Quickest Line Outage Detection and Identification

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***Abstract*—A method is proposed to detect and identify power system transmission line outages in near real-time. The method exploits the statistical properties of the small random fluctuations in electricity generation and demand that a power system is subject to as time evolves. To detect and identify transmission line outages, a linearized incremental small-signal power system model is used in conjunction with high-speed synchronized voltage phase angle measurements obtained from phasor measurement units. By monitoring the statistical properties of voltage phase angle time-series, line outages are detected and identified using techniques borrowed from the theory of quickest change detection. As illustrated through case studies, the proposed method is effective in detecting and identifying single- and double-line outages in an accurate and timely fashion.**

I. Introduction

To monitor real-time operational reliability, power system operators rely heavily on a model of the system obtained offline; this model contains the transmission network topology, parameters, and historical and forecasted power generation and demand [1], [2]. Thus, the validity and accuracy of online studies are contingent upon the accuracy of the system model used, which is, in turn, heavily dependent upon accurate records and telemetry from remotely monitored circuit breakers. Erroneous records or telemetry, including knowledge of transmission line statuses, have contributed to numerous major North American blackouts [1], [2]. For example, in the 2011 San Diego blackout, operators could not detect that certain lines were overloaded or close to being overloaded because the network model was not up-to-date, causing state estimator results to be inaccurate [2]. Therefore, in order to update the model used in operational reliability studies in a timely manner to reflect current system conditions, there exists an impetus to develop efficient and robust online tools to detect and identify network topology changes. The development of such tools to identify transmission line outages and, in general, changes in network topology, can be enabled by the widespread deployment of phasor measurement units (PMUs). Unlike current system measurements, PMUs measure voltage and current phasors at a very high speed (usually 30 measurements per second) [3], and phasors measured at different locations by different devices are time-synchronized [4].

In this paper, by exploiting the statistical properties of voltage phase angle measurements obtained from PMUs, we propose a method to detect and identify line outages in near real-time based on the theory of quickest change detection (QCD). In QCD, a decision maker observes a sequence of random variables, the probability distribution of which changes abruptly due to some event that occurs at an unknown time, such as a line outage in the present setting. The objective is to detect this change in distribution as quickly as possible subject to a fixed rate of false alarm before the change [5].

We model the incremental changes in net power injection at each bus as independent random variables, the probability distribution of which is determined by random fluctuations in generation and demand. Then, we use an incremental small-signal power flow model to relate the probability distributions of the power injections and the voltage angle measurements via a linear mapping. A decision maker observes the sequence of incremental changes in bus voltage phase angle measurements obtained from PMUs, the statistical properties of which change due to a line outage. By feeding this data sequentially to a QCD algorithm, we can detect the change in distribution. Furthermore, in the proposed QCD-based framework, a sequence of observations is classified into one among multiple hypotheses based on statistical properties of the observed measurements [6]. Hence, the true line outage is identified among a set of credible line outages, and the network topology can be updated in a timely manner.

Issues of inaccurate state estimation results arising from topology errors have been apparent for many years [7]. Numerous approaches have been proposed to detect and identify topology errors in the context of state estimation [8], [9], [10]. The specific issue of determining external system topology errors was explored in [11]. Owing to the potential improvement in situational awareness offered by the widespread deployment of PMUs, recent work has focused on external system line-outage detection and/or identification by taking advantage of voltage phase angle measurements [12], [13], [14]. These rely on the phase angle difference between two sets of PMU voltage phasor measurements obtained before and after the event, and proceed to identify the line outage location via hypothesis testing [12], sparse vector estimation [13], or mixed-integer nonlinear optimization [14]. Existing approaches, however, do not exploit the fact that the line outage is persistent, i.e., once a line outage occurs, it remains as so until it is detected and brought back into service. In our QCD-based framework, the persistence of the fault is exploited to detect the line outage with lower rate of false alarms and misdetections.

This paper builds on our preliminary work reported in [15], providing extensions in several directions. First, in the line-outage model, we expound on the voltage phase angle difference between measurement sets acquired immediately before and after the outage occurs. Second, we modify the QCD

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algorithm to exploit this single-sample change component. Additionally, we demonstrate the effectiveness of the proposed method on the IEEE 118-bus test system, with respect to both single- and double-line outages. Finally, we extend the proposed line outage detection method to the case in which phase angle measurements are not available at all buses.

The remainder of this paper is organized as follows. In Section II, the power system model adopted in this work is described. The statistical line-outage model is developed in Section III. In Section IV, we outline the QCD-based line outage identification algorithms that we propose in this paper. Subsequently, we extend the QCD-based algorithm to detect and identify double-line outages in Section V. In Section VI, we illustrate the proposed ideas via case studies involving the IEEE 118-bus system. In Section VII, we extend the QCD-based algorithm to accommodate the case in which phase angle measurements are not available at all buses. Finally, in Section VIII, we offer concluding remarks and directions for future research.

II. Power System Model

Consider a power system represented by a graph consisting of a set of N nodes denoted by $\mathcal{V} = \{1,\dots,N\}$, each one corresponding to a bus, and a set of L edges denoted by $\mathcal{E}$, i.e., for $m,n \in \mathcal{V}$, $(m,n) \in \mathcal{E}$, there exists a transmission line between buses m and n. With a slight abuse of notation, we also denote by (m,n) the transmission line between buses m and n. At time t, let $V_i(t)$ and $\theta_i(t)$, respectively, denote the voltage magnitude and phase angle at bus i. Also let $P_i(t)$ and $Q_i(t)$, respectively, denote the net active and reactive power injection at bus i. Then, the quasi-steady-state behavior of a power system can be described by the power flow equations, which can be written compactly as real and reactive power balance components at each bus i as follows:

$$P_i(t) = p_i(\theta_1(t),\dots,\theta_N(t), V_1(t),\dots,V_N(t)), \quad (1)$$
$$Q_i(t) = q_i(\theta_1(t),\dots,\theta_N(t), V_1(t),\dots,V_N(t)), \quad (2)$$

where the dependence on network parameters, such as line series and shunt impedances, is implicitly considered in the functions $p_i(\cdot)$ and $q_i(\cdot)$. In the remainder of this section, we describe a linearized incremental small-signal power flow model that arises from the sampled PMU measurements. We also obtain statistical models that describe the PMU measurements of voltage phase angles as they relate to bus active power injections.

A. Linearized Incremental Power Flow Model

Let $P_i[k]$ and $Q_i[k]$, respectively, denote active and reactive power injections in bus i at time instant $t = k\Delta t$, $k = 0,1,2,\dots$, $\Delta t > 0$, i.e., $P_i[k] = P_i(k\Delta t)$ and $Q_i[k] = Q_i(k\Delta t)$. Then, variations in the active and reactive power injections in bus i between pairs of consecutive sampling times $2k\Delta t$ and $(2k+1)\Delta t$ are defined as $\Delta P_i[k] = P_i[2k+1] - P_i[2k]$ and $\Delta Q_i[k] = Q_i[2k+1] - Q_i[2k]$, respectively. Suppose synchronized voltage phasor measurements in all buses are collected using PMUs each Δt unit of time. Then, let $V_i[k]$ and $\theta_i[k]$ denote sampled PMU measurements of the voltage magnitude and phase angle for bus i, $V_i(t)$ and $\theta_i(t)$, at $t = k\Delta t$, respectively. Further, define variations in voltage magnitudes and phase angles between pairs of consecutive sampling times $2k\Delta t$ and $(2k+1)\Delta t$, $k = 0,1,2,\dots$, as $\Delta V_i[k] = V_i[2k+1] - V_i[2k]$ and $\Delta\theta_i[k] = \theta_i[2k+1] - \theta_i[2k]$, respectively.

Next, suppose a solution to (1)–(2) exists at $(\theta_i[2k], V_i[2k], P_i[2k], Q_i[2k])$, $i = 1,\dots,N$, i.e.,

$$P_i[2k] = p_i(\theta_1[2k],\dots,\theta_N[2k], V_1[2k],\dots,V_N[2k]),$$
$$Q_i[2k] = q_i(\theta_1[2k],\dots,\theta_N[2k], V_1[2k],\dots,V_N[2k]),$$

for each bus i. Assume that, for each bus i, $p_i(\cdot)$ and $q_i(\cdot)$ are continuously differentiable with respect to each θ_i and V_i at $\theta_i[2k]$ and $V_i[2k]$, $i = 1,\dots,N$. Then, assuming that $\Delta\theta_i[k]$ and $\Delta V_i[k]$ are sufficiently small, we can approximate $\Delta P_i[k]$ and $\Delta Q_i[k]$ via a first-order Taylor series expansion of (1)–(2) as

$$\begin{aligned} \Delta P_i[k] &\approx \sum_{j=1}^{N} a_{ij}[2k]\Delta\theta_j[k] + \sum_{j=1}^{N} b_{ij}[2k]\Delta V_j[k], \\ \Delta Q_i[k] &\approx \sum_{j=1}^{N} c_{ij}[2k]\Delta\theta_j[k] + \sum_{j=1}^{N} d_{ij}[2k]\Delta V_j[k], \end{aligned} \quad (3)$$

where

$$a_{ij}[2k] = \frac{\partial p_i}{\partial \theta_j},\ b_{ij}[2k] = \frac{\partial p_i}{\partial V_j},$$
$$c_{ij}[2k] = \frac{\partial q_i}{\partial \theta_j},\ d_{ij}[2k] = \frac{\partial q_i}{\partial V_j},$$

for each bus i, all evaluated at $(\theta_i[2k], V_i[2k])$, $i = 1,\dots,N$.

A standard assumption used in analysis of transmission systems is that, for each bus i, $a_{ij}[2k]$ and $d_{ij}[2k]$ are much larger than $b_{ij}[2k]$ and $c_{ij}[2k]$, for all $j = 1,\dots,N$ [16]. This effectively decouples (3) so that variations in active power injections primarily affect bus voltage angles, while variations in reactive power injections mainly affect bus voltage magnitudes. For analysis purposes, we assume the decoupling assumption holds and only consider $\Delta P_i[k] \approx \sum_{j=1}^{N} a_{ij}[2k]\Delta\theta_j[k]$. Further, under the so-called DC assumptions,[1] namely (i) the system is lossless, (ii) $V_i[k] = 1$ per unit (p.u.) for all $i \in \mathcal{V}$, k, and (iii) $\theta_m[k] - \theta_n[k] << 1$ for all k and for $m,n \in \mathcal{V}$, $a_{ij}[k]$ simply becomes the negative of the imaginary part of the $(i,j)^{th}$ entry of the network admittance matrix constructed while neglecting transmission line resistances [16]. Under these assumptions, then, $a_{ij}[2k]$ becomes a function of only the network, i.e., $a_{ij}[2k] = a_{ij}$, for all k under the same topology.

Without loss of generality, we set the system reference angle to be that of the voltage phase angle at bus 1 in the system, i.e., $\theta_1(t) = 0$, $t \geq 0$. Then, for each $i \in \mathcal{V}$, $i \neq 1$, under the DC assumptions, the small-signal model in (3) can be approximated as follows:

$$\Delta P_i[k] \approx \sum_{j\in\mathcal{V}, j\neq 1} a_{ij}\Delta\theta_j[k]. \quad (4)$$

[1] Note that even though we employ DC assumptions, the resultant model in (4) in this paper is not the conventional DC power flow model.

TABLE I
PARAMETER VALUES FOR 3-BUS SYSTEM SHOWN IN FIG. 1.

P_2	P_3	$X_{(1,2)}$	$X_{(2,3)}$	$X_{(1,3)}$
-1	-0.9	0.0504	0.0372	0.0636

Define vectors $\Delta P[k] \in \mathbb{R}^{N-1}$ and $\Delta\theta[k] \in \mathbb{R}^{N-1}$, the entries of which are $\Delta P_i[k]$ and $\Delta\theta_i[k]$, respectively, for $i \in \mathcal{V}$, $i \neq 1$. Then, (4) can be written in matrix form as

$$\Delta P[k] \approx H_0 \Delta\theta[k]. \tag{5}$$

B. Statistical Model

Small variations in the real power injection vector, $\Delta P[k]$, can be attributed to random fluctuations in electricity consumption and the subsequent response of some generators in the system. Hence, we model the entries in $\Delta P[k]$ as independent and identically distributed (i.i.d.) with a joint Gaussian probability density function (pdf), i.e., $\Delta P[k] \sim \mathcal{N}(0, \Sigma)$.

Since the statistics of $\Delta P[k]$ are known, we consider $\Delta P[k]$ as the independent variable to the system described in (5), where $\Delta\theta[k]$ is the observation that depends on $\Delta P[k]$; thus we rewrite (5) as

$$\Delta\theta[k] \approx M_0 \Delta P[k], \tag{6}$$

where $M_0 = H_0^{-1}$. Consequently, for the system under normal operation (i.e., prior to line-outage event), $\Delta\theta[k] \sim f_0$, where

$$f_0 = \mathcal{N}\left(0, M_0 \Sigma M_0^T\right). \tag{7}$$

Example 1 (Three-Bus System): In this example, we illustrate the modeling concepts introduced above with the lossless system shown in Fig. 1, where $X_{(m,n)}$ is the imaginary part of the impedance of the line connecting buses m and n. The parameter values are listed in Table I and, unless otherwise stated, all quantities are in per unit. For ease of notation, we suppress the dependence on time instant k. Then, the nonlinear real power balance equations are

$$\begin{aligned} P_1 &= \frac{V_1 V_2}{X_{(1,2)}} \sin(\theta_1 - \theta_2) + \frac{V_1 V_3}{X_{(1,3)}} \sin(\theta_1 - \theta_3), \\ P_2 &= \frac{V_2 V_1}{X_{(1,2)}} \sin(\theta_2 - \theta_1) + \frac{V_2 V_3}{X_{(2,3)}} \sin(\theta_2 - \theta_3), \\ P_3 &= \frac{V_3 V_1}{X_{(1,3)}} \sin(\theta_3 - \theta_1) + \frac{V_3 V_2}{X_{(2,3)}} \sin(\theta_3 - \theta_2). \end{aligned} \tag{8}$$

Since bus 1 is the reference bus with $\theta_1 = 0$, we remove the first equation from (8). Furthermore, under the DC assumptions, the model in (8) can be approximated by a small-signal linear incremental model of the form in (5), i.e., $\Delta P[k] \approx H_0 \Delta\theta[k]$, where

$$H_0 = \begin{bmatrix} \frac{1}{X_{(1,2)}} + \frac{1}{X_{(2,3)}} & -\frac{1}{X_{(2,3)}} \\ -\frac{1}{X_{(2,3)}} & \frac{1}{X_{(1,3)}} + \frac{1}{X_{(2,3)}} \end{bmatrix}, \tag{9}$$

where the network topology is encoded into H_0. ■

III. LINE OUTAGE MODEL

Suppose, at time $t = t_f$, an outage occurs in the line (m, n). We assume the loss of line (m, n) does not cause islands to form in the post-event system, i.e., the underlying graph representing the power system remains connected. Further, we assume the loss of line (m, n) is a persistent event, i.e., line (m, n) is not returned to service in the time frame that we consider for line outage detection. In fact, in the algorithms described in Section IV, we take advantage of this persistent change. In this section, we describe the linear incremental model due to the persistent change, followed by special consideration for the single-sample change present in some cases.

A. Persistent Change

Suppose a persistent outage occurs in line (m, n) at time $t = t_f$, where $(2\gamma - 1)\Delta t \leq t_f < 2\gamma \Delta t$, for some $\gamma > 0$. For $k \geq \gamma$, the matrix H_0 in (5) changes to a new matrix $H_{(m,n)}$. Without loss of generality, we can write the post-outage matrix $H_{(m,n)}$ as the sum of the pre-outage matrix H_0 and some perturbation matrix $\Delta H_{(m,n)}$, i.e., $H_{(m,n)} = H_0 + \Delta H_{(m,n)}$. Then, we get the following post-outage equation:

$$\Delta P[k] \approx H_{(m,n)} \Delta\theta[k], \tag{10}$$

for $k \geq \gamma$. Since H_0 has the same sparsity structure as the graph Laplacian of the network, we conclude that the only non-zero terms in the matrix $\Delta H_{(m,n)}$ are $\Delta H_{(m,n)}[n, n] = -\Delta H_{(m,n)}[m, n] = \Delta H_{(m,n)}[m, m] = -\Delta H_{(m,n)}[m, n] = -1/X_{(m,n)}$, where $X_{(m,n)}$ is the imaginary part of the impedance of the outaged line. Thus, the matrix $\Delta H_{(m,n)}$ is a rank-one matrix and can be expressed as

$$\Delta H_{(m,n)} = -\frac{1}{X_{(m,n)}} r_{(m,n)} r_{(m,n)}^T, \tag{11}$$

where $r_{(m,n)} \in \mathbb{R}^{N-1}$ is a vector with the $(m-1)^{th}$ entry equal to 1, the $(n-1)^{th}$ entry equal to -1, and all other entries equal to 0.

By considering $\Delta\theta[k]$ in (10) as the observation, we can invert the relation in (10), and write

$$\Delta\theta[k] \approx M_{(m,n)} \Delta P[k], \ \ k \geq \gamma, \tag{12}$$

where $M_{(m,n)} = H_{(m,n)}^{-1} = M_0 + \Delta M_{(m,n)}$. Thus, after the outage of line (m, n), $\Delta\theta[k] \sim f_{(m,n)}^{\sigma}$, where

$$f_{(m,n)}^{\sigma} = \mathcal{N}\left(0, M_{(m,n)} \Sigma M_{(m,n)}^T\right), \tag{13}$$

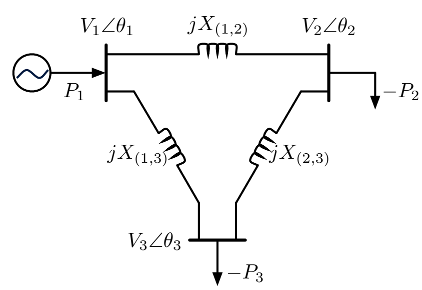

Fig. 1. Network topology for 3-bus system.

for $k \geq \gamma$. Note that if line (m,n) is not restored, this change is *persistent*. Using the matrix inversion lemma (see, e.g., [17]) to avoid repeated matrix inversions for each possible line outage, we obtain

$$\Delta M_{(m,n)} = \beta_{(m,n)}\ s_{(m,n)}\ s_{(m,n)}^T,$$

where $\beta_{(m,n)} = 1/(X_{(m,n)} - r_{(m,n)}^T H_0^{-1} r_{(m,n)})$ and $s_{(m,n)} = H_0^{-1} r_{(m,n)}$.

Example 2 (Three-Bus System): In this example, we consider again the 3-bus system in Fig. 1. Consider the outage of line $(2,3)$; then, in accordance with (10)–(11) for $m = 2$ and $n = 3$, we have that $H_{(2,3)} = H_0 + \Delta H_{(2,3)}$, where

$$\Delta H_{(2,3)} = -\frac{1}{X_{(2,3)}} r_{(2,3)} r_{(2,3)}^T$$

is a rank-one matrix. In this case, $r_{(2,3)} = [1, -1]^T$. Then $M_{(2,3)} = H_{(2,3)}^{-1}$. Using the matrix inversion lemma, we obtain that $M_{(2,3)} = H_0^{-1} + \beta_{(2,3)} s_{(2,3)} s_{(2,3)}^T$, where

$$\beta_{(2,3)} = \frac{1}{X_{(2,3)} - r_{(2,3)}^T H_0^{-1} r_{(2,3)}},\ \ s_{(2,3)} = H_0^{-1} r_{(2,3)},$$

with H_0 as given in (9). ■

B. Instantaneous Change

In Section III-A, we considered the case in which a line outage occurs at time $t = t_f$, where $(2\gamma - 1)\Delta t \leq t_f < 2\gamma\Delta t$. In such a case, the system behavior is fully described by the model in (12) and (13). If, however, the outage occurs at time $t = t_f$, where $2\gamma\Delta t \leq t_f < (2\gamma + 1)\Delta t$, then we must also consider an instantaneous change that affects only one incremental sample, namely $\Delta\theta[\gamma] = \theta[2\gamma + 1] - \theta[2\gamma]$.

Since $\theta[2\gamma]$ is obtained from the pre-outage system, while $\theta[2\gamma+1]$ is obtained from the post-outage one, the incremental change at $k = \gamma$, i.e., $\Delta\theta[\gamma]$, is not fully described by the model in (10). Instead, given the nonlinear real power flow equation for each bus i, $i \in \mathcal{V}$, as defined in (1), prior to the outage of line (m,n), we have that

$$P_i[k] = p_i[k],\ \ k \leq 2\gamma, \tag{14}$$

where the dependence on the pre-outage network topology and parameters is implicitly considered in $p_i(\cdot)$. On the other hand, in the post-outage scenario involving line (m,n) (assuming the outage is persistent), and similar to the pre-outage power flow equation in (1), we have that

$$P_i(t) = p_i'(\theta_1(t), \ldots, \theta_N(t), V_1(t), \ldots, V_N(t)), \tag{15}$$

for $t > t_f$, where the dependence on the post-outage network topology are implicitly considered in $p_i'(\cdot)$. Given (15), analogous to (14), we have that

$$P_i[k] = p_i'[k],\ \ k \geq 2\gamma + 1. \tag{16}$$

The incremental change in net active power injection between the two measurement samples immediately prior to and following the line outage, i.e., between $k = 2\gamma$ and $k = 2\gamma+1$, can be expressed as

$$\Delta P_i[\gamma] = P_i[2\gamma + 1] - P_i[2\gamma]. \tag{17}$$

Substituting (14) and (16) into (17) and adding and subtracting $p_i[\gamma + 1]$, we obtain

$$\Delta P_i[\gamma] = \Delta P_i^{\mu}[\gamma] + \Delta P_i^{\sigma}[\gamma], \tag{18}$$

where

$$\Delta P_i^{\mu}[\gamma] = p_i'[2\gamma + 1] - p_i[2\gamma + 1], \tag{19}$$

and

$$\Delta P_i^{\sigma}[\gamma] = p_i[2\gamma + 1] - p_i[2\gamma]. \tag{20}$$

Let $\Delta P^{\sigma}[\gamma] = [\Delta P_2^{\sigma}[\gamma], \Delta P_3^{\sigma}[\gamma], \ldots, \Delta P_N^{\sigma}[\gamma]]^T$. Following the development in Sections II-A, it follows that

$$\Delta P^{\sigma}[\gamma] \approx H_0 \Delta\theta[\gamma]. \tag{21}$$

Next, consider the $\Delta P_i^{\mu}[\gamma]$ component in (19). Under the same DC approximations used to derive (4), the line flow across line (m,n) under the pre-outage network topology can be approximated by

$$P_{(m,n)}[2\gamma + 1] \approx \frac{1}{X_{(m,n)}} \left(\theta_m[2\gamma + 1] - \theta_n[2\gamma + 1]\right).$$

The line outage, in turn, can be simulated by adding appropriate injections, $P_{(m,n)}[2\gamma+1]$ and $-P_{(m,n)}[2\gamma+1]$, at buses m and n, respectively (see, e.g., [16, Ch.11]). Define ΔP^{μ} as an $(N-1)$-dimensional vector, the $(m-1)^{th}$ and $(n-1)^{th}$ entries of which are $P_{(m,n)}[2\gamma+1]$ and $-P_{(m,n)}[2\gamma+1]$, respectively, and all other entries are 0. Then, we obtain that

$$\Delta P^{\mu}[\gamma] = P_{(m,n)}[2\gamma + 1] r_{(m,n)}, \tag{22}$$

where $r_{(m,n)}$ is defined as in (11). With the total variation $\Delta P[\gamma] = \Delta P^{\mu}[\gamma] + \Delta P^{\sigma}[\gamma]$, combined with expressions developed in (21) and (22), we obtain that, at $k = \gamma$,

$$\Delta\theta[\gamma] \approx M_0 \left(\Delta P[\gamma] - P_{(m,n)}[2\gamma + 1] r_{(m,n)}\right), \tag{23}$$

which indicates that the mean of $\Delta\theta[\gamma]$ is $-M_0 P_{(m,n)}[2\gamma + 1] r_{(m,n)}$, not simply 0, as in all sample instants $k < \gamma$. Thus, at $k = \gamma$, $\Delta\theta[k] \sim f_{(m,n)}^{\mu}$, where

$$f_{(m,n)}^{\mu} = \mathcal{N}\big(-M_0 P_{(m,n)}[2\gamma + 1] r_{(m,n)}, M_0 \Sigma M_0^T\big). \tag{24}$$

C. Summary and Problem Statement

Suppose an outage involving line (m,n) occurs at time t_f between PMU sampling times t_1 and t_2; then, from the analysis in Sections II, III-A and III-B, we have that

$$\Delta\theta[k] \approx \begin{cases} M_0 \Delta P[k], & \text{if } k \leq \gamma - 1, \\ M_{(m,n)} \Delta P[k], & \text{if } k \geq \gamma + 1. \end{cases} \tag{25}$$

At time instant $k = \gamma$, two cases are considered, depending on t_1 and t_2. If $t_1 = (2\gamma - 1)\Delta t$ and $t_2 = 2\gamma\Delta t$,

$$\Delta\theta[k] \approx M_{(m,n)} \Delta P[k],$$

and if $t_1 = 2\gamma\Delta t$ and $t_2 = (2\gamma + 1)\Delta t$,

$$\Delta\theta[k] \approx M_0 \left(\Delta P[\gamma] - P_{(m,n)}[2\gamma + 1] r_{(m,n)}\right).$$

Also, from the earlier analysis, the pdf's of $\Delta\theta[k]$ are

$$\Delta\theta[k] \sim \begin{cases} f_0, & \text{if } k \leq \gamma - 1, \\ f_{(m,n)}^{\sigma}, & \text{if } k \geq \gamma + 1, \end{cases} \tag{26}$$

where f_0 and $f^{\sigma}_{(m,n)}$ are described in (7) and (13), respectively. For $k = \gamma$,

$$\Delta\theta[k] \sim \begin{cases} f^{\sigma}_{(m,n)}, & \text{if } t_1 = (2\gamma - 1)\Delta t, \\ f^{\mu}_{(m,n)}, & \text{if } t_1 = 2\gamma\Delta t, \end{cases} \tag{27}$$

where $f^{\mu}_{(m,n)}$ is described in (24).

Given the model in (26)–(27), our objective in this paper is to devise algorithms for detection and identification of line outages based on the probability distributions of the $\Delta\theta[k]$'s, which can be computed from PMU measurements. To this end, we assume that small variations in the real power injection vector, $\Delta P[k]$, are attributed to random fluctuations in end-user electricity consumption (and the corresponding response of generators in the system). Hence, it is reasonable to conclude that the probability distribution of these active power variations would not be affected by system topology changes. Thus, in conjunction with the model described in (26)–(27), the problem of line outage detection reduces to a problem of detecting a change in the probability distribution of the random observation vector sequence $\{\Delta\theta[k]\}_{k\geq 1}$.

IV. Single-Line Outage Identification Via Quickest Change Detection

In the setting described in Section III, the goal is to detect the change in the probability distribution of the sequence $\{\Delta\theta[k]\}_{k\geq 1}$ as quickly as possible while maintaining a certain level of detection accuracy, which is usually captured in the false alarm rate. This is a well studied problem in statistical signal processing known as quickest change detection (QCD). Next, we provide a precise mathematical description of this problem and the QCD algorithm that we will use to detect a line outage. We refer the readers to [5] for a survey on QCD theory and algorithms; also see [18] and [19].

We assume that the sequence $\{\Delta\theta[k]\}_{k\geq 1}$ of random vectors is available to a decision maker. In the base case, prior to any outages, we have that $\Delta\theta[k] \sim f_0$. At some random time t_f, which is a priori unknown, an outage occurs in line (m, n). Then, the pdf of the sequence $\{\Delta\theta[k]\}_{k\geq 1}$ changes from f_0 to f_1. For purposes of line outage detection, if $t_f \in [2\gamma\Delta t, (2\gamma + 1)\Delta t)$, then $f_1 = f^{\mu}_{(m,n)}$ at the line outage instant, and $f_1 = f^{\sigma}_{(m,n)}$ beyond the point of change, as defined in (24) and (13), respectively. On the other hand, if $t_f \in [(2\gamma - 1)\Delta t, 2\gamma\Delta t)$, then $f_1 = f^{\sigma}_{(m,n)}$ at and after the line outage instant. In both cases, the objective is to detect this transition from f_0 to f_1 in the pdf of the $\{\Delta\theta[k]\}$'s as quickly as possible. Mathematically, the objective is to find a stopping time, τ, on the sequence, i.e., a discrete random variable taking positive integer values such that the decision to stop at time k is a function of observations until time k. In the absence of a change, we would like $\mathbb{E}[\tau]$ to be as large as possible, i.e., avoid false alarms. On the other hand, once the change occurs, we would like $\mathbb{E}[\tau]$ to be as small as possible. A popular formulation in the literature, due to Pollak, that captures the above trade-off is [20]:

$$\begin{aligned} &\min_{\tau} \ \sup_{\gamma\geq 1} \mathbb{E}_{\gamma}[\tau - \gamma | \tau \geq \gamma] \\ &\text{subject to } \ \mathbb{E}_{\infty}[\tau] \geq \beta, \end{aligned} \tag{28}$$

where $\mathbb{E}_{\gamma}$ denotes the expectation with respect to probability measure when change occurs at point γ, $\mathbb{E}_{\infty}$ denotes the corresponding expectation when the change never occurs, and $\beta > 0$ is the given constraint on the mean time to false alarm.

In the remainder of this section we discuss two algorithms to detect a single-line outage. We first devise a test for the persistent change described in Section III-A, and then modify it slightly to include the single-sample change described in Section III-B. Both tests are based on the fundamental idea from information theory that for any two densities f and g, we have that

$$D(f \parallel g) := \int f(x) \log \frac{f(x)}{g(x)} dx \geq 0, \tag{29}$$

with equality if and only if $f = g$. The quantity $D(f \parallel g)$ is called the Kullback-Leibler (KL) divergence, and it plays a fundamental role in the theory of quickest change detection [18].

A. Persistent Change Detection

Here, we describe a test to detect and identify a single-line outage by exploiting the persistent shift described in Section III-A, i.e., the post-outage pdf is assumed to be $f_1 = f^{\sigma}_{(m,n)}$ for all instants at and after the change, as given in (13). Thus, the distribution of the observation at the change point is not taken into account in the algorithm design.

Suppose both the pre- and post-outage pdfs f_0 and $f^{\sigma}_{(m,n)}$ are known. Then, a popular algorithm in the literature that enjoys some optimality properties with respect to Pollak's formulation in (28) is the Cumulative Sum (CuSum) algorithm [21]. To describe the CuSum algorithm, a sequence of statistics is computed as[2]

$$W_{(m,n)}[k+1] = \left(W_{(m,n)}[k] + \log \frac{f^{\sigma}_{(m,n)}(\Delta\theta[k+1])}{f_0(\Delta\theta[k+1])} \right)^{+}, \tag{30}$$

where $W_{(m,n)}[0] = 0$. Denote by τ_{C} the time at which the CuSum algorithm declares a line outage; then,

$$\tau_{\mathrm{C}} = \inf\{k \geq 1 : W_{(m,n)}[k] > A\}.$$

Before any line outage, the mean of the log likelihood ratio is negative, due to (29). As a result, $W_{(m,n)}[k+1]$ would remain close to or at 0 prior to the outage. On the other hand, after an outage, the mean of the log likelihood ratio is positive, again due to (29). As a result $W_{(m,n)}[k+1]$ increases unboundedly after the outage of line (m, n). Hence, the CuSum algorithm declares the occurrence of an outage in line (m, n) the first time that $W_{(m,n)}[k]$ reaches a pre-determined threshold A. The threshold A can be chosen to control the mean time to false alarm in the formulation in (28). If a larger mean time to false alarm is required, then A is set to a larger value, and vice-versa.

In the setting we consider in this paper, since the line in which the outage has occurred is unknown, the post-outage pdf of $\Delta\theta$ is also unknown. However, since the single-line outage can occur in, at most, L ways, the post-outage distribution

[2] $(x)^{+} = x$ if $x \geq 0$, otherwise $(x)^{+} = 0$.

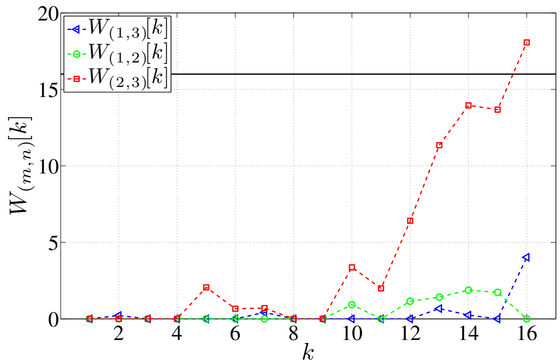

Fig. 2. A particular realization of $W_{(m,n)}[k]$ statistics for outage of $(2, 3)$ in Example 3.

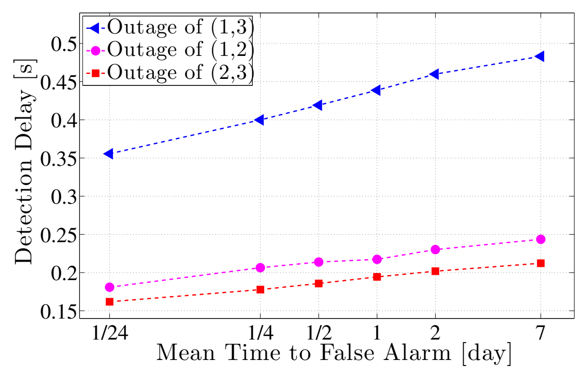

Fig. 3. Average detection delay vs. mean time to false alarm for the three-bus system in Example 3.

is known to belong to the finite set $\{f^\sigma_{(m,n)},\ (m,n) \in \mathcal{E}\}$. In this context, we can apply the *generalized likelihood ratio test* (GLRT) approach. In this approach, we compute L CuSum statistics in parallel, one for each post-outage scenario, and declare a change the first time a change is detected in any one of the parallel CuSum tests. In other words, we compute (30) for each line (m, n) in the system, with $W_{(m,n)}[0] = 0$, and stop at

$$\tau_{\max} = \inf\left\{k \geq 1 : \max_{(m,n)\in\mathcal{E}} W_{(m,n)}[k] > A\right\}. \tag{31}$$

1) Choice of Threshold A: Regarding the stopping time $\tau_{\max}$, if all the observations have density function f_0, then in order to ensure $\mathbb{E}_\infty[\tau_{\max}] \geq \beta$, we set

$$A = \log L\beta; \tag{32}$$

see [22] for a proof. On the other hand, if the fault occurs at line (m, n), and the post-outage observations have density function $f^\sigma_{(m,n)}$, then the average detection delay is

$$\sup_{\gamma\geq 1} \mathbb{E}_\gamma[\tau_{\max} - \gamma | \tau_{\max} \geq \gamma] = \mathbb{E}_1[\tau_{\max}] \sim \frac{\log\beta}{D(f^\sigma_{(m,n)} \parallel f_0)},$$

as $\beta \to \infty$ [22].

2) Line Outage Identification: The algorithm in (31) can be used for line outage identification as well. One option is to use the following intuitive technique. Let $(\hat{m}, \hat{n})$ denote the index of the line to be identified as outaged; then

$$(\hat{m}, \hat{n}) = \arg\max_{(m,n)\in\mathcal{E}} W_{(m,n)}[\tau_{\max}]. \tag{33}$$

Thus, the line for which the associated CuSum statistic is the highest at the time of stopping is declared as the outaged line. We note that this technique of isolating the fault is different from the more complicated one that is employed in [23].

3) Probability of False Isolation: Let the outage occur in line $(m, n) \in \mathcal{E}$ and let the change occur at time $\gamma = 1$; then we define the probability of false isolation (PFI) for the stopping time $\tau_{\max}$ as

$$\alpha_{(m,n)} = \mathsf{P}_1\big((\hat{m}, \hat{n}) \neq (m, n)|\text{ fault in line } (m, n)\big).$$

In other words, at $\tau_{\max}$, the PFI refers to the probability that the outage in line (m, n) is falsely identified as outage in another line. In addition to the average detection delay, the PFI can be used as a metric to gauge the effectiveness of the line outage detection and identification algorithm.

Next, we illustrate the ideas presented above to detect and identify line outages in a small power system example.

Example 3 (Three-Bus System): We apply the GLRT algorithm in (30)–(31) to detect and identify a line outage in our running example of the three-bus system in Fig. 1. As in Example 2, we consider the outage of line $(2, 3)$. In order to simulate PMU measurements of slight fluctuations in active power injection at each bus, we create the following power injection time-series data. The injection at bus i, denoted by P_i, is

$$P_i[k] = P^0_i[k] + \sigma v[k], \tag{34}$$

where $P^0_i[k]$ is the nominal power injection in bus i at instant k (see values in Table I), and $v[k]$ is a pseudorandom value drawn from standard normal distributions with 0-mean and standard deviation $\sigma = 0.5$. The variation component, $\sigma v[k]$, represents random fluctuations in electricity consumption. For each set of bus injection data, we solve the nonlinear power flow equations in (1)–(2), with the slack bus (bus 1 in this example) absorbing all power imbalances, to obtain the sequence of phase angle "measurements" $\{\theta[k]\}_{k\geq 1}$. In this example, we assume the random fluctuations at buses 2 and 3 are uncorrelated, so that Σ is a diagonal matrix.

To illustrate the algorithm in (31), we simulate the outage of line $(2, 3)$ with $\gamma = 10$. Using the GLRT, we execute three CuSum tests in parallel, corresponding to each possible line outage. In Fig. 2, typical progressions of $W_{(m,n)}[k]$ are plotted. We note that prior to line outage, i.e., for $k < \gamma$, all three $W_{(m,n)}[k]$ statistics remain close to zero, as expected. After the line outage, the statistics $W_{(1,3)}[k]$ and $W_{(1,2)}[k]$, corresponding to CuSum tests for outages in lines $(1, 3)$ and $(1, 2)$, respectively, still remain close to zero. On the other hand, $W_{(2,3)}[k]$, corresponding to the CuSum test for outage of line $(2, 3)$, increases in value beginning at $k = \gamma$ until it crosses the threshold $A = 16$ at $k = 16$. Thus, using (33), we conclude that the faulty line in this example is line $(2, 3)$.

To illustrate the tradeoff between detection delay and mean time to false alarm, assuming a PMU sampling rate of 30

TABLE II
FALSE ISOLATION PROBABILITY OBTAINED VIA SIMULATION FOR THREE-BUS SYSTEM IN EXAMPLE 3.

β [day]	1/24	1/4	1/2	1	2	7
Line $(1,3)$ outage	0.0060	0.0045	0.0070	0.0015	0.0030	0.0020
Line $(1,2)$ outage	0.0040	0.0026	0.0016	0.0012	0.0018	0.0014
Line $(2,3)$ outage	0.0014	0.0006	0.0002	0.0006	0.0008	0.0002

measurements per second, we choose the threshold A using (32) to satisfy mean time to false alarms of 1 hour, 6 hours, half a day, 1 day, 2 days, and 1 week. For each A and possible line outage, and by setting $\gamma = 1$, we simulate 5001 random sample paths and obtain the average detection delay. In Fig. 3, we plot the simulated average detection delay, $\sup_{\gamma \geq 1} \mathbb{E}_\gamma[\tau_{\max} - \gamma | \tau_{\max} \geq \gamma]$, against the logarithm of the mean time to false alarm, $\log \beta$, for each post-outage scenario. In each case, we also enumerate the number of times a line outage is falsely isolated, and obtain the simulated PFI, as reported in Table II. We note that the line outage identification algorithm in (33) is able to achieve very low PFI. ■

4) Effect of Sudden Generation/Load Change: The line outage detection and identification algorithm in (31) and (33) relies on the change in the pdf's of $\Delta\theta[k]$, which occurs due to a line outage, as described in (26). The pdf's, in turn, depend on (i) the statistical properties of random fluctuations in load and generation, and (ii) H_0 (M_0) in the pre-outage case and $H_{(m,n)}$ ($M_{(m,n)}$) after an outage in line (m,n). The latter depends only on the network topology and is invariant to changes in generation and load. With respect to the statistical properties of random fluctuations in load and generation, if some event affects such properties, i.e., $\Delta P[k] \sim \mathcal{N}(0, \Sigma')$, for $k > \gamma$, then we would need to recompute $f^\sigma_{(m,n)} = \mathcal{N}(0, M_{(m,n)} \Sigma' M^T_{(m,n)})$, for each credible outage in line (m,n). On the other hand, as long as statistical properties of $\Delta P[k]$ do not change, a sudden change in generation/load, which coincides with a line outage, would not affect the line outage detection and identification algorithm.

Example 4 (Three-Bus System): Again, we consider the three-bus system in Fig. 1, and as in Example 3, we apply the algorithm in (30)–(31) to detect and identify a line outage that coincides with a sudden change in power injections. To this end, we obtain simulated power injection data using (34) when there is a simultaneous line outage and sudden change in P^0_3 from -0.9 p.u. to -0.2 p.u. at $\gamma = 1$, with $\sigma = 0.5$, both before and after the fault. In Table III, we report the PFIs and average detection delays obtained from 5001 random sample paths. We note that, indeed, these results are similar to those obtained in Example 3, where we did not apply coincident load change with the line outage. ■

TABLE III
SIMULATION RESULTS FOR SIMULTANEOUS LINE OUTAGE AND LOAD CHANGE IN THREE-BUS SYSTEM IN EXAMPLE 4.

β [day]	1/24	1/4	1/2	1	2	7
	Lines $(1,3)$ outage					
PFI	0.0088	0.0042	0.0046	0.0050	0.0032	0.0036
Detection Delay [s]	0.3903	0.4440	0.4670	0.4812	0.4929	0.5310
	Lines $(1,2)$ outage					
PFI	0.0090	0.0056	0.0060	0.0050	0.0030	0.0018
Detection Delay [s]	0.1952	0.2171	0.2237	0.2329	0.2395	0.2557
	Lines $(2,3)$ outage					
PFI	0.0020	0.0014	0.0014	0.0004	0.0004	0.0006
Detection Delay [s]	0.1634	0.1786	0.1847	0.1944	0.2042	0.2125

B. Instantaneous Change Detection

In Section III-B, we described a mean shift in $\Delta\theta[k]$ that only affects the sample at $k = \gamma$, in the case that the fault occurs at time $t_f \in [2\gamma\Delta t, (2\gamma+1)\Delta t)$, namely $\Delta\theta[\gamma] \sim f^\mu_{(m,n)}$; we now incorporate this effect in our algorithm. To this end, let

$$U_{(m,n)}[k+1] = \log \frac{f^\mu_{(m,n)}(\Delta\theta[k+1])}{f_0(\Delta\theta[k+1])}; \tag{35}$$

then the mean of this random variable is positive at the change point; see (29). Next, we modify the CuSum algorithm from Section IV-A to also consider the mean shift, as follows:

$$W^*_{(m,n)}[k+1] = \max \Bigg\{ U_{(m,n)}[k+1], \left(W^*_{(m,n)}[k] + \log \frac{f^\sigma_{(m,n)}(\Delta\theta[k+1])}{f_0(\Delta\theta[k+1])} \right)^+ \Bigg\}, \tag{36}$$

where $W^*_{(m,n)}[0] = 0$. The above equation can be derived in the same way as the classical CuSum algorithm is derived using the maximum likelihood principle.

The modified CuSum algorithm in (35)–(36) is equivalent to the standard one in (30), except at the change point. The modified statistic, $W^*_{(m,n)}[k+1]$, replaces $W_{(m,n)}[k+1]$ in (30), and we proceed with the same protocol as before for detecting and identifying line outages.

Given the modified CuSum algorithm described above, we would expect the quantity $U_{(m,n)}[k+1]$, as defined in (35), to only affect $W^*_{(m,n)}[\gamma]$; this is indeed true. Extensive simulations for low false alarm rates (the range of false alarms we are interested in for this problem) have revealed that the modified algorithm does not provide significant gain as compared to the algorithm (31) presented in Section IV-A. As a result, in the remainder of the paper, we focus on the algorithm in (31).

V. DOUBLE-LINE OUTAGE IDENTIFICATION VIA QUICKEST CHANGE DETECTION

So far, in this paper, we have considered algorithms to detect and identify single-line outages. While simultaneous double-line outages are rare occurrences, they can be impactful. Suppose, instead of the single-line outage in line (m,n), simultaneous credible outage (i.e., one that does not island the system) of lines $\ell_i = (m,n)$ and $\ell_j = (u,v)$ occurs at $t = t_f$, where $(2\gamma-1)\Delta t \leq t_f < 2\gamma\Delta t$ or $2\gamma\Delta t \leq t_f < (2\gamma+1)\Delta t$. Assuming that the outage is persistent, then the post-outage observations $\Delta\theta[k]$ can be described as

$$\Delta\theta[k] \approx M_{\{\ell_i,\ell_j\}} \Delta P[k], \; k \geq \gamma+1, \tag{37}$$

TABLE IV
FALSE ISOLATION PROBABILITY OBTAINED VIA SIMULATION FOR SINGLE-LINE OUTAGES IN 118-BUS SYSTEM.

β [day]	1/24	1/4	1/2	1	2	7
Line $(54,55)$ outage	0.0088	0.0044	0.0026	0.0022	0.0010	0.0012
Line $(63,59)$ outage	0	0	0	0	0	0
Line $(64,65)$ outage	0	0	0	0	0	0
Line $(65,68)$ outage	0	0	0	0	0	0

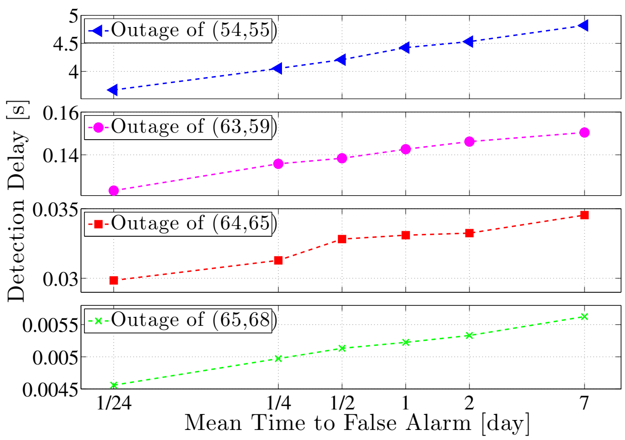

Fig. 4. IEEE 118-bus system: average detection delay vs. mean time to false alarm.

where $M_{\{\ell_i,\ell_j\}} = H^{-1}_{\{\ell_i,\ell_j\}}$, and $H_{\{\ell_i,\ell_j\}} = H_0 + \Delta H_{(m,n)} + \Delta H_{(u,v)}$. Thus, after the outage of lines (m,n) and (u,v), $\Delta\theta[k] \sim f^{\sigma}_{\{\ell_i,\ell_j\}}$, where

$$f^{\sigma}_{\{\ell_i,\ell_j\}} = \mathcal{N}\left(0, M_{\{\ell_i,\ell_j\}}\Sigma M^T_{\{\ell_i,\ell_j\}}\right), \;\; k \geq \gamma+1. \quad (38)$$

As in the single-line outage scenario, we can avoid repeated matrix inversion operations by invoking the matrix inversion lemma for low-rank matrix perturbations.

In addition to computing $W_{(m,n)}[k+1]$ statistics in (30) for all credible single-line outages, we also compute

$$W_{\{\ell_i,\ell_j\}}[k+1] = \left(W_{\{\ell_i,\ell_j\}}[k] + \log\frac{f^{\sigma}_{\{\ell_i,\ell_j\}}(\Delta\theta[k+1])}{f_0(\Delta\theta[k+1])}\right)^+,$$

for each credible double-line outage in lines ℓ_i and ℓ_j. The outaged lines are identified similar to (33), with the additional CuSum statistics considered.

VI. Case Studies

In this section, we further illustrate the proposed single- and double-line outage detection and identification algorithms on the IEEE 118-bus test system. We use the simulation tool MATPOWER [24] throughout to obtain relevant voltage angles by repeatedly solving AC power flow solutions of the system, at each time step k, corresponding to synthetic power injection profiles generated using (34), with $\sigma = 0.03$. Assuming these random fluctuations are uncorrelated, then, Σ is a diagonal matrix with each diagonal entry being 0.0018. To simulate the

TABLE V
FALSE ISOLATION PROBABILITY AND DETECTION DELAY OBTAINED VIA SIMULATION FOR DOUBLE-LINE OUTAGES IN 118-BUS SYSTEM.

β [day]	1/24	1/4	1/2	1	2	7
	Lines $(23,24)$ and $(65,68)$ outage					
PFI	0	0	0	0	0	0
Detection Delay [ms]	0.666	0.999	1.066	0.6660	0.7326	0.8658
	Lines $(56,59)$ and $(59,61)$ outage					
PFI	0.037	0.018	0.014	0.018	0.025	0.015
Detection Delay [s]	2.945	3.263	3.366	3.463	3.549	3.797

worst-case detection delay, we choose $\gamma = 1$. As in the three-bus case in Example 3, assuming a PMU sampling rate of 30 measurements per second, the threshold A is chosen using (32) to satisfy mean time to false alarms of 1 hour, 6 hours, half a day, 1 day, 2 days, and 1 week.

A. Single-Line Outage Detection

In this case study, we consider credible single-line outages (i.e., those that do not island the system) in the IEEE 118-bus test system; in particular, we focus on the simulated outage of lines $(54,55)$, $(63,59)$, $(64,65)$, and $(65,68)$. These represent a range in performance with regard to average detection delay and probability of false isolation. For each outage and threshold A, we simulate 5001 random paths. Parallel CuSums, as described in (30), are computed for all credible single-line outages in the system. Statistics are accumulated until the condition in (31) is met. In Fig. 4, we plot the detection delay versus false alarm trade-off for the four possible outages. In Table IV, we report the PFIs obtained by enumerating the number of sample paths for which the wrong line outage is identified. For three of the four outages considered, we obtain perfect isolation. In the case of line $(54,55)$ outage, the PFIs are still quite low, as desired.

B. Double-Line Outage Detection

We simulate 1001 random paths for two outage cases: (i) lines $(23,24)$ and $(65,68)$ outage, and (ii) lines $(56,59)$ and $(59,61)$ outage. These two outage scenarios are chosen since the former has the shortest detection delay, while the latter has the longest. Parallel CuSums are computed for each of the credible single- and double-line outages. In Table V, the PFIs indicate the probability that neither of the two actual outaged lines are identified by the algorithm in (33). Average detection delays for each case are also reported in Table V. While we find that the proposed double-line outage detection approach remains manageable here, for larger systems, the combinatorial nature of adding parallel CuSums becomes intractable.

VII. Line Outage Detection and Identification with a Subset of Measurements

Incentives to invest in the deployment of a measurement infrastructure are driven by preliminary demonstrations of its potential benefits in monitoring, protection, and control capabilities (see, e.g., [25]). Moreover, today, in addition to PMU installations, synchronous phasor measurement capabilities are available as standard features in many protective

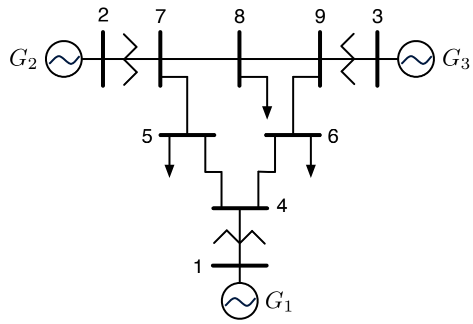

Fig. 5. Network topology for WECC 3-machine 9-bus system.

TABLE VI
FALSE ISOLATION PROBABILITY OBTAINED VIA SIMULATION FOR THREE-BUS SYSTEM IN EXAMPLE 3.

β [day]	1/24	1/4	1/2	1	2	7
Line $(8, 9)$ outage	0.0068	0.0096	0.0070	0.0086	0.0056	0.0060
Line $(7, 8)$ outage	0.0128	0.0132	0.0112	0.0110	0.0134	0.0108
Line $(6, 9)$ outage	0	0	0	0	0.0002	0
Line $(5, 7)$ outage	0	0.0002	0	0	0.0002	0
Line $(4, 6)$ outage	0.0002	0.0002	0.0002	0	0	0
Line $(4, 5)$ outage	0.002	0.0014	0.0008	0.0006	0.0008	0.0012

relays, meters, and fault recorders [26]. Thus, so far, we have assumed that PMU measurements are available at all buses; however, present-day power systems are still far from having such a rich set of available phasor measurement devices. With respect to this, our proposed method can be easily extended to the case in which phase angle measurements are not available at all buses.

Let $\mathcal{O} \subseteq \mathcal{V}$ represent the set of $N_\mathcal{O}$ observable buses for which phase angle measurements are available. Define $\Delta\theta_\mathcal{O}[k]$ to be an $N_\mathcal{O}$-dimensional vector, the entries of which are $\Delta\theta_i[k]$, for $i = 1, \dots, N_\mathcal{O}$. Suppose the observation $\Delta\theta_i[k]$ is obtained at bus j, where $j \in \mathcal{V}$. Now, define an $N_\mathcal{O} \times N$ observation matrix C the i^{th} row of which contains zeros everywhere except in the j^{th} entry, which is equal to 1. Then, we can write

$$\Delta\theta_\mathcal{O}[k] = C\Delta\theta[k]. \tag{39}$$

Furthermore, by substituting (6) into (39), we obtain that the pre-outage observations $\Delta\theta_\mathcal{O}[k] \sim f_0$, where

$$f_0 = \mathcal{N}\left(0, CM_0\Sigma M_0^T C^T\right), \tag{40}$$

Suppose a persistent outage in line (m, n) occurs at time $t = t_f$, where $(2\gamma - 1)\Delta t \le t_f < 2\gamma\Delta t$ or $2\gamma\Delta t \le t_f < (2\gamma + 1)\Delta t$. By substituting (12) into (39), the post-outage observations $\Delta\theta_\mathcal{O}[k] \sim f^\sigma_{(m,n)}$, where

$$f^\sigma_{(m,n)} = \mathcal{N}\left(0, CM_{(m,n)}\Sigma M^T_{(m,n)} C^T\right), \;\; k \ge \gamma + 1. \tag{41}$$

In order to detect and identify single-line outages with a subset of measurement locations, we can utilize the proposed QCD-based algorithm in (31) and (33) along with the updated pre- and post-outage pdf's in (40) and (41).

Example 5 (3-Machine 9-Bus System): We consider the WECC 3-machine, 9-bus system model (see, e.g., [24]), the topology of which is shown in Fig. 5. We simulate active power injection fluctuations at each bus i using (34) with $\sigma = 0.03$. We assume that voltage phase measurements are available at buses 3, 5, 6, 7, and 9 only. For each outage and threshold A, choosen to satisfy mean time to false alarms of 1 hour, 6 hours, half a day, 1 day, 2 days, and 1 week, we simulate 5001 random sample paths. In Fig. 6, we illustrate the detection delay versus false alarm tradeoff for all credible single-line outages. In Table VI, we report the PFIs obtained for each outage. ■

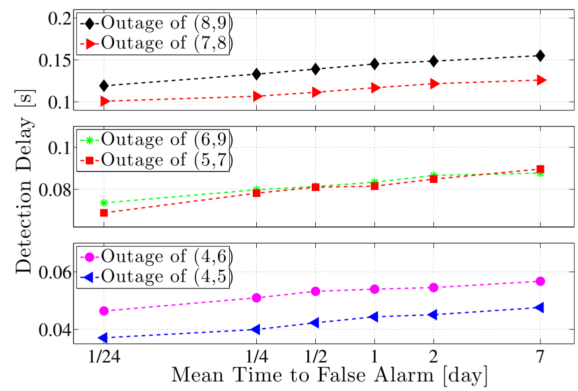

Fig. 6. Average detection delay vs. mean time to false alarm for the 3-machine 9-bus system in Example 5.

VIII. Concluding Remarks

In this paper, we proposed a method to detect and identify transmission line outages that exploits the statistical properties of voltage phase angle measurements obtained from PMUs in real-time. We assumed that the incremental variations in net power injection at each bus are independent random variables, which are related to the voltage phase angles via a linear mapping resulting from an incremental small-signal power flow model. By processing voltage angle measurements sequentially, we employed a QCD algorithm to detect and identify line outages.

Future work includes design of optimal QCD-based algorithms for line outage detection using fewer PMUs as well as multiple-line outage detection with reduced computational burden.

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