# Measurement-based Optimal Power Flow with Linear Power-flow Constraint for DER Dispatch

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***Abstract*—This paper proposes a measurement-based method to obtain optimal power-flow (OPF) solutions that optimize distribution-system operations by dispatching active- and reactive-power outputs of distributed energy resources (DERs). Central to the proposed method is the estimation of a linear power-flow model from synchronized voltage and power-injection data collected from distribution-level phasor measurement units (D-PMUs). The estimated model is then incorporated into an OPF problem as an equality constraint. We formulate a quadratic cost function that enables co-optimization of DER active- and reactive-power costs, voltage deviations away from prescribed reference levels, as well as active- and reactive-power deviations from desired setpoints. Via numerical simulations of the IEEE 33-bus distribution test system, we demonstrate that the proposed measurement-based method yields sufficiently accurate solutions compared to model-based OPF solutions. Furthermore, we highlight the adaptability of the proposed method in case an accurate network model is not available.**

## I. Introduction

Integration of distributed energy resources (DERs), such as rooftop solar and battery storage, helps to alleviate environmental concerns of conventional fossil fuel-based generation. Moreover, DERs can be coordinated to provide grid support to the transmission system in the form of frequency regulation and reactive-power support [1], [2]. However, due to the wider range of possible operating points brought forth by DERs, power quality may be compromised for end-customers of electricity [3]. Reliable and efficient operation of the integrated power grid has motivated recent research into developing DER management systems [2]. Such systems generally require an accurate and up-to-date network model, which may not be available to system operators in real time [4]. Furthermore, optimal DER control requires repeated solutions of (possibly nonconvex) AC optimal power-flow (OPF) problems constrained by nonlinear power-balance constraints, which may be computationally burdensome in practical field implementation, especially with rapidly varying operating points. Instead, in this paper, we embed a linearized power-flow constraint in the OPF problem. Compared to the full-blown AC OPF problem, the proposed method incurs lower computational burden to obtain sufficiently accurate OPF solutions. Furthermore, we propose to estimate the linear power-flow model using synchronized voltage and power measurements collected from distribution-level phasor measurement units (D-PMUs). In so doing, the proposed approach is amenable to real-world implementation of DER management systems because it does not rely on an accurate offline network model, and it adapts to the evolving operating point.

We focus our review of relevant literature on measurement-based methods for online applications in distribution system operations. As a promising technology for such applications, D-PMUs provide time-synchronized voltage- and current-phasor measurements with phase angle accuracy of 0.01°; and they stream the phasor data through a standardized communication interface (e.g., IEEE C37.118) at intervals in the sub-second range [5]. Recent research in the use of high-resolution phasor data in distribution systems places emphasis on improving monitoring and observability (see, e.g., [6], [7]). Additional applications of D-PMU data include detection and identification of abnormal events such as voltage sags and high-impedance faults [8], [9]. Furthermore, high-resolution PMU data can enable the estimation of evolving parameters and operating points within the power grid via, e.g., a linearized power-flow model [10] and pertinent voltage-to-power sensitivities [11]. Furthermore, several research efforts were made recently to compute the power-grid topology using only measurements via estimation approaches [4], [12], [13]. In addition to estimation and monitoring applications, measurement-based DER management and control have received interest in recent years. For example, methods to regulate distribution-system voltages were developed in [14]–[16] by computing voltage-sensitivities from measurements. Furthermore, [17] proposes a measurement-based method to regulate the power exchange between the distribution and transmission systems by provisioning active power from DERs.

In this paper, we propose a measurement-based approach to approximate OPF solutions aimed at optimizing several aspects of distribution-system operations. In contrast to the grid-topology estimation in [4], [12], [13], we estimate a linear distribution-system power-flow model from online measurements of bus voltages and active- and reactive power injections via partial least-square (PLS) estimation, similar to [10]. The estimated model contains key characteristics of the actual system without explicitly recovering the grid topology. We extend the work in [10] by incorporating the estimated linear power-flow model into the OPF problem as an equality constraint to compute optimal DER setpoints. Unlike [14]–[17], the proposed formulation achieves combined objectives of minimizing (i) DER active- and reactive-power costs and (ii) deviations of bus voltages and bus injections away from their respective setpoints.

## II. Preliminaries

In this section, we establish notation, describe power-flow models, and formulate the AC OPF problem. We also motivate the need for a measurement-based approach.

### A. Network and Power-flow Models

Consider a distribution system with $N$ buses collected in the set $\mathcal{N} = \{1, \ldots, N\}$. Distribution lines are represented by two directed edges and collected in the set of edges $\mathcal{L} := \{(i,j)\} \subseteq \mathcal{N} \times \mathcal{N}$. Suppose measurements of pertinent system variables are sampled at time $t = k\Delta t$, $k = 0, 1, \ldots$, where $\Delta t$ is the time interval between consecutive samples. Let $V_{i,[k]}$ and $\theta_{i,[k]}$ denote, respectively, the voltage magnitude and phase-angle at bus $i$ and discrete time step $k$. Also let $P^{\text{load}}_{i,[k]}$ and $Q^{\text{load}}_{i,[k]}$ denote, respectively, the active- and reactive-power load at bus $i$ and time step $k$. Similarly, let $P^{\text{gen}}_{i,[k]}$ and $Q^{\text{gen}}_{i,[k]}$ denote, respectively, the active- and reactive-power generation at bus $i$ and time step $k$. Furthermore, collect voltage phase-angles and magnitudes at time step $k$ in vector $x_{[k]} = [\theta_{1,[k]}, \ldots, \theta_{N,[k]}, V_{1,[k]}, \ldots, V_{N,[k]}]^{\text{T}}$. Also collect active- and reactive-power loads in vector $y^{\text{load}}_{[k]} = [P^{\text{load}}_{1,[k]}, \ldots, P^{\text{load}}_{N,[k]}, Q^{\text{load}}_{1,[k]}, \ldots, Q^{\text{load}}_{N,[k]}]^{\text{T}}$. Similarly, collect active- and reactive-power generation in vector $y_{[k]} = [P^{\text{gen}}_{1,[k]}, \ldots, P^{\text{gen}}_{N,[k]}, Q^{\text{gen}}_{1,[k]}, \ldots, Q^{\text{gen}}_{N,[k]}]^{\text{T}}$. Then, distribution-system power-flow equations at time step $k$ can be compactly expressed as

$$y_{[k]} = g(x_{[k]}) + y^{\text{load}}_{[k]}, \tag{1}$$

where $g : \mathbb{R}^{2N} \to \mathbb{R}^{2N}$. In (1), the dependence on network topology and associated parameters (such as circuit breaker status and line impedances) is implicitly considered in $g(\cdot)$. Let $x_{[k]} = x_{[k-1]} + \Delta x_{[k]}$, where $x_{[k-1]}$ denotes the operating point at the previous time step $k-1$. Then, according to (1),

$$y_{[k]} = g(x_{[k-1]} + \Delta x_{[k]}) + y^{\text{load}}_{[k]}. \tag{2}$$

Suppose $g(\cdot)$ is continuously differentiable with respect to $x$, and assume that $\Delta x_{[k]}$ is sufficiently small. Then we can approximate $y_{[k]}$ as

$$y_{[k]} \approx g(x_{[k-1]}) + J_{[k]}\Delta x_{[k]} + y^{\text{load}}_{[k]}, \tag{3}$$

where

$$J_{[k]} = \left.\frac{\partial g}{\partial x}\right|_{x_{[k-1]}} \tag{4}$$

is the Jacobian matrix of the power-flow equations evaluated at the previous operating point $x_{[k-1]}$. Recall that $\Delta x_{[k]} = x_{[k]} - x_{[k-1]}$ and further substitute this into (3) to yield

$$y_{[k]} \approx J_{[k]} x_{[k]} + c_{[k]}, \tag{5}$$

where $c_{[k]} = g(x_{[k-1]}) + y^{\text{load}}_{[k]} - J_{[k]} x_{[k-1]}$. We note that, in the model-based framework, the evaluation of both $c_{[k]}$ and $J_{[k]}$ relies on the up-to-date operating point at time step $k-1$ as well as an accurate network model of the distribution system.

Power-flow analysis is a ubiquitous tool in power system planning and operations. Exact and approximate variants are commonly used in state estimation, contingency analysis, and OPF problems, to name a few.

### B. AC Optimal Power-flow Problem Formulation

The AC OPF problem minimizes a desired cost function (e.g., cost of generation, transmission losses, deviations away from references, etc.) subject to nonlinear power-balance and other operational constraints. It is formulated as [18]

$$\begin{aligned} \min_{x_{[k]}, y_{[k]}} \quad & f(x_{[k]}, y_{[k]}), \\ \text{s.t.} \quad & y_{[k]} = g(x_{[k]}) + y^{\text{load}}_{[k]}, \\ & x_{\min} \leq x_{[k]} \leq x_{\max}, \\ & y_{\min} \leq y_{[k]} \leq y_{\max}, \end{aligned} \tag{6}$$

where $f : \mathbb{R}^{2N} \times \mathbb{R}^{2N} \to \mathbb{R}$ is the objective function to be minimized; $x_{\min}$ and $x_{\max}$ represent, respectively, minimum and maximum voltage phase-angle and magnitude limits; and $y_{\min}$ and $y_{\max}$ represent, respectively, minimum and maximum allowable active- and reactive-power generation. Entries of $y_{\min}$ and $y_{\max}$ corresponding to DERs represent their minimum and maximum output. The nonlinear OPF problem in (6) is a nonconvex optimization program and is consequently NP-hard to solve [18].

### C. Problem Statement

The solution of the conventional OPF problem relies on an offline network model with accurate topology and parameters, which may not be available due to insufficient telemetry from, e.g., circuit breakers, in the distribution system. Furthermore, the nonconvex nature of the AC OPF problem in (6) may pose significant computational hurdles for the utility operator, especially with rapidly varying operating point. Recognizing that the nonlinear power-flow equations in (6) cause the optimization problem to be nonconvex, we seek to replace them with a linear relationship. Furthermore, with a view for practical field deployment where an accurate network model may not be available, we use online measurements of nodal voltages and injections to compute linear sensitivities that make up the linear power-balance constraint. The proposed method accomplishes the dual objectives of (i) eradicating the reliance on an accurate offline model, and (ii) reducing the computational burden in solving the optimal operating point.

## III. Measurement-based OPF Approach

In this section, we formulate the proposed measurement-based approach to obtain the linearized power-flow model in (5). The relevant linear sensitivities are estimated via the partial least-squares (PLS) method without relying on an offline system network model. Subsequently, we incorporate the measurement-based power-flow model into the the OPF problem by replacing the nonlinear power-flow constraint with the estimated linear one.

### A. Estimation of Linearized Power-flow Model

Based on the linearized power-flow model in (5), we hypothesize that there exists $H_{[k]} \in \mathbb{R}^{(2N+1)\times 2N}$ that satisfies the following relationship:

$$y^{\text{T}}_{[k]} \approx \begin{bmatrix} x^{\text{T}}_{[k]} & 1 \end{bmatrix} H_{[k]}, \tag{7}$$

which is equivalent to (5) with $H_{[k]} = [J_{[k]}, c_{[k]}]^{\mathrm{T}}$. In order to eradicate the reliance on the accurate network model, we propose to estimate the entries of $H_{[k]}$ using only online measurements. To this end, suppose that $M+1$ measurements of bus voltage angles and magnitudes, $x_{[k-M]}, \ldots, x_{[k]}$, and active- and reactive-power injections, $y_{[k-M]}, \ldots, y_{[k]}$, are available. Further suppose that the linear power-flow model remains approximately constant over the $M$ measurement samples. Assuming that $M > 2N$, we stack up $M$ instances of (7) to yield the following over-determined system:

$$B_{[k]} = A_{[k]} H_{[k]}, \tag{8}$$

where matrices $A_{[k]}$ and $B_{[k]}$ are composed of online measurements, and they are given by

$$A_{[k]} = \begin{bmatrix} x_{[k-M]}^{\mathrm{T}} & 1 \\ \vdots & \vdots \\ x_{[k]}^{\mathrm{T}} & 1 \end{bmatrix}, \quad B_{[k]} = \begin{bmatrix} y_{[k-M]}^{\mathrm{T}} \\ \vdots \\ y_{[k]}^{\mathrm{T}} \end{bmatrix}. \tag{9}$$

Since (8) is over-determined, we can obtain the ordinary least-squares estimate for $H_{[k]}$ as

$$\widehat{H}_{[k]} \approx (A_{[k]}^{\mathrm{T}} A_{[k]})^{-1} A_{[k]}^{\mathrm{T}} B_{[k]}. \tag{10}$$

In our setting, the bus voltage phase-angles and magnitudes in $A_{[k]}$ are highly correlated due to similar evolving patterns in operating-point changes, which may result in ill-conditioned regressor matrix in (10). A more suitable solution approach for regression problems with collinearity in $A_{[k]}$ is the PLS method. The central idea is to project so-called *latent features* or key components in $A_{[k]}$ and $B_{[k]}$ onto lower-dimensional latent matrices $T_{[k]}$ and $U_{[k]}$, respectively, which best model the relationship in (8). The decompositions of $A_{[k]}$ and $B_{[k]}$ are performed in a way that they satisfy [19]

$$T_{[k]} = A_{[k]} G_{[k]}, \quad B_{[k]} = U_{[k]} L_{[k]}^{\mathrm{T}}. \tag{11}$$

where $T_{[k]}$ and $U_{[k]}$ are latent matrices with extracted components from matrices $A_{[k]}$ and $B_{[k]}$, and $G_{[k]}$ and $L_{[k]}$ are loading matrices. Through the nonlinear iterative partial least squares (NIPALS) algorithm, columns of $A_{[k]}$ and $B_{[k]}$ are chosen to populate $T_{[k]}$ and $U_{[k]}$, respectively, so as to maximize the covariance between them. This is reflected in the loading matrices $G_{[k]}$ and $L_{[k]}$. Then, PLS performs regression on the latent variables instead of the original ones. With the over-determined regression model $U_{[k]} = T_{[k]} \Theta_{[k]}$, $\Theta_{[k]}$ can be estimated via ordinary LSE as

$$\Theta_{[k]} = (T_{[k]}^{\mathrm{T}} T_{[k]})^{-1} T_{[k]}^{\mathrm{T}} U_{[k]}. \tag{12}$$

Substituting $U_{[k]} = T_{[k]} \Theta_{[k]}$ into (11), we get

$$B_{[k]} = T_{[k]} \Theta_{[k]} L_{[k]}^{\mathrm{T}} = A_{[k]} G_{[k]} \Theta_{[k]} L_{[k]}^{\mathrm{T}}. \tag{13}$$

Finally, recognizing that (13) is equivalent to (8) with $H_{[k]} = G_{[k]} \Theta_{[k]} L_{[k]}^{\mathrm{T}}$, substitution of (12) into the above yields the PLS estimate of $H_{[k]}$ in (8) as

$$\widehat{H}_{[k]} \approx G_{[k]} (T_{[k]}^{\mathrm{T}} T_{[k]})^{-1} T_{[k]}^{\mathrm{T}} U_{[k]} L_{[k]}^{\mathrm{T}}. \tag{14}$$

Interested readers may refer to [19] for background on the PLS method and details of the NIPALS algorithm.

### B. Quadratic OPF Problem Formulation

The conventional OPF problem in (6) is nonconvex due to the nonlinear power-flow equality constraint. To overcome the challenges involved with solving the nonconvex optimization problem, we leverage online measurements to estimate linear sensitivities to construct the linear power-flow model in (7) and incorporate this into (6) as follows:

$$\begin{aligned} \min_{x_{[k]}, y_{[k]}} \quad & f(x_{[k]}, y_{[k]}), \qquad (15) \\ \text{s.t.} \quad & y_{[k]} = \widehat{H}_{[k]}^{\mathrm{T}} \begin{bmatrix} x_{[k]} \\ 1 \end{bmatrix}, \\ & x_{\min} \le x_{[k]} \le x_{\max}, \\ & y_{\min} \le y_{[k]} \le y_{\max}. \end{aligned}$$

In the above, as described in Section III-A, the matrix $\widehat{H}_{[k]}$ is estimated using online measurements collected at time steps $k-M, \ldots, k$. In practical implementations, $\widehat{H}_{[k]}$ may be the most recent estimate available, or it may be computed using (5) from a model if one is at hand.

In our setup, we utilize a quadratic cost function that optimizes DER active- and reactive-power injections in order to minimize a weighted combination of: (i) cost of DER active- and reactive-power generation, (ii) deviations of voltage phase-angles and magnitudes from their reference values, and (iii) deviations of nodal active- and reactive-power injections from their setpoints. To capture the above, we fix the cost function in (15) as

$$\begin{aligned} f(x_{[k]}, y_{[k]}) = y_{[k]}^{\mathrm{T}} \Upsilon y_{[k]} + (x_{[k]} - x^{\circ})^{\mathrm{T}} \Psi (x_{[k]} - x^{\circ}) \\ + (y_{[k]} - y^{\circ})^{\mathrm{T}} \Phi (y_{[k]} - y^{\circ}), \qquad (16) \end{aligned}$$

where $x^{\circ}$ and $y^{\circ}$ denote desired setpoints for voltages and injections, respectively. In (16), $\Upsilon = \mathrm{diag}(\upsilon_1, \ldots, \upsilon_{2N})$, $\Psi = \mathrm{diag}(\psi_1, \ldots, \psi_{2N})$, and $\Phi = \mathrm{diag}(\varphi_1, \ldots, \varphi_{2N})$ are diagonal weight matrices with non-negative entries, i.e., $\upsilon_i, \psi_i, \varphi_i \ge 0$, $i = 1, \ldots, 2N$. The cost function in (16) is general in the sense that weighting matrices $\Upsilon$, $\Psi$, and $\Phi$, respectively, enforce minimization in the cost of power injections, voltage-phasor deviations, and power-injection deviations. With positive semidefinite weight matrices, (15) is a quadratic optimization problem with linear constraints where the solution can be efficiently computed using standard convex optimization solvers, e.g., [20]. Furthermore, by estimating the linearized power-flow model in (7) from online measurements, the up-to-date operating conditions and system configuration are accurately accounted for, as opposed to an offline network model which may not reflect operating-point changes and switch re-configurations in real time.

## IV. Case Studies

We perform numerical simulations of the IEEE 33-bus test system (see, e.g., [21]) in MATPOWER [22]. The single-line diagram of the test system is shown in Fig. 1. We assume that measurements of voltage phasors and active- and reactive-power injections are available from D-PMUs installed at all buses. Four DERs are connected to buses

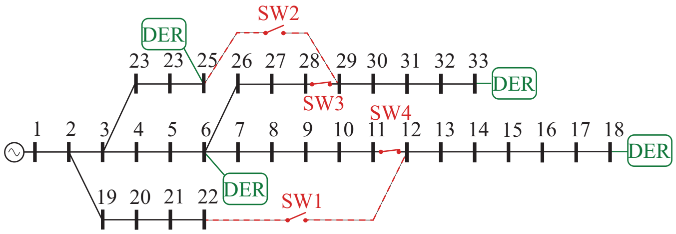


Fig. 1: IEEE 33-bus test system with 4 DERs installed at buses 6, 18, 25, and 33. Switches SW1 and SW2 are normally open, and switches SW3 and SW4 are normally closed.

$i \in \mathcal{D} = \{6, 18, 25, 33\}$, and their active- and reactive-power outputs are, respectively, constrained within $P_i^{\text{gen}} \in [-0.15, 0.15]$ p.u. and $Q_i^{\text{gen}} \in [-0.15, 0.15]$ p.u. to represent DERs with four-quadrant operation, similar to e.g., [14], [17]. For all other buses $i \in \mathcal{N} \setminus \mathcal{D}$, $P_i^{\text{gen}} = Q_i^{\text{gen}} = 0$. In order to estimate the linear power-flow model in (7), we perturb the active- and reactive-power components of loads around their nominal values with random Gaussian distributed variations of zero mean and $0.05\%$ standard deviation relative to the nominal load values. Subsequent case studies use $M = 200$ measurements in order to have a sufficiently over-determined regression problem in (8).

We compare OPF solutions obtained via three methods: (i) model-based AC OPF in (6), (ii) model-based OPF with linearized power-flow equality constraint in (5), and (iii) proposed measurement-based OPF in (15) with estimated linear equality constraint. For all three, we use a standard personal computer and solve the OPF problem with CVX, a toolbox for convex programs [20]. The AC OPF is solved via its convex relaxation, which is exact for radial systems (see, e.g., [18]).

## *A. Accurate System Model*

We benchmark the proposed measurement-based OPF method against the model-based methods assuming that an accurate network model is available. We intend to compute DER active- and reactive-power setpoints to simultaneously (i) minimize voltage-magnitude deviations from reference levels (1 p.u. in our case studies), (ii) regulate the substation active-power supply, and (iii) minimize cost of active- and reactive power injections. To this end, we set the diagonal entries in weight matrix $\Psi$ such that $\psi_1, \ldots, \psi_N = 0$ (corresponding to voltage phase-angles) and $\psi_{N+1}, \ldots, \psi_{2N} = 1$ (corresponding to voltage magnitudes). Furthermore, we set appropriate entries in $x^\circ$ to reference voltage magnitude 1 p.u., i.e.,[1] $x^\circ = [0_N^{\mathrm{T}}, 1_N^{\mathrm{T}}]^{\mathrm{T}}$. Diagonal entries in weight matrix $\Upsilon$ are set to 1 to reflect equal cost for all active- and reactive-power injections. Finally, in $\Phi$, $\varphi_1$ is set to 1 while $\varphi_i = 0$ for $i = 2, \ldots, 2N$, and the first entry in $y^\circ$ is set to the substation active-power reference of 0.1 p.u.

Figure 2 plots bus voltage magnitudes $V_i$, bus active-power injections $P_i^{\text{gen}}$, and reactive-power injections $Q_i^{\text{gen}}$. From a visual inspection of Fig. 2, we observe that the proposed measurement-based method provides close approximations to

[1] $0_N$ and $1_N$ denote the length-$N$ vectors of all 0s and 1s, respectively

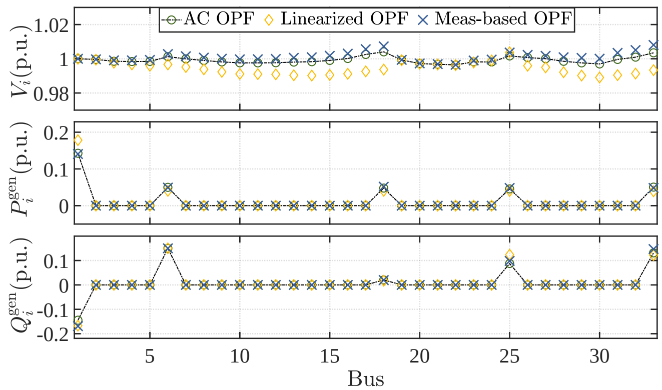


Fig. 2: Accurate system model: OPF results obtained via (i) model-based AC OPF, (ii) model-based linearized OPF, and (iii) proposed measurement-based OPF. Top pane: bus voltages $V_i$; Middle pane: active-power injections $P_i^{\text{gen}}$; Bottom pane: reactive-power injections $Q_i^{\text{gen}}$.

TABLE I: Accurate system model: comparison of cost-function, relative-error, and computation-time metrics among (i) model-based AC OPF, (ii) model-based linearized OPF, and (iii) proposed measurement-based OPF.

| | Cost function (p.u.) | Voltage-magnitude error (%) | Active-power error (%) | Reactive-power error (%) | Execution time (s) |
|---|---|---|---|---|---|
| AC OPF | 0.0118 | — | — | — | 7.297 |
| Linearized OPF | 0.0182 | 0.510 | 17.497 | 16.141 | 2.094 |
| Meas-based OPF | 0.0124 | 0.190 | 4.799 | 6.766 | 2.000 |

the benchmark solution obtained from the model-based AC OPF. Aggregate mean percent errors in voltage magnitude, active-power injection, and reactive-power injection are reported, respectively, in columns 3, 4, and 5 of Table I. Additionally, as shown in column 2 of Table I, we note that the cost function evaluated from the measurement-based solution is very close to that of the benchmark AC OPF one. Finally, column 6 highlights that the use of linear power-flow constraint significantly reduces the execution time needed to obtain the OPF solution. We hypothesize that the time difference between the model-based OPF with linearized power-flow constraint and the measurement-based one may be due to numerical properties of the iterative solver. Also note that the injection at bus 1 deviates slightly from the specified setpoint of 0.1 p.u. We can easily enforce this by weighing it more heavily in the cost function.

## *B. Inaccurate System Model*

In this case study, the system topology in Fig. 1 is reconfigured by closing SW1 and SW2 to establish electrical connection between buses 22 and 12, and buses 25 and 29, respectively. Simultaneously, SW3 and SW4 are opened to disconnect the lines between buses 28 and 29, and buses 11 and 12, respectively. In addition, active- and reactive-power loads grow by 25%. However, these changes are not updated in the system model used by the model-based AC OPF. The cost function is set up as in the previous case study in Section IV-A. In Fig. 3 and Table II, we report simulation results obtained from model-based AC OPF with the updated system model, the same with the old inaccurate model from

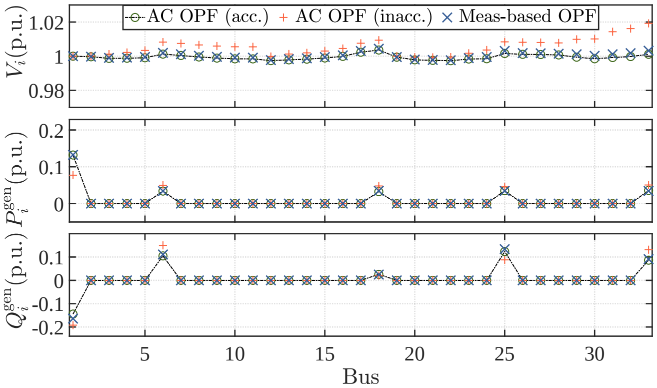


Fig. 3: Inaccurate system model: OPF results obtained via (i) model-based AC OPF with accurate system model, (ii) model-based AC OPF with inaccurate system model, and (iii) proposed measurement-based OPF. Top pane: bus voltages $V_i$; Middle pane: active-power injections $P_i^{\rm gen}$; Bottom pane: reactive-power injections $Q_i^{\rm gen}$.

TABLE II: Inaccurate system model: comparison of cost-function, relative-error, and computation-time metrics among (i) model-based AC OPF with accurate system model, (ii) model-based AC OPF with inaccurate system model, and (iii) proposed measurement-based OPF.

| | Cost function (p.u.) | Voltage-magnitude error (%) | Active-power error (%) | Reactive-power error (%) | Execution time (s) |
|---|---|---|---|---|---|
| AC OPF (acc.) | 0.0060 | — | — | — | 6.234 |
| AC OPF (inacc.) | 0.0171 | 0.774 | 41.439 | 36.152 | 7.297 |
| Meas-based OPF | 0.0061 | 0.172 | 3.216 | 5.745 | 1.218 |

Section IV-A, as well as the proposed measurement-based method. The results clearly indicate the superiority of the measurement-based method, which is adaptive to the network reconfiguration and operating-point change. It provides a much more accurate solution compared to the model-based AC OPF with the inaccurate model.

## V. Concluding Remarks

In this paper, we present a measurement-based method to determine optimal DER active- and reactive-power dispatch in distribution systems. We estimate a linear power-flow model from D-PMU measurements of voltage phasors and active- and reactive-power injections using PLS estimation. We further incorporate the estimated model into an OPF problem as a linear equality constraint. The OPF problem minimizes a quadratic cost function that combines (i) DER active- and reactive-power costs and (ii) deviations of bus voltages and injections away from reference setpoints. Simulations of the IEEE 33-bus test system demonstrate that the proposed measurement-based method yields accurate OPF solutions compared to model-based AC OPF, while incurring lower computational cost. Furthermore, we demonstrate the adaptability of the proposed method in case an accurate offline system model is not available. The measurement-based OPF method is potentially useful in various applications aimed at controlling distribution systems and highlights the benefits of communication-based DER control. Compelling avenues for future work include (i) online implementation, (ii) extension to unbalanced networks, (iii) estimation using fewer measurements.

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