_s = 0.741 \Omega, L_s = 20 \text{ mH}, R_e = 0.0 \Omega, L_e = 38.5 \text{ mH},$$

$$\tau_f = 0.01 \text{ s}, D_p = 120 \text{ N\cdotm}\cdot\text{s}/\text{rad}, \omega_N = \omega_g^0 = 376.99 \text{ rad/s}, U_{\infty} = 6.6 \text{ kV}, u_{dc} = 13 \text{ kV},$$

rated grid frequency is 60 Hz, rated ac side voltage is 6.6 kV, and rated synchronverter capacity is 1 MVA.

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