Enabling Peer-to-peer Transactions in Measurement-based Distribution System Market

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***Abstract*—This paper presents a measurement-based electricity market structure to establish peer-to-peer (P2P) transactions along with imports from or exports to the upstream network. A key benefit of the proposed P2P market is that participants therein can fully express their proclivities by setting their individual preferences for buying and selling partners independently. Moreover, resulting P2P transactions satisfy power flow constraints of the underlying distribution system without needing an offline network model. Instead, we estimate a linear sensitivity model mapping bus voltages to injections using *only* online measurements collected from P2P market participants, which is then embedded as an equality constraint in an optimal power flow (OPF) problem. The OPF problem minimizes total cost of P2P transactions incurred to market participants capturing network usage fees, buying/selling preferences, net import/export cost, and operation cost. The optimal solution of the OPF problem comprises the P2P transactions (specifying partners, quantity, and price for each trade), the optimal dispatch, as well as locational marginal prices at buses where measurements are collected. Via numerical simulations involving a 22-bus test system, we demonstrate the effectiveness of the proposed method to establish P2P transactions that respect individual preferences; we also validate notable properties pertaining to the trade prices.**

***Index Terms*—Buying and selling preferences, distribution system, electricity market, locational marginal pricing, peer-to-peer transactions, synchrophasors**

I. Introduction

Critical in the transition to a low-carbon electric energy future is extensive deployment of distributed energy resources (DERs), such as distributed and renewable generation and flexible demand, in power distribution networks, to replace large-scale fossil fuel power plants. Under suitable control and coordination, DERs can help to improve system performance by contributing to, e.g., voltage support and congestion management [1]. At the same time, distribution-level electricity trading practices are needed to incentivize DERs in providing grid support and to compensate them through a fair pricing scheme instead of payments based on fixed or time-of-use rates only [2]. Of particular interest at the distribution level is to facilitate potentially many pair-wise peer-to-peer (P2P) transactions of electricity at a given time, where each transaction specifies the trading quantity and price between two particular market participants (e.g., an energy producer and an energy consumer) [3].

A notable benefit of P2P markets over pool-based market structures is their ability to respond to market participant preferences regarding electricity source and trading partners [4]. P2P markets have been developed using various approaches, including game theory, contract networks, auction theory, and convex optimization (see, e.g., [5], [6] for comprehensive review). Amongst optimization-based P2P markets, user preferences are typically handled by including them in the objective function of a multi-bilateral economic dispatch problem [7]–[11]. In [7], [8], only the economic layer of the P2P market is considered without any mechanism to ensure that the resulting P2P transactions satisfy the physical constraints of the underlying electrical network. This purely economic model is extended to include the physical layer by incorporating P2P trades into an optimal power flow (OPF) problem, where power flow constraints may be expressed through the branch flow model [9], power transfer distribution factors [10], or the DC power flow approximation [11]. Although the approaches in [9]–[11] are effective at constraining market outcomes to feasible P2P transactions within limitations of the grid, they require offline knowledge of an accurate and up-to-date network model, which may not be available in practice. The advancement of online measurement technologies tailored for distribution systems, such as distribution-level phasor measurement units (D-PMUs), motivates the development of data-driven electricity markets. Recent work in [12] offers one such method to calculate locational marginal prices (LMPs) as part of the optimal solution of an OPF problem formulated without relying on any offline knowledge of the distribution network, but it does not consider P2P transactions.

In this paper, we extend the method in [12] to formulate a measurement-based OPF problem that establishes a set of optimal P2P transactions, specifying the partners, quantity, and price for each trade, in addition to obtaining the optimal DER dispatch and the associated LMPs. We embed market participant preferences into the objective function of the OPF problem in the form of penalties for potential pairwise P2P transactions, which is minimized along with net import/export costs and DER generation costs. Distinct from [7]–[11], our proposed formulation enables market participants to fully express their proclivities by independently setting their buying and selling preferences in trades with a single partner. For example, a market participant who owns a non-carbon-emitting DER may be averse to buying electricity from the owner of a carbon-emitting DER but may have no aversion to selling electricity to that same individual. Furthermore, unlike [9]–[11], the resulting P2P transactions and optimal dispatch satisfy power flow constraints *without* relying on an offline network model. Instead, we estimate a linear sensitivity model mapping voltages to injections using only online measurements collected from buses with market participants (see, e.g., [13], [14]). Finally, the proposed method prevents arbitrage in resulting P2P transactions by explicitly limiting an individual's electricity sales to be less than its production. Via numerical simulations involving a 22-bus test system, we demonstrate the effectiveness of the proposed method in establishing P2P transactions that respect individual preferences. Simulations also validate notable properties that are pertinent to the trade prices.

II. Preliminaries

In this section, we describe the distribution system model along with market participants therein. We also outline the online estimation of a linear sensitivity model that approximates the nonlinear power flow equations typically constructed using prior offline knowledge of the distribution network.

A. Market Participants

Consider a distribution system with $N+1$ buses collected in the set $\mathcal{N} = \{0, 1, \ldots, N\}$, where bus 0 is the slack bus connected to the substation with fixed and known voltage. Within the set $\mathcal{N}$, G buses are connected to dispatchable DERs collected in the set $\mathcal{G} \subseteq \mathcal{N} \setminus \{0\}$. Let V_i and θ_i denote, respectively, the voltage magnitude and phase angle at bus $i \in \mathcal{N}$; and let P_i^{d} and Q_i^{d} denote, respectively, the active- and reactive-power demand arising from the aggregate non-dispatchable load at bus $i \in \mathcal{N}$. Also let P_g^{g} denote the active-power generation arising from the aggregate dispatchable DER at bus $g \in \mathcal{G}$. Further let $\mathcal{E} \subseteq \mathcal{N} \setminus \{0\}$ represent the set of E buses that are connected to so-called market participants that may take part in P2P transactions. Without loss of generality, we assume that all DERs are market participants so that $\mathcal{G} \subseteq \mathcal{E}$.

B. Estimated Linear Power Flow Model

In order to constrain nodal voltage magnitudes at the buses connected to market participants, i.e., those contained in the set $\mathcal{E}$, they must be equipped with D-PMUs that provide measurements of voltage phasors and complex power injections. For the sake of containing notational burden, we further assume that *only* buses connected to market participants are equipped with D-PMUs. Collect voltage phase angles and magnitudes of measured buses in vectors $\theta = [(\theta_\ell)_{\ell\in\mathcal{E}}]^{\mathrm{T}}$ and $V = [(V_\ell)_{\ell\in\mathcal{E}}]^{\mathrm{T}}$, respectively. Also collect active- and reactive-power demand at measured buses in vectors $P^{\mathrm{d}} = [(P_\ell^{\mathrm{d}})_{\ell\in\mathcal{E}}]^{\mathrm{T}}$ and $Q^{\mathrm{d}} = [(Q_\ell^{\mathrm{d}})_{\ell\in\mathcal{E}}]^{\mathrm{T}}$, respectively. Net active- and reactive-power injections are then respectively $P = MP^{\mathrm{g}} - P^{\mathrm{d}}$ and $Q = -Q^{\mathrm{d}}$, where $M \in \mathbb{R}^{E\times G}$ is a matrix of 0s and 1s that maps entries in $\mathcal{G}$ to corresponding bus indices in $\mathcal{E}$. Measured values of the variables defined above are distinguished by $\widehat{\cdot}$ placed above the corresponding variables.

Suppose pertinent system variables are sampled at time $t = k\Delta t$, $k = 0, 1, \ldots$, where Δt is the sampling interval (in the range of several seconds or less [15]). Then we collect measured nodal voltage phase angles and magnitudes at time step k in $\widehat{x}_{[k]} = [\widehat{\theta}_{[k]}^{\mathrm{T}}, \widehat{V}_{[k]}^{\mathrm{T}}]^{\mathrm{T}}$. Further collect the measured net nodal power injections at time step k in $\widehat{y}_{[k]} = [\widehat{P}_{[k]}^{\mathrm{T}}, \widehat{Q}_{[k]}^{\mathrm{T}}]^{\mathrm{T}}$. We then hypothesize that the measured injections and voltages are related linearly as $\widehat{y}_{[k]} = J_{[k]}\widehat{x}_{[k]} + c_{[k]}$, or equivalently

$$\widehat{y}_{[k]}^{\mathrm{T}} = \begin{bmatrix} \widehat{x}_{[k]}^{\mathrm{T}} & 1 \end{bmatrix} H_{[k]} \tag{1}$$

where $H_{[k]} = [J_{[k]}, c_{[k]}]^{\mathrm{T}}$. By recording a minimum of $2E$ most recent samples and stacking the corresponding instances of (1) while assuming $H_{[k]}$ remains constant across these samples, we can easily apply the ordinary least squares (OLS) algorithm to obtain an estimate of $H_{[k]}$. However, due to potential correlation amongst voltage measurements at various buses, the OLS algorithm may lead to ill-conditioned regressor matrices. Recent work in [14] suggests that the partial least squares (PLS) algorithm is effective to provide meaningful estimates of $H_{[k]}$ in the presence of collinearity. We refer interested readers to [14] for further details on the estimation algorithm and its performance. For the purpose of this paper, similar to [12], it suffices to assume that an up-to-date estimate $\widehat{H} = [\widehat{J}, \widehat{c}]^{\mathrm{T}}$ is available from applying the PLS algorithm to sufficiently many recently obtained measurements. We will also find it helpful to decompose $\widehat{J}$ and $\widehat{c}$ as follows:

$$\widehat{J} = \begin{bmatrix} \widehat{J}^{P\theta} & \widehat{J}^{PV} \\ \widehat{J}^{Q\theta} & \widehat{J}^{QV} \end{bmatrix}, \quad \widehat{c} = \begin{bmatrix} \widehat{c}^{P} \\ \widehat{c}^{Q} \end{bmatrix}. \tag{2}$$

III. Peer-to-peer Electricity Market

This section incorporates P2P transactions within a measurement-based OPF problem embedding the estimated sensitivity model from Section II-B in place of the typical nonlinear power flow equations constructed from prior knowledge of the network topology and its parameters. Also included in this section are optimality conditions of the OPF problem.

A. Problem Formulation

Let $S^{\mathrm{P2P}} \in \mathbb{R}^{G\times E}$ denote the nonnegative P2P transaction matrix where the (g,ℓ) entry represents the active power sold by the DER at bus g to market participant at bus ℓ. We employ three ways to penalize P2P transactions based on participants' obligation to the distribution system operator (DSO) as well as their buying and selling preferences.

i) *Obligation to DSO.* Let $\gamma^{\mathrm{dso}} > 0$ represent a network usage fee rate, which may be imposed by the DSO uniformly on all P2P transactions in S^{P2P} for using the underlying electrical network.
ii) *Buyer Preferences.* Let matrix $\Gamma^{\mathrm{b}} \in \mathbb{R}^{G\times E}$ comprise buying preferences of market participants, where the (g,ℓ) entry therein represents the penalty applied by the market participant at bus ℓ to the DER at bus g.
iii) *Seller Preferences.* Let matrix $\Gamma^{\mathrm{s}} \in \mathbb{R}^{G\times E}$ comprise selling preferences of DERs, where the (g,ℓ) entry therein represents the subsidy applied by the DER at bus g to the market participant at bus ℓ.

We note that the seller and buyer preferences need not coincide with each other in a single P2P transaction. For example, suppose the DER at bus g is a diesel generator but the participant at bus ℓ prefers to buy electricity only from renewable sources. To capture this preference, the participant at bus ℓ could set the

(g,ℓ) entry in Γ^{b} to a large positive value, whereas the same entry in Γ^{s} may be small or zero to reflect the ambivalence of the DER at bus g with respect to selling power to the market participant at bus ℓ. On the other hand, a DER may prefer to sell its power to a low-income household either altruistically or to, e.g., qualify for certain government subsidies. In this case, the DER could set the pertinent entry in Γ^{s} to a positive value, whereas the same entry in Γ^{b} may be zero reflecting an absence of preference on the part of the buyer. In practice, each market participant would declare its buying preferences in the corresponding column of Γ^{b}, while each DER would declare its selling preferences in the corresponding row of Γ^{s}. This is an important distinction in our proposed formulation over prior work so as to enable market participants to fully express their individual proclivities.

Now let $S^{\mathrm{imp}} \in \mathbb{R}^E$ comprise imports from the upstream network to the E market participants in the distribution network; and let $S^{\mathrm{exp}} \in \mathbb{R}^G$ comprise exports to the upstream network from the G DERs. Further let $\pi^{\mathrm{imp}} \geq 0$ ($\pi^{\mathrm{exp}} \geq 0$) denote the uniform price at which all market participants (DERs) would buy (sell) power from (to) the upstream network. With the notation for P2P transactions, imports, and exports established above, we formulate the following OPF problem to minimize the collective total costs of all market participants:

$$
\begin{aligned}
\underset{\Omega}{\text{minimize}}\quad & \gamma^{\mathrm{dso}}\mathbb{1}_G^{\mathrm{T}}S^{\mathrm{P2P}}\mathbb{1}_E + \mathrm{Tr}((\Gamma^{\mathrm{b}}-\Gamma^{\mathrm{s}})^{\mathrm{T}}S^{\mathrm{P2P}}) \\
& + \pi^{\mathrm{imp}}\mathbb{1}_E^{\mathrm{T}}S^{\mathrm{imp}} - \pi^{\mathrm{exp}}\mathbb{1}_G^{\mathrm{T}}S^{\mathrm{exp}} + C(P^{\mathrm{g}}) && \text{(3a)}\\
\text{subject to}\quad & MP^{\mathrm{g}} - P^{\mathrm{d}} = \widehat{J}^{P\theta}\theta + \widehat{J}^{PV}V + \widehat{c}^{P},\ (\lambda), && \text{(3b)}\\
& -Q^{\mathrm{d}} = \widehat{J}^{Q\theta}\theta + \widehat{J}^{QV}V + \widehat{c}^{Q},\ (\mu), && \text{(3c)}\\
& MP^{\mathrm{g}} + (S^{\mathrm{P2P}})^{\mathrm{T}}\mathbb{1}_G + S^{\mathrm{imp}} \\
& \qquad = P^{\mathrm{d}} + M(S^{\mathrm{P2P}}\mathbb{1}_E + S^{\mathrm{exp}}),\ (\beta), && \text{(3d)}\\
& P^{\mathrm{g}} \geq S^{\mathrm{P2P}}\mathbb{1}_E + S^{\mathrm{exp}},\ (\rho), && \text{(3e)}\\
& S^{\mathrm{P2P}} \geq \mathbb{0}_{G\times E},\ (\Sigma), && \text{(3f)}\\
& S^{\mathrm{imp}} \geq \mathbb{0}_E,\ (\zeta), && \text{(3g)}\\
& S^{\mathrm{exp}} \geq \mathbb{0}_G,\ (\xi), && \text{(3h)}\\
& \underline{V} \leq V \leq \overline{V},\ (\nu^-,\nu^+), && \text{(3i)}\\
& \underline{P}^{\mathrm{g}} \leq P^{\mathrm{g}} \leq \overline{P}^{\mathrm{g}},\ (\phi^-,\phi^+), && \text{(3j)}
\end{aligned}
$$

where penalties for P2P transactions (expressed through network usage fees and buying/selling preferences), net import/export cost, as well as the operation cost of DERs $C(P^{\mathrm{g}})$ are minimized in the objective function (3a) subject to operational constraints in (3b)–(3j). Decision variables of (3) collected in $\Omega = \{S^{\mathrm{P2P}}, S^{\mathrm{imp}}, S^{\mathrm{exp}}, P^{\mathrm{g}}, \theta, V\}$ include P2P transactions (trading partners and quantities), imports and exports, DER active-power setpoints, and voltage phase angles and magnitudes of only buses with measurements (equivalently buses connected to market participants). As such, the problem does not optimize over buses without measurements in $\mathcal{N}\setminus\mathcal{E}$. Equality constraints include estimated linear sensitivity-based active- and reactive-power flow equations in (3b) and (3c), respectively. Imposed in (3d) is the nodal active-power balance at all buses with measurements considering injections from DERs and purchases as well as withdrawals by loads and due to sales. The inequality constraint pertinent to the P2P transaction matrix S^{P2P} in (3e) ensures that total sales from each participant with a DER do not exceed its production so as to prevent arbitrage while ensuring individual preferences have meaningful impact on the optimal dispatch. Furthermore, inequality constraints in (3f)–(3h) impose nonnegative entries in S^{P2P}, S^{imp}, and S^{exp}. Finally, nodal voltage magnitudes and active-power outputs of DERs are confined to their minimum and maximum limits through (3i)–(3j).

B. Optimality Conditions

The optimality conditions for the OPF problem in (3) are established through Karush-Kuhn-Tucker (KKT) conditions. Below, we first formulate the Lagrangian of the problem in (3), from which KKT conditions are then derived.

1) Lagrangian Function: The Lagrangian of (3) is

$$
\begin{aligned}
\mathcal{L} = {} & \gamma^{\mathrm{dso}}\mathbb{1}_G^{\mathrm{T}}S^{\mathrm{P2P}}\mathbb{1}_E + \mathrm{Tr}((\Gamma^{\mathrm{b}}-\Gamma^{\mathrm{s}})^{\mathrm{T}}S^{\mathrm{P2P}}) \\
& + \pi^{\mathrm{imp}}\mathbb{1}_E^{\mathrm{T}}S^{\mathrm{imp}} - \pi^{\mathrm{exp}}\mathbb{1}_G^{\mathrm{T}}S^{\mathrm{exp}} + C(P^{\mathrm{g}}) \\
& + \lambda^{\mathrm{T}}(\widehat{J}^{P\theta}\theta + \widehat{J}^{PV}V + \widehat{c}^{P} - MP^{\mathrm{g}} + P^{\mathrm{d}}) \\
& + \mu^{\mathrm{T}}(\widehat{J}^{Q\theta}\theta + \widehat{J}^{QV}V + \widehat{c}^{Q} + Q^{\mathrm{d}}) \\
& + \beta^{\mathrm{T}}(P^{\mathrm{d}} + M(S^{\mathrm{P2P}}\mathbb{1}_E + S^{\mathrm{exp}}) \\
& - MP^{\mathrm{g}} - (S^{\mathrm{P2P}})^{\mathrm{T}}\mathbb{1}_G - S^{\mathrm{imp}}) \\
& - \rho^{\mathrm{T}}(P^{\mathrm{g}} - S^{\mathrm{P2P}}\mathbb{1}_E - S^{\mathrm{exp}}) \\
& - \mathrm{Tr}(\Sigma^{\mathrm{T}}S^{\mathrm{P2P}}) - \zeta^{\mathrm{T}}S^{\mathrm{imp}} - \xi^{\mathrm{T}}S^{\mathrm{exp}} \\
& + (\nu^-)^{\mathrm{T}}(\underline{V} - V) + (\nu^+)^{\mathrm{T}}(V - \overline{V}) \\
& + (\phi^-)^{\mathrm{T}}(\underline{P}^{\mathrm{g}} - P^{\mathrm{g}}) + (\phi^+)^{\mathrm{T}}(P^{\mathrm{g}} - \overline{P}^{\mathrm{g}}). \qquad (4)
\end{aligned}
$$

2) KKT Conditions: Denote the optimal Lagrangian by $\mathcal{L}^\star$ and distinguish the optimal values taken by decision variables and Lagrange multipliers of (3) with superscript $\star$. Then the KKT conditions include stationarity conditions, complementary slackness conditions, primal feasibility, and dual feasibility. Stationarity conditions are given by the following:

$$
\frac{\partial\mathcal{L}^\star}{\partial P^{\mathrm{g}\star}} = \frac{\partial C(P^{\mathrm{g}\star})}{\partial P^{\mathrm{g}\star}} - M^{\mathrm{T}}(\lambda^\star + \beta^\star) - \rho^\star + \phi^{+\star} - \phi^{-\star} = \mathbb{0}_G, \tag{5}
$$

$$
\begin{aligned}
\frac{\partial\mathcal{L}^\star}{\partial S^{\mathrm{P2P}\star}} = {} & \Gamma^{\mathrm{b}} - \Gamma^{\mathrm{s}} - \mathbb{1}_G\beta^{\star\mathrm{T}} - \Sigma^\star \\
& + (\mathbb{1}_G\gamma^{\mathrm{dso}} + M^{\mathrm{T}}\beta^\star + \rho^\star)\mathbb{1}_E^{\mathrm{T}} = \mathbb{0}_{G\times E},
\end{aligned} \tag{6}
$$

$$
\frac{\partial\mathcal{L}^\star}{\partial S^{\mathrm{imp}\star}} = \mathbb{1}_E\pi^{\mathrm{imp}} - \beta^\star - \zeta^\star = \mathbb{0}_E, \tag{7}
$$

$$
\frac{\partial\mathcal{L}^\star}{\partial S^{\mathrm{exp}\star}} = -\mathbb{1}_G\pi^{\mathrm{exp}} + M^{\mathrm{T}}\beta^\star - \rho^\star - \xi^\star = \mathbb{0}_G, \tag{8}
$$

$$
\frac{\partial\mathcal{L}^\star}{\partial \theta^\star} = (\widehat{J}^{P\theta})^{\mathrm{T}}\lambda^\star + (\widehat{J}^{Q\theta})^{\mathrm{T}}\mu^\star = \mathbb{0}_E, \tag{9}
$$

$$
\frac{\partial\mathcal{L}^\star}{\partial V^\star} = (\widehat{J}^{PV})^{\mathrm{T}}\lambda^\star + (\widehat{J}^{QV})^{\mathrm{T}}\mu^\star + \nu^{+\star} - \nu^{-\star} = \mathbb{0}_E. \tag{10}
$$

The optimal solution also satisfies the following complementary slackness conditions:

$$
\rho_g^\star(P_g^{\mathrm{g}\star} - e_g^{\mathrm{T}}S^{\mathrm{P2P}\star}\mathbb{1}_E - S_g^{\mathrm{exp}\star}) = 0,\ g\in\mathcal{G}, \tag{11}
$$

$$
\Sigma^\star_{(g,\ell)}S^{\mathrm{P2P}\star}_{(g,\ell)} = 0,\ g\in\mathcal{G},\ \ell\in\mathcal{E}, \tag{12}
$$

$$
\zeta_\ell^\star S_\ell^{\mathrm{imp}\star} = 0,\ \ell\in\mathcal{E},\ \xi_g^\star S_g^{\mathrm{exp}\star} = 0,\ g\in\mathcal{G}, \tag{13}
$$

$$
\nu_\ell^{-\star}(\underline{V}_\ell - V_\ell^\star) = 0,\ \nu_\ell^{+\star}(V_\ell^\star - \overline{V}_\ell) = 0,\ \ell\in\mathcal{E}, \tag{14}
$$

$$\phi_g^{-\star}(\underline{P}_g^{\mathrm{g}} - P_g^{\mathrm{g}\star}) = 0,\ \phi_g^{+\star}(P_g^{\mathrm{g}\star} - \overline{P}_g^{\mathrm{g}}) = 0,\ g \in \mathcal{G}, \quad (15)$$

where e_g denotes the g-th standard basis vector, and $S^{\mathrm{P2P}\star}_{(g,\ell)}$ and $\Sigma^\star_{(g,\ell)}$ respectively denote the (g,ℓ) entries in $S^{\mathrm{P2P}\star}$ and $\Sigma^\star$. Finally, the optimal solution satisfies the primal feasibility conditions delineated by (3b)–(3j) as well as the dual feasibility conditions as follows:

$$\rho_g^\star, \Sigma^\star_{(g,\ell)}, \zeta_\ell^\star, \xi_g^\star, \nu_\ell^{-\star}, \nu_\ell^{+\star}, \phi_g^{-\star}, \phi_g^{+\star} \geq 0,\ g \in \mathcal{G},\ \ell \in \mathcal{E}. \quad (16)$$

IV. Electricity Pricing

This section presents results and properties pertaining to locational marginal prices and P2P trading prices derived from the optimality conditions of the market problem in (3).

A. Locational Marginal Prices

In general, LMPs represent the rate of change of total optimal cost in (3a) due to incremental changes in demand at different buses in the system [16]. Mathematically, the active-power LMP at a particular bus is the first derivative of optimal Lagrangian in (4) with respect to the active-power load at that bus [16]. Using the chain rule in calculus and the optimality conditions in (5)–(16), it is straightforward to show that the active-power LMP at bus $\ell \in \mathcal{E}$ is the ℓ-th entry in $\lambda^\star + \beta^\star$.

Lemma 1. Suppose that at optimal solution of the problem in (3), $\{S^{\mathrm{P2P}\star}, S^{\mathrm{imp}\star}, S^{\mathrm{exp}\star}, P^{\mathrm{g}\star}, \theta^\star, V^\star\}$, limits on voltage magnitudes are not binding so that $\nu^{+\star} = \nu^{-\star} = \mathbb{0}_E$. Then $\lambda^\star = \mu^\star = \mathbb{0}_E$, so the active-power LMP at bus $\ell \in \mathcal{E}$ simplifies as the ℓ-th entry in $\beta^\star$.

Proof. With $\nu^{+\star} = \nu^{-\star} = \mathbb{0}_E$, we can stack (9)–(10) into a single matrix-vector equation as follows:

$$\begin{bmatrix} \widehat{J}^{P\theta} & \widehat{J}^{Q\theta} \\ \widehat{J}^{PV} & \widehat{J}^{QV} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \lambda^\star \\ \mu^\star \end{bmatrix} = \widehat{J}^{\mathrm{T}} \begin{bmatrix} \lambda^\star \\ \mu^\star \end{bmatrix} = \begin{bmatrix} \mathbb{0}_E \\ \mathbb{0}_E \end{bmatrix}. \quad (17)$$

Provided that the estimation of linear sensitivities described in Section II-B yields correct results, $\widehat{J}$ essentially approximates the Schur complement of the block corresponding to measured buses in the full power flow Jacobian matrix. Since the Jacobian matrix is evaluated at a valid operating point, we can assume that $\widehat{J}$ and its matrix transpose are invertible. Thus, (17) has only the trivial solution of $\lambda^\star = \mu^\star = \mathbb{0}_E$. □

B. Peer-to-peer Transaction Prices

Given the optimal solution of (3), we set the trade price of the P2P transaction from the DER at bus $g \in \mathcal{G}$ to the market participant at bus $\ell \in \mathcal{E}$ as

$$\pi^\star_{(g,\ell)} = \frac{\partial C(P^{\mathrm{g}\star})}{\partial P_g^{\mathrm{g}\star}} + \phi_g^{+\star} - \phi_g^{-\star} - \gamma^s_{(g,\ell)}, \quad (18)$$

where $\gamma^{\mathrm{s}}_{(g,\ell)}$ denotes the (g,ℓ) entry in Γ^{s}. To motivate the trade price in (18), consider first the special case where minimum and maximum limits on the output of the DER at bus g are not binding at the optimal solution, i.e., $\phi_g^{+\star} = \phi_g^{-\star} = 0$. The trade price then simplifies as the marginal cost of generation of the DER at bus g less the subsidy (if any) that it offers to the market participant at bus ℓ. In the case where the upper limit on the output of the DER at bus g is binding, the positive-valued $\phi_g^{+\star}$ provides a price signal incentivizing further investment in capacity in proportion to the marginal benefit of increasing $\overline{P}_g^{\mathrm{g}}$. On the other hand, positive-valued $\phi_g^{-\star}$ results from the lower limit on the output of the DER at bus g being binding because, e.g., it represents must-run generation. Here, the trade price would be less than the marginal cost of generation of the DER at bus g. The market participant at bus ℓ is also responsible for paying the network usage fee of $\gamma^{\mathrm{dso}} S^{\mathrm{P2P}\star}_{(g,\ell)}$ to the DSO in addition to their payment of $\pi^\star_{(g,\ell)} S^{\mathrm{P2P}\star}_{(g,\ell)}$ to the DER at bus g.

Lemma 2. Suppose that at the optimal solution of the problem in (3), $\{S^{\mathrm{P2P}\star}, S^{\mathrm{imp}\star}, S^{\mathrm{exp}\star}, P^{\mathrm{g}\star}, \theta^\star, V^\star\}$, limits on voltage magnitudes are not binding so that $\nu^{+\star} = \nu^{-\star} = \mathbb{0}_E$. Then the trade price of a nonzero P2P transaction from the DER at bus $g \in \mathcal{G}$ to the market participant at bus $\ell \in \mathcal{E}$, $\pi^\star_{(g,\ell)}$, satisfies

$$\pi^{\mathrm{exp}} - \gamma^{\mathrm{s}}_{(g,\ell)} \leq \pi^\star_{(g,\ell)} \leq \pi^{\mathrm{imp}} - \gamma^{\mathrm{dso}} - \gamma^{\mathrm{b}}_{(g,\ell)}, \quad (19)$$

where $\gamma^{\mathrm{s}}_{(g,\ell)}$ and $\gamma^{\mathrm{b}}_{(g,\ell)}$ denote the (g,ℓ) entries in Γ^{s} and Γ^{b}, respectively.

Proof. We rearrange (5) and express the g-th entry therein as

$$\frac{\partial C(P^{\mathrm{g}\star})}{\partial P_g^{\mathrm{g}\star}} + \phi_g^{+\star} - \phi_g^{-\star} = \lambda_g^\star + \beta_g^\star + \rho_g^\star, \quad (20)$$

$$\pi^\star_{(g,\ell)} = \lambda_g^\star + \beta_g^\star + \rho_g^\star - \gamma^{\mathrm{s}}_{(g,\ell)}, \quad (21)$$

$$\pi^\star_{(g,\ell)} = \beta_g^\star + \rho_g^\star - \gamma^{\mathrm{s}}_{(g,\ell)}, \quad (22)$$

where the second equality results from subtracting both sides by $\gamma^{\mathrm{s}}_{(g,\ell)}$ and substituting (18), and the third equality holds because $\lambda_g^\star = 0$ when voltage limits are not binding, by Lemma 1. With (22) in place, we first provide the proof for the lower bound in (19). Take the g-th entry in (8) to get $\beta_g^\star = \pi^{\mathrm{exp}} + \rho_g^\star + \xi_g^\star$, which we substitute into (22) to yield

$$\pi^\star_{(g,\ell)} = \pi^{\mathrm{exp}} + 2\rho_g^\star + \xi_g^\star - \gamma^{\mathrm{s}}_{(g,\ell)}. \quad (23)$$

In the above, since $\xi_g^\star \geq 0$ and $\rho_g^\star \geq 0$ by dual feasibility, we obtain the lower bound in (19), as desired.

To establish the upper bound on the trade price in (19), we extract the (g,ℓ) entry in (6) to get

$$\gamma^{\mathrm{b}}_{(g,\ell)} - \gamma^{\mathrm{s}}_{(g,\ell)} - \beta_\ell^\star - \Sigma^\star_{(g,\ell)} + \gamma^{\mathrm{dso}} + \beta_g^\star + \rho_g^\star = 0, \quad (24)$$

where $\Sigma^\star_{(g,\ell)}$ denotes the (g,ℓ) entry in $\Sigma^\star$. Furthermore, by complementary slackness, the nonzero trade from DER $g \in \mathcal{G}$ to market participant $\ell \in \mathcal{E}$ taking place at the optimal solution of (3) implies that $\Sigma^\star_{(g,\ell)} = 0$. Then (24) simplifies as

$$\gamma^{\mathrm{b}}_{(g,\ell)} - \gamma^{\mathrm{s}}_{(g,\ell)} - \beta_\ell^\star + \gamma^{\mathrm{dso}} + \beta_g^\star + \rho_g^\star = 0, \quad (25)$$

which we rearrange to get

$$\pi^\star_{(g,\ell)} = \beta_\ell^\star - \gamma^{\mathrm{dso}} - \gamma^{\mathrm{b}}_{(g,\ell)}, \quad (26)$$

where we have made use of (22). Now take the ℓ-th entry of (7) to get $\beta_\ell^\star = \pi^{\mathrm{imp}} - \zeta_\ell^\star$. Further recognizing that $\zeta_\ell^\star \geq 0$ by dual feasibility, we get that $\beta_\ell^\star \leq \pi^{\mathrm{imp}}$, which we substitute into (26) to yield the upper bound in (19), as desired. □

The bounds on the trade price given by (19) imply that any nonzero P2P transaction resulting from the optimal solution

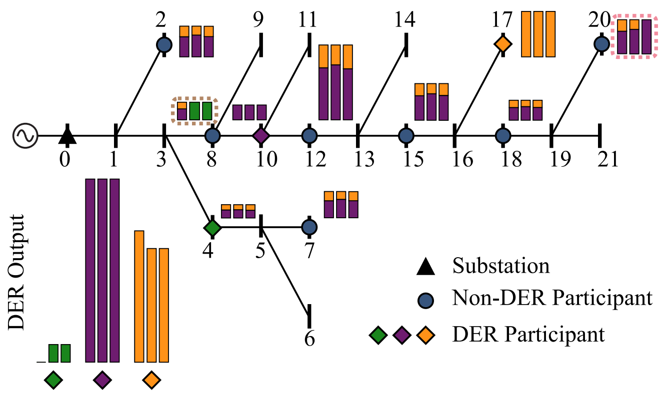

Fig. 1. One-line diagram of the 22-bus distribution test system modified to include market participants at buses in $\mathcal{E} = \{2, 4, 7, 8, 10, 12, 15, 17, 18, 20\}$ and, amongst these, DERs at buses in $\mathcal{G} = \{4, 10, 17\}$. Stacked bars near a market participant specify the portion of its demand (captured by total height) sourced from each DER with (i) no buying or selling preferences (left), (ii) buying preferences only (middle), and (iii) buying *and* selling preferences (right). Buying preferences in Cases (ii)–(iii) consist of a penalty applied by the market participant at bus 8 to purchases from DERs at buses 10 and 17 (see brown box), and selling preferences in Case (iii) consist of a subsidy offered by the DER at bus 10 to the market participant at bus 20 (see pink box). DER outputs corresponding to each case are shown in bottom left.

of (3) would be mutually beneficial (or at worst neutral) for both the seller (i.e., DER at bus g) and the buyer (i.e., market participant at bus ℓ) relative to the alternative option of transacting with the upstream network under payments based on fixed or time-of-use rates. This is inclusive of the impact of network usage fees proportional to γ^{dso}, which are paid by the buyer and hence present in the upper bound of (19). Equivalently speaking, the bounds in (19) imply that no market participant (DER) would buy (sell) power for more (less) than the import (export) price, unless as a result of their own buying (selling) preferences captured by $\gamma^{\mathrm{b}}_{(g,\ell)}$ ($\gamma^{\mathrm{s}}_{(g,\ell)}$).

V. Case Studies

In this section, we present numerical simulations involving a 22-bus distribution system (see, e.g., [17]) modified to include market participants and DERs, as shown in Fig. 1 with parameter values listed in Appendix A. We use MATPOWER [18] to obtain simulated nodal voltage and power injection measurements at the buses connected to market participants, from which the linear sensitivity model is estimated as described in Section II-B using the NIPALS algorithm in MVARTOOLS [19]. The optimization problem in (3) is constructed using YALMIP [20] and solved with Gurobi [21]. Simulations are performed in MATLAB R2022b on a personal computer with an Intel i5-7500 processor at 3.40 GHz and 16 GB RAM. Simulation results demonstrate the effectiveness of market participant preferences in influencing the optimal DER dispatch and P2P transactions, along with the validity of the pricing properties presented in Section IV.

A. Effect of Buying and Selling Preferences

We demonstrate the impact of market participant preferences on optimal P2P transactions and DER dispatch, as summarized in Fig. 1 and Table I. We present results for three cases differing in market participant preferences, where case (i) represents a benchmark P2P scenario without any buying or selling preferences, i.e., $\Gamma^{\mathrm{b}} = \Gamma^{\mathrm{s}} = \mathbb{0}_{G\times E}$. In case (ii), the market participant at bus 8 expresses buying preferences for electricity produced by the DER at bus 4 and so penalizes purchases from DERs at buses 10 and 17 in setting $\gamma^{\mathrm{b}}_{(10,8)} = \gamma^{\mathrm{b}}_{(17,8)} = 5\,[¢/\mathrm{kWh}]$, with all other entries in Γ^{b} being zero. In Case (iii), in addition to the buying preferences deployed in Case (ii), the DER at bus 10 applies a subsidy to the market participant at bus 20 by setting $\gamma^{\mathrm{s}}_{(10,20)} = 1\,[¢/\mathrm{kWh}]$ with all other entries in Γ^{s} being zero.

TABLE I
Trade price, $\pi^{\star}_{(g,\ell)}$ $[¢/\mathrm{kWh}]$, for market participant at bus ℓ to buy from DER at bus g, where Cases (i)–(iii) are consistent with results depicted in Fig. 1.

Non-DER Market Participant ℓ	Case (i)		Case (ii)		Case (iii)	
	DER g	Price	DER g	Price	DER g	Price
$2, 7, 12, 15, 18$	$10, 17$	4.53	$10, 17$	4.53	$10, 17$	4.53
8	$10, 17$	4.53	4	5.00	4	5.00
20	$10, 17$	4.53	$10, 17$	4.53	10	3.53

As shown in Fig. 1, in Case (i) with no preferences whatsoever, the more expensive DER at bus 4 is not dispatched because the cheaper DERs at buses 10 and 17 are able to serve all the demand. Correspondingly, as reported in column 3 in Table I, all trade prices are equal. With the buying preferences imposed in Case (ii), the market participant at bus 8 sources all its power from the DER at bus 4, as highlighted by the brown-coloured box in Fig. 1. At the same time, the buyer at bus 8 incurs a higher trade price for preferring to serve its demand with the more expensive DER at bus 4, as reported in column 5 in Table I. In Case (iii), the market participant at bus 20 sources all its power from the DER at bus 10 (highlighted by the pink-coloured box in Fig. 1) and incurs a lower trade price (column 7 in Table I), reflecting the subsidy provided by the DER at bus 10. Finally, it is worth noting that, without the option of P2P transactions, all market participants would buy their power from the upstream network at the import price of $10\,[¢/\mathrm{kWh}]$, leading to greater cost of electricity.

B. Effect of Export Price

Persisting with buying and selling preferences used in Case (iii), we further modify the market configuration by doubling the export price to $\pi^{\mathrm{exp}} = 6\,[¢/\mathrm{kWh}]$ in order to uncover the impact of nonzero exports on the optimal solution of the market problem in (3). All DERs produce power up to their respective maximum limits $\overline{P}^{\mathrm{g}}_{g}$, $g \in \mathcal{G}$, as shown by the horizontal bars on the right side of Fig. 2. Any excess power not used to serve the demand of market participants is sold to the upstream network. This is because the new export price is greater than the maximum possible marginal cost for all three DERs (at $\overline{P}^{\mathrm{g}}$). As in Case (iii), the market participant at bus 8 buys power solely from the DER at bus 4 in accordance with buying preferences, and the participant at bus 20 buys solely from the DER at bus 10 in response to the subsidy offered. All other market participants buy power from all three DERs. All nonzero P2P transactions

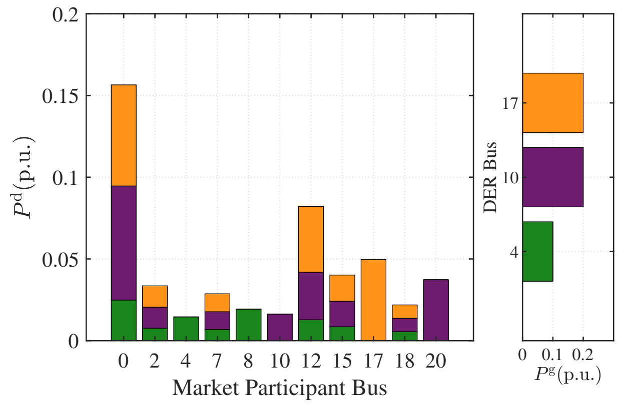

Fig. 2. P2P transactions with buyer preferences, seller subsidies, and exports. Stacked vertical bars represent the total power bought by market participants, with colour specifying the source DER. The bar for bus 0 represents exports, i.e., power sold by DERs to the upstream network. DER outputs are shown on the right with horizontal bars of the corresponding colour.

are priced at $\pi^\star_{(g,\ell)} = \pi^{\rm exp} = 6\,[¢/\rm kWh]$, with the exception of $\pi^\star_{(10,20)} = 5\,[¢/\rm kWh]$ where the reduced price is consistent with the subsidy $\gamma^{\rm s}_{(10,20)} = 1\,[¢/\rm kWh]$. In agreement with the lower bound in (19), when it becomes profitable for DERs to export power to the upstream network, then the lowest price they would accept for P2P transactions is equal to the export price less any subsidies they elect to offer to certain market participants.

VI. Concluding Remarks

In this paper, we presented a measurement-based market structure to facilitate peer-to-peer transactions in distribution systems where individual preferences have a direct impact on the optimal DER dispatch. By considering fixed prices associated with importing (exporting) power from (to) the upstream network, the proposed model allows for seamless integration with the widely adopted feed-in tariffs. By embedding a linear sensitivity model estimated from D-PMU measurements into the constraints of an OPF problem, voltage limits at the subset of buses equipped with D-PMUs are satisfied without the need for any offline knowledge of the distribution network topology. Using this model, we propose a mechanism for pricing P2P trades and show that these trades are beneficial to both buyer and seller while taking into account transaction costs and preferences. We present numerical simulations involving a 22-bus distribution test system to demonstrate the impact of individual preferences on optimal DER dispatch as well as on bounds for the trade prices. Promising directions for future work include distributed solution of both linear model estimation and optimization as well as extensions to multi-interval markets and combined markets for energy and flexibility.

Appendix

A. Parameters for Case Studies in Section V

1) Imports and Exports: $\pi^{\rm imp} = 10\,[¢/\rm kWh]$, $\pi^{\rm exp} = 3\,[¢/\rm kWh]$ in Section V-A and $\pi^{\rm exp} = 6\,[¢/\rm kWh]$ in Section V-B.

2) Network Usage Fee: $\gamma^{\rm dso} = 0.01\,[¢/\rm kWh]$.

3) DERs at Buses in {4, 10, 17}: The DER cost function in (3a) is given by $C(P^{\rm g}) = (P^{\rm g})^{\rm T}{\rm diag}(a)(P^{\rm g}) + bP^{\rm g}$, where $a = [0.1, 0.1, 0.1]\,[¢/\rm kWh^2]$ and $b = [5, 4, 4.5]\,[¢/\rm kWh]$; $\underline{P}^{\rm g} = [0, 0, 0]^{\rm T}\,[\rm kW]$ and $\overline{P}^{\rm g} = [100, 200, 200]^{\rm T}\,[\rm kW]$.

References

[1] S. Xu, Y. Xue, and L. Chang, "Review of power system support functions for inverter-based distributed energy resources–standards, control algorithms, and trends," *IEEE Open Journal of Power Electronics*, vol. 2, pp. 88–105, 2021.
[2] N. Good, K. A. Ellis, and P. Mancarella, "Review and classification of barriers and enablers of demand response in the smart grid," *Renewable and Sustainable Energy Reviews*, vol. 72, pp. 57–72, 2017.
[3] W. Tushar, T. K. Saha, C. Yuen, D. Smith, and H. V. Poor, "Peer-to-peer trading in electricity networks: An overview," *IEEE Trans. Smart Grid*, vol. 11, no. 4, pp. 3185–3200, 2020.
[4] C. Oliveira, D. F. Botelho, T. Soares, A. S. Faria, B. H. Dias, M. A. Matos, and L. W. de Oliveira, "Consumer-centric electricity markets: A comprehensive review on user preferences and key performance indicators," *Electric Power Systems Research*, vol. 210, p. 108088, 2022.
[5] S. Aggarwal, N. Kumar, S. Tanwar, and M. Alazab, "A survey on energy trading in the smart grid: Taxonomy, research challenges and solutions," *IEEE Access*, vol. 9, pp. 116 231–116 253, 2021.
[6] T. Capper, A. Gorbatcheva, M. A. Mustafa, M. Bahloul, J. M. Schwidtal, R. Chitchyan, M. Andoni, V. Robu, M. Montakhabi, I. J. Scott, C. Francis, T. Mbavarira, J. M. Espana, and L. Kiesling, "Peer-to-peer, community self-consumption, and transactive energy: A systematic literature review of local energy market models," *Renewable & sustainable energy reviews*, vol. 162, p. 112403, 2022.
[7] E. Sorin, L. Bobo, and P. Pinson, "Consensus-based approach to peer-to-peer electricity markets with product differentiation," *IEEE Trans. Power Syst.*, vol. 34, no. 2, pp. 994–1004, 2019.
[8] T. Baroche, F. Moret, and P. Pinson, "Prosumer markets: A unified formulation," in *IEEE Milan PowerTech*, 2019, pp. 1–6.
[9] G. van Leeuwen, T. AlSkaif, M. Gibescu, and W. van Sark, "An integrated blockchain-based energy management platform with bilateral trading for microgrid communities," *Applied Energy*, vol. 263, p. 114613, 2020.
[10] M. Khorasany, Y. Mishra, and G. Ledwich, "A decentralized bilateral energy trading system for peer-to-peer electricity markets," *IEEE Trans. Ind. Electron.*, vol. 67, no. 6, pp. 4646–4657, 2020.
[11] T. Chernova and E. Gryazina, "Peer-to-peer market with network constraints, user preferences and network charges," *International Journal of Electrical Power & Energy Systems*, vol. 131, p. 106981, 2021.
[12] R. Khatami, S. Nowak, and Y. C. Chen, "Measurement-based locational marginal pricing in active distribution systems," in *IEEE Power & Energy Society General Meeting*, 2022.
[13] S. Nowak, L. Wang, and Y. C. Chen, "Measurement-based optimal power flow with linear power-flow constraint for DER dispatch," in *North American Power Symposium*, 2019, pp. 1–5.
[14] S. Nowak, Y. C. Chen, and L. Wang, "Measurement-based optimal DER dispatch with a recursively estimated sensitivity model," *IEEE Trans. Power Syst.*, vol. 35, no. 6, pp. 4792–4802, Nov. 2020.
[15] A. Von Meier, E. Stewart, A. McEachern, M. Andersen, and L. Mehrmanesh, "Precision micro-synchrophasors for distribution systems: A summary of applications," *IEEE Trans. Smart Grid*, vol. 8, no. 6, pp. 2926–2936, Nov. 2017.
[16] R. Khatami and M. Parvania, "Continuous-time locational marginal price of electricity," *IEEE Access*, vol. 7, pp. 129 480–129 493, Sep. 2019.
[17] M. Ramalinga Raju, K. Ramachandra Murthy, and K. Ravindra, "Direct search algorithm for capacitive compensation in radial distribution systems," *International Journal of Electrical Power & Energy Systems*, vol. 42, no. 1, pp. 24–30, 2012.
[18] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 12–19, 2011.
[19] bitjungle, "MVARTOOLS," 2001. [Online]. Available: https://github.com/bitjungle/MVARTOOLS
[20] J. Löfberg, "Yalmip : A toolbox for modeling and optimization in MATLAB," in *CACSD Conference*, Taipei, Taiwan, 2004.
[21] Gurobi Optimization, LLC, "Gurobi Optimizer Reference Manual," 2023. [Online]. Available: https://www.gurobi.com