Impact of EV Charging Load Uncertainty as Gaussian Mixtures Model on System Performance

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***Abstract*—This paper proposes a method to assess the impact of the uncertainty in electric vehicle (EV) charging loads on system static performance reflected through the resulting uncertainty in system states, i.e., bus voltage magnitude and phase-angles. Historical EV charging data is fit to a Gaussian mixtures model (GMM), which can capture generic probability distributions. The GMM is then propagated through a linearized power flow model to yield a probabilistic characterization of the voltage magnitudes and phase-angles. As a direct consequence of the linear model approximation, the resulting characterization is also a GMM, the parameters of which can be computed in closed form. Numerical simulations involving the IEEE 33-bus distribution test system demonstrate the effectiveness of the proposed method to assess uncertainty in system states.**

I. Introduction

Owing to greater energy efficiency and lower emissions of electric vehicles (EVs) as compared with conventional internal combustion engine-based vehicles, EVs are gaining global popularity in the automotive industry [1]. For example, EVs are expected to contribute to 10% of the total load demand in Great Britain by 2030 [2]. Most EVs are used in cities, and they connect to the low-voltage distribution system for charging purposes [1]. The charging loads are uncertain due to the inherently unpredictable nature of human travel behaviour. Coupled with high levels of EV penetration leading to potentially heavy loading, the uncertainty in EV charging loads may cause the distribution system to exceed prescribed performance requirements, such as minimum and maximum bus voltage magnitudes and line flows, in an unpredictable fashion [3]. This paper develops an analytically tractable method to determine the probability with which system static state variables (bus voltage magnitudes and phase-angles) lie within certain ranges given a probabilistic Gaussian mixtures model (GMM) of EV charging loads synthesized from power or energy consumption data collected at charging stations.

Deterministic power flow analysis is a fundamental tool used by power engineers to compute the voltage phase-angles and magnitudes at all buses given a particular instance of generation and load. However, a single-snapshot solution of the power flow problem does not offer insight into the probabilistic system performance in the presence of uncertainty arising from, e.g., EV charging loads. To address this, in probabilistic power flow analysis, sources of uncertainty are modelled as random variables, which results in the power flow solution also being represented by random variables. Both numerical and analytical methods have been proposed to solve the probabilistic power flow problem.

Numerical methods for solving the probabilistic power flow problem typically require thousands (or more) repeated solutions of the nonlinear power flow problem, where the uncertain independent variables therein, such as the load demand, are sampled from a given probability distribution [4]. Such methods can capture the impact of generic distributions with complex correlation structures, but they are generally impractical for large-scale distribution systems due to the computation burden involved. On the other hand, analytical methods aim to directly compute the probability density function (PDF) or cumulative distribution fuction (CDF) of the random variables of interest without repeated solutions of the nonlinear power flow problem. These typically prescribe a particular PDF or CDF to the uncertainty in independent variables of the power flow problem [5]. Common models include the Gaussian distribution for load, beta distribution for solar irradiance, and Weibull distribution for wind speed [5]. The input uncertainty would then be propagated through (typically) linearized power flow equations to the system state variables via convolution- or cumulant-based methods [6]. Although these methods are less computationally expensive than numerical methods, they may be prone to reduced accuracy as the PDFs are approximated as truncated series expansions. Furthermore, the uncertainty in independent variables of the power flow problem may not fit a well-defined PDF.

Modelling uncertainty in EV charging loads is challenging as it depends on numerous factors, including arrival and departure times, the number of EVs at the station, charging voltage and current levels, state of charge (SOC), battery capacity, and charging duration. With respect to the aforementioned features, a Poisson PDF is used to model EV charging start times [7], and a uniform PDF is used to model the SOC [8]. Furthermore, PDFs for EV charging loads can be inferred via travel statistics and battery characteristics [9]–[11]. Another viable option, given high-fidelity power and energy measurements collected at charging stations, is to synthesize a probabilistic model for EV charging loads *directly* from measured data. For this, the Gaussian mixture model (GMM) is an attractive candidate as it can capture generic distributions while retaining the analytical tractability associated with Gaussian PDFs. The central idea behind the GMM is to represent a generic multimodal PDF as a convex combination of several unimodal Gaussian PDFs. By using a sufficient number of Gaussian PDFs and by adjusting their means and covariances as well as the weights in the convex combination, generic PDFs can be approximated to arbitrary accuracy [12]. The GMM has been used to approximate uncertainty in solar photovoltatic generation [13], wind farm output [14], and load demand [15]. With respect to EVs, [16] uses GMMs to predict a user's departure time and the associated energy consumption based on their known arrival time, but the uncertainty in EV charging loads is not directly modelled with the GMM.

Given the review of pertinent literature above, this paper's contributions are as follows. We directly fit historical measurements of EV charging loads to a multivariate GMM obtained as the solution of a maximum likelihood estimation problem. Each Gaussian component is then propagated through a linearized system power flow model. The result is a probabilistic characterization of the uncertainty in bus voltage phase-angles and magnitudes, which is also described by a GMM, owing to the linear model approximation. Furthermore, the parameters in the resulting GMM can be obtained in closed form with little computational overhead. We can further assess the impact of uncertainty in EV charging loads by evaluating the probability with which voltage magnitudes lie within certain (permissible) ranges. The effectiveness of using GMMs to represent EV charging loads is verified using a dataset from the California Institute of Technology (Caltech) [17], and the proposed uncertainty propagation method is then verified via numerical case studies involving the IEEE 33-bus test system [18].

II. Preliminaries

In this section, we present the nonlinear power flow equations with a focus on distribution networks. We further derive a linearized power flow model to be used later.

A. Power Flow Model

Consider an AC distribution system with buses collected in set $\mathcal{N} = \{1, \ldots, N\}$. At each bus, the power flow equations relate the bus voltage magnitude and phase-angle to active- and reactive-power injections at all other buses in the system. For each bus $i \in \mathcal{N}$, let V_i denote the voltage magnitude, θ_i the voltage phase-angle, P_i the net active-power load, and Q_i the net reactive-power load. Without loss of generalization, we assume bus 1 is the substation with fixed voltage magnitude, and it sets the phase-angle reference. Then, the power balance at bus $i \in \mathcal{N}^- = \{2, \ldots, N\}$ can be expressed as follows:

$$0 = V_i \sum_{k=1}^{n} V_k \left(G_{ik} \cos\left(\theta_i - \theta_k\right) + B_{ik} \sin\left(\theta_i - \theta_k\right)\right) + P_i, \quad (1)$$

$$0 = V_i \sum_{k=1}^{n} V_k \left(G_{ik} \sin\left(\theta_i - \theta_k\right) - B_{ik} \cos\left(\theta_i - \theta_k\right)\right) + Q_i, \quad (2)$$

where G_{ik} and B_{ik} are the real and imaginary parts of the (i, k) entry in the network admittance matrix, respectively. To contain notational burden, we assume that there is at most one EV charging load at each bus $i \in \mathcal{N}^-$. In practice, this may be the aggregate load of several (or even many) EV charging stations. Collect all L buses that are connected to EV charging stations in the set $\mathcal{L} \subseteq \mathcal{N}^-$. To explicitly consider the uncertainty in power injections arising from EV charging loads, we distinguish between fixed and uncertain load withdrawals in (1)–(2), as follows:

$$P_i = \overline{P}_i + \widetilde{P}_i, \quad (3)$$

$$Q_i = \overline{Q}_i + \widetilde{Q}_i, \quad (4)$$

where $\overline{P}_i$ and $\overline{Q}_i$ are respectively the fixed active- and reactive-power loads, at bus i, and $\widetilde{P}_i$ and $\widetilde{Q}_i$ are respectively the uncertain active- and reactive-power loads at bus i.

Let $y = [P_2, \ldots, P_N, Q_2, \ldots, Q_N]^{\mathrm{T}}$ contain net active- and reactive-power loads, and let system state variable $x = [\theta_2, \ldots, \theta_N, V_2, \ldots, V_N]^{\mathrm{T}}$ contain bus voltage phase-angles and magnitudes. Then (1)–(2) can be expressed compactly as

$$0 = g(x) + y = g(x) + \overline{y} + C\widetilde{y}, \quad (5)$$

where $g : \mathbb{R}^{2(N-1)} \mapsto \mathbb{R}^{2(N-1)}$, x is an unknown to be solved, and $\overline{y}$ and $\widetilde{y}$ are the (known) fixed and uncertain loads, respectively. Also, C is a matrix of 1s and 0s that maps the buses with EV charging stations in $\mathcal{L}$ to corresponding indices in $\mathcal{N}^-$.

B. Linearized Model

Suppose (5) is solved with nominal uncertain load $\widetilde{y} = \widetilde{y}^\star$, leading to the nominal power flow solution $x^\star$. Then, from (5), we have

$$0 = g(x^\star) + \overline{y} + C\widetilde{y}^\star. \quad (6)$$

We can take the first-order Taylor series expansion of (5) around $x^\star$ to get

$$0 = g(x^\star) + J(x - x^\star) + \overline{y} + C\widetilde{y}, \quad (7)$$

where $J = \frac{dg}{dx}|_{x^\star}$ is the Jacobian matrix of the power flow equations. Further rearrange (6) and substitute the resultant into (7) to get

$$0 = J(x - x^\star) + C(\widetilde{y} - \widetilde{y}^\star), \quad (8)$$

The Jacobian matrix evaluated at $x^\star$ is guaranteed to be invertible if the power flow converges to that solution. Thus, we can rearrange (7) as

$$x = H\widetilde{y} + c, \quad (9)$$

where

$$H = -J^{-1}C, \quad c = J^{-1}C\widetilde{y}^\star + x^\star, \quad (10)$$

are the linear- and constant-term coefficients of a linearized power flow model evaluated at $x^\star$.

III. Uncertainty Analysis

We decompose the problem of assessing EV charging load uncertainty on system performance into two distinct tasks. The first is to fit historical EV charging load data to a GMM. Next, we propagate the GMM through the linearized model in (9) to compute the uncertainty in x.

A. GMM for EV Charging Loads

In general, the uncertainty in an EV charging load $\widetilde{y}$ is not Gaussian due to various physical (e.g., state and rate of charge) and human (e.g., arrival and departure times) factors. Instead, we approximate the uncertainty in $\widetilde{y}$ as a GMM and show that it indeed provides a realistic representation of the uncertainty in EV charging loads via numerical case studies in Section IV.

We model the uncertainty in $\widetilde{y}$ as convex combination of $K+1$ multivariate Gaussian PDFs of the following form:

$$\widetilde{y} \sim \sum_{k=0}^{K} f_{\widetilde{y},k}(\widetilde{y}) = \sum_{k=0}^{K} \omega_{\widetilde{y},k} \mathcal{N}(\mu_{\widetilde{y},k}, \Sigma_{\widetilde{y},k}), \tag{11}$$

where $\omega_{\widetilde{y},k}$ is the weight given to the kth component of the multivariate GMM, and

$$\mathcal{N}(\widetilde{y}|\mu_{\widetilde{y},k},\Sigma_{\widetilde{y},k}) = \frac{1}{(2\pi)^{L/2}} \frac{1}{|\Sigma_{\widetilde{y},k}|^{1/2}} \cdot \exp\left(-\frac{1}{2}(\widetilde{y}-\mu_{\widetilde{y},k})^{\mathrm{T}} \Sigma_{\widetilde{y},k}^{-1} (\widetilde{y}-\mu_{\widetilde{y},k})\right), \tag{12}$$

with $\mu_{\widetilde{y},k}$ and $\Sigma_{\widetilde{y},k}$ respectively denoting the mean and covariance of the kth Gaussian PDF. Furthermore, for the GMM to be well-defined, the following must hold:

$$0 \leq \omega_{\widetilde{y},k} \leq 1, \; k = 0, \ldots, K, \tag{13}$$

$$\sum_{k=0}^{K} \omega_{\widetilde{y},k} = 1. \tag{14}$$

Now, consider a historical dataset $\mathcal{Z} = \cup_{\ell \in \mathcal{L}} \mathcal{Z}_\ell$, where Z_ℓ contains EV charging load data for bus $\ell \in \mathcal{L}$. Within such a dataset, the uncertain EV charging load at bus ℓ is either zero, corresponding to the event that the charging station is idle, or nonzero, corresponding to the event that the charging station is actively charging a vehicle. Accordingly, we divide the dataset $\mathcal{Z}$ into $\mathcal{Z}^0$ and $\mathcal{Z}^\emptyset$ respectively, containing data points where all charging loads are zero and where at least one charging load is nonzero. These events are mutually exclusive, so that $\mathcal{Z}^0 \cap \mathcal{Z}^\emptyset = \emptyset$, and $\Pr(\widetilde{y} = \mathbb{0}_L) + \Pr(\widetilde{y} \neq \mathbb{0}_L) = 1$. In (11), we represent the event in which all EV charging loads are zero by the first term in the sum $f_{\widetilde{y},0}(\widetilde{y})$, and the remaining terms in the sum capture the events in which at least one charging load is nonzero. We next discuss the specifics of how to obtain the terms in (11).

1) Zero-charging Event: We approximate the zero-charging event as the following Gaussian PDF:

$$f_{\widetilde{y},0}(\widetilde{y}) = \omega_{\widetilde{y},0} \mathcal{N}(\mathbb{0}_L, \Sigma_{\widetilde{y},0}), \tag{15}$$

where $\omega_{\widetilde{y},0} = \Pr(\widetilde{y} = \mathbb{0}_L) = |\mathcal{Z}^0|/|\mathcal{Z}|$ is the weight associated with the zero-charging event, and $\Sigma_{\widetilde{y},0}$ is a diagonal matrix and diagonal entries are set to be very small.

2) Nonzero-charging Events: Given adequate historical data points of nonzero-charging events in $\mathcal{Z}^\emptyset$, we can fit the following GMM to the data:

$$f_{\widetilde{y}}^{\emptyset}(\widetilde{y}) = \sum_{k=1}^{K} f_{\widetilde{y},k}^{\emptyset}(\widetilde{y}) = \sum_{k=1}^{K} \omega_{\widetilde{y},k}^{\emptyset} \mathcal{N}(\mu_{\widetilde{y},k}, \Sigma_{\widetilde{y},k}), \tag{16}$$

where parameters $\omega_{\widetilde{y},k}^{\emptyset}$, $\mu_{\widetilde{y},k}$, and $\Sigma_{\widetilde{y},k}$, $k = 1, \ldots, K$, are obtained as the solution of a maximum likelihood estimation (MLE) problem. Interested readers may refer to [19] for details of the expectation maximization (EM) algorithm to solve the MLE problem. Furthermore, we apply the Bayesian information criterion (BIC) to determine the number of Gaussian components, i.e., K, to represent the uncertainty in EV charging loads sufficiently well. The procedure involved with applying the BIC is provided in [19]. To ensure that the area under the final GMM (inclusive of zero-charging and nonzero-charging events), we scale coefficients $\omega_{\widetilde{y},k}^{\emptyset}$ obtained from the EM algorithm by the total probability of nonzero-charging events, as follows:

$$\omega_{\widetilde{y},k} = \omega_{\widetilde{y},k}^{\emptyset} \cdot \Pr(\widetilde{y} \neq \mathbb{0}_L), \; k = 1, \ldots, K, \tag{17}$$

where $\Pr(\widetilde{y} \neq \mathbb{0}_L) = |\mathcal{Z}^\emptyset|/|\mathcal{Z}|$.

B. Uncertainty Propagation

The uncertain EV charging load $\widetilde{y}$ is modelled as the multivariate GMM in (11), where the parameters therein are obtained by fitting the model to a historical dataset $\mathcal{Z}$. Furthermore, given the linearized power flow model in (9), our goal is to obtain a PDF that describes the uncertainty in system states, i.e., bus voltage phase-angles and magnitudes collected in x. Since Gaussian PDFs are closed under linear transformation and we can apply the superposition principle for linear systems, the PDF describing uncertainty in x is also a GMM, which can be described as follows:

$$x \sim \sum_{k=0}^{K} \omega_{x,k} \mathcal{N}(\mu_{x,k}, \Sigma_{x,k}), \tag{18}$$

where, for $k = 0, \ldots, K$, the weight, mean, and covariance matrix of the kth Gaussian PDF are respectively given by

$$\omega_{x,k} = \omega_{\widetilde{y},k}, \tag{19}$$

$$\mu_{x,k} = H\mu_{\widetilde{y},k} + c, \tag{20}$$

$$\Sigma_{x,k} = H\Sigma_{\widetilde{y},k} H^{\mathrm{T}}, \tag{21}$$

all in closed form [20]. Furthermore, the CDF can be obtained by integrating (18), as follows:

$$F_x \sim \sum_{k=0}^{K} \omega_{x,k} \int \mathcal{N}(\mu_{x,k}, \Sigma_{x,k}). \tag{22}$$

IV. Case Studies

In this section, we demonstrate the effectiveness of modelling EV charging loads as a multivariate GMM with historical charging data obtained from Caltech [17]. We then demonstrate propagation of uncertainty from EV charging loads to system states via numerical case studies involving the IEEE 33-bus test system [18].

TABLE I: GMM component parameters for EV charging data

Component k	Weight $\omega_{\widetilde{y},k}$	$\mu_{\widetilde{y},k}$	$\Sigma_{\widetilde{y},k}$
0	0.051	$\begin{bmatrix}0\\0\end{bmatrix}$	$\begin{bmatrix}0.073 & 0\\0 & 0.073\end{bmatrix}$
1	0.158	$\begin{bmatrix}4.906\\6.845\end{bmatrix}$	$\begin{bmatrix}2.367 & 0.271\\0.271 & 1.886\end{bmatrix}$
2	0.282	$\begin{bmatrix}5.475\\5.074\end{bmatrix}$	$\begin{bmatrix}2.623 & 0.597\\0.597 & 2.631\end{bmatrix}$
3	0.261	$\begin{bmatrix}1.618\\1.602\end{bmatrix}$	$\begin{bmatrix}1.989 & 0.130\\0.130 & 1.865\end{bmatrix}$
4	0.248	$\begin{bmatrix}4.048\\3.530\end{bmatrix}$	$\begin{bmatrix}2.619 & 1.264\\1.264 & 3.374\end{bmatrix}$

A. Fitting Historical EV Charging Data to GMM

We consider a subset of the Caltech dataset related to the charging loads at 1 pm of two EV charging stations from January 1, 2018 to May 25, 2021. Within the dataset that we consider, the zero-charging event where both charging stations are idle occurs with probability $\Pr(\widetilde{y} = \mathbb{0}_2) = 0.051$, implying that nonzero-charging events where at least one charging station is active occur with probability $\Pr(\widetilde{y} \neq \mathbb{0}_2) = 0.949$. As described in Section III-A, we model the zero-charging events by a narrow Gaussian PDF, the parameters for which are reported as component $k = 0$ in Table I with corresponding weight $\omega_{\widetilde{y},0} = \Pr(\widetilde{y} = \mathbb{0}_2)$. For the nonzero-charging events, we apply the EM algorithm to the subset of data where at least one of the two charging stations is actively charging a vehicle. We use the built-in MATLAB command `aicbic` to evaluate the BIC and determine that four Gaussian components are sufficient to approximate variations in the charging loads. We further use MATLAB command `fitgmdist` to determine the parameters in the 4-component GMM. These are reported in Table I for $k = 1, \ldots, 4$, where the mean and covariance matrix of each Gaussian component are in columns 3 and 4, respectively. The weights reported in column 2 have been scaled according to (17) to reflect the fact that the nonzero-charging events make up the only portion of the dataset we consider. To verify that the GMM represents a realistic model for the EV charging load data, in Fig. 1, we plot intensity maps representing probability distributions derived from the continuous-valued GMM (Fig. 1a) with parameters given in Table I and the original discrete-valued dataset (Fig. 1b). Based on a visual inspection, we indeed find that the two plots are very similar. We also observe that the uncertainty in the two EV charging loads are positively correlated.

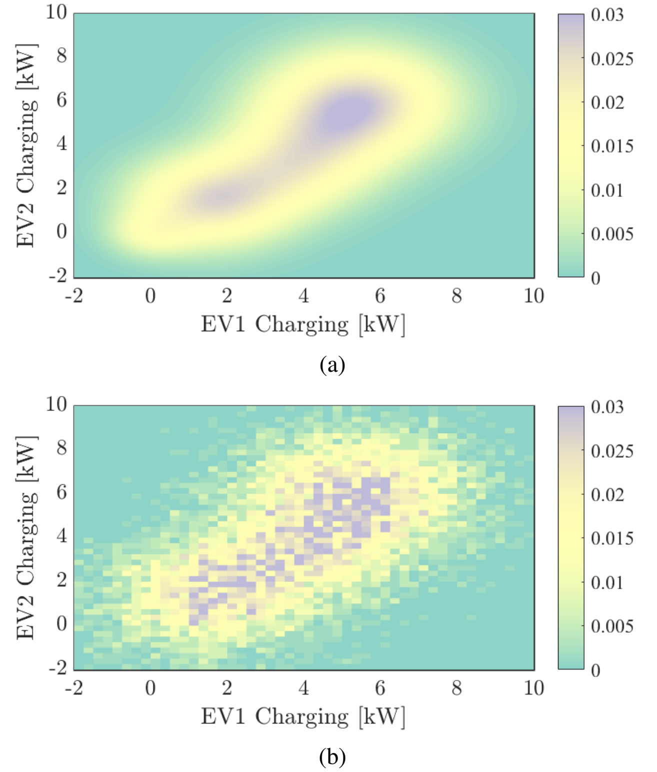

Fig. 1: Probability distribution of two EV charging loads derived from (a) the continuous-valued GMM and (b) the discreted-valued dataset.

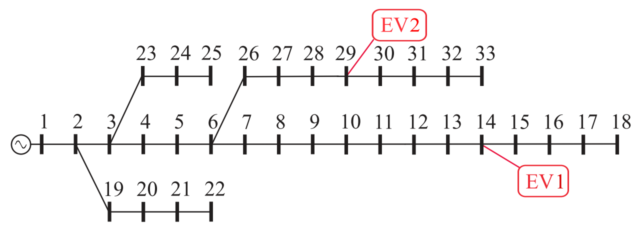

Fig. 2: 33-bus test system with EV charging stations at buses 14 and 29.

B. Propagating Uncertainty to System States

As depicted in Fig. 2, we modify the IEEE 33-bus test system to include uncertain EV charging loads at buses 14 and 29 corresponding to charging stations considered in Section IV-A. To mimic a case with greater EV penetration, the charging loads used to obtain the 5-component GMM in Table I are scaled by $10\times$. We then convert the actual EV charging load values to per-unit quantities using a system power base of 10 MVA. Accordingly, the mean values in column 3 of Table I are scaled by 10^{-3} and the entries in covariance matrices in column 4 are scaled by 10^{-6}. We linearize the power flow equations for the test system and apply the uncertainty propagation method described in Section III-B. Particularly, the GMM describing the uncertainty in voltage phase-angles and magnitudes due to that in EV charging loads can be obtained using (18), where the parameters therein can be computed in closed form. In Fig. 3a, we project the resulting multivariate GMM and plot the marginal PDF of the voltage magnitude at bus 3 as the red trace. The marginal CDF obtained from (22) is similarly plotted as the red trace in Fig. 3b. To verify the effectiveness of the proposed method, we use 1046 EV charging load values from the dataset directly and solve the nonlinear power flow equations for the corresponding voltage phase-angles and magnitudes. In Fig. 3a, we plot a histogram of voltage magnitudes at bus 3 in blue colour and resulting probabilities are plotted in Fig. 3b also in blue colour. Indeed, we observe that the analytical method described in Section III yields a good approximation to the actual variations in bus 3 voltage magnitude. For further verification, in Fig. 4,

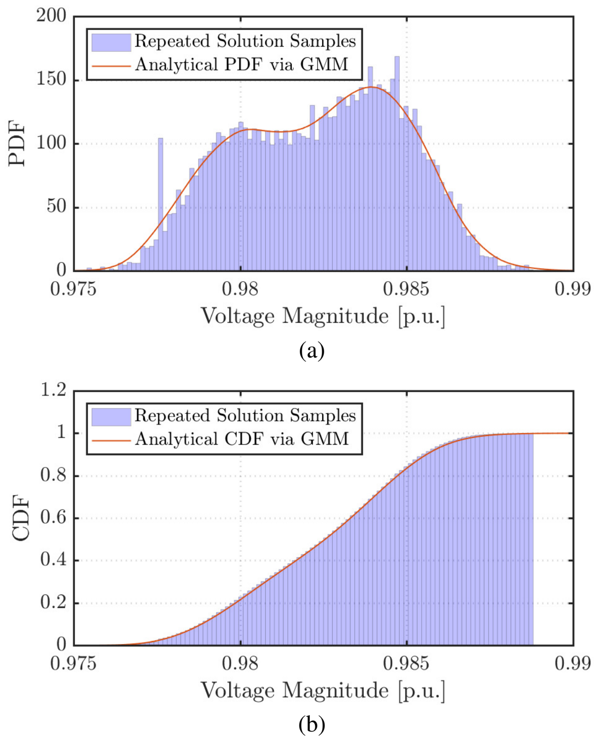

Fig. 3: Comparison of the (a) probability density/mass function and (b) probability of bus 3 voltage magnitude values obtained via (i) repeated solutions of the nonlinear power flow equations given historical EV charging data and (ii) the proposed analytical GMM-based method.

we plot the maximum absolute error (MAE) in CDFs of voltage magnitudes at all buses. As an example, the MAE for bus 3 is the maximum absolute mismatch between the blue-coloured histogram and the red trace in Fig. 3b.

V. Concluding Remarks

In this paper, we synthesized a probabilistic GMM of the uncertainty in EV charging loads directly from historical measurements of power or energy consumption at charging stations. We then presented an analytical method to estimate the uncertainty in bus voltage magnitudes and phase-angles arising from that in EV charging loads modelled as a GMM. The resulting characterization is also a GMM, the parameters in which can be obtained in closed form. The utility of the proposed method was demonstrated via numerical case studies involving the IEEE 33-bus test system. Avenues for future work include evaluating the proposed method with other datasets and incorporating GMMs into a chance-constrained optimal distributed energy resource dispatch problem.

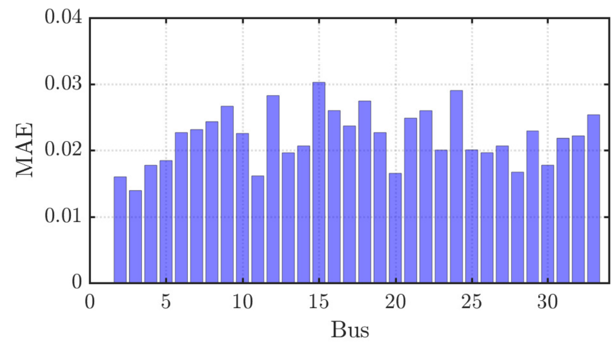

Fig. 4: MAE between repeated nonlinear solutions samples and analytical CDF via GMM for each bus.

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