A Data-driven Convex-optimization Method for Estimating Load Changes

Abdullah Al-Digs†, Bo Chen‡, Sairaj V. Dhople§, Yu Christine Chen¶
†‡¶Department of Electrical and Computer Engineering, The University of British Columbia, Canada
§Department of Electrical and Computer Engineering, University of Minnesota, Twin Cities, USA
Emails: †aldigs@ece.ubc.ca, ‡cbhp1993@ece.ubc.ca, §sdhople@umn.edu, ¶chen@ece.ubc.ca

***Abstract*—This paper presents an optimization-based method to detect the occurrence, estimate the magnitude, and identify the location of load changes in the power system. The proposed method relies on measurements of only frequency at the output of synchronous generators along with a reduced-order power system dynamical model that captures locational effects of load disturbances on generator frequency dynamics. These locational aspects are retained in the estimation model by incorporating linearized power-flow balance into differential equations that describe synchronous-generator dynamics. The sparsity structure of load-change disturbances is leveraged so that only a limited number of measurements are needed to estimate load changes. Furthermore, a convex relaxation of the problem ensures that it can be solved online in a computationally efficient manner. Time-domain simulations involving the Western Electricity Coordinating Council 9-bus test system demonstrate the accuracy of the proposed method.**

***Index Terms*—Convex optimization, event detection, load change estimation.**

I. Introduction

In this paper, we propose to simultaneously detect the occurrence, estimate the magnitude, and identify the location of load-change disturbances in the power system using online measurements of frequencies collected at synchronous-generator buses in conjunction with a reduced-order power system dynamical model. The ability to estimate load (or injection) changes is particularly relevant for operations and controls tasks in future power networks given that the widespread integration of rapidly varying renewable generation would likely result in large and frequent excursions away from the steady-state operating point. The proposed method acknowledges system electromechanical dynamics by adopting a third-order model for each synchronous generator, capturing its rotor-angle position, electrical frequency, and turbine-governor dynamics. We further incorporate linearized power-flow equations into the generator dynamical models, thereby reducing the standard power-system differential-algebraic-equation (DAE) model into one that contains only differential equations. In this way, pertinent attributes of the network topology are embedded within the system dynamical model, and the locational effects of different load disturbances on frequency response are captured. Moreover, by leveraging the sparsity structure of load changes in the system, we accomplish the estimation task using only generator frequency measurements sampled at a single snapshot in time. We further utilize a well-known convex relaxation of the sparse-signal estimation problem that can be solved with computationally efficient algorithms. Taken together, the proposed framework provides timely estimates of both magnitude and location of potentially fast/large transients in loads or injections.

Timely detection and identification of disturbances in the power system is crucial to maintain its operational reliability. Most prior art has addressed the problem of detecting and identifying transmission-line outages [1]–[4], power-quality disturbances [5]–[7], and cascading events leading to blackouts [8]–[11]. With respect to load estimation, nonintrusive load monitoring has been widely studied to analyze load-state changes at the individual household appliance level [12]–[14]. Unlike these, our work focuses on estimating load changes in the bulk transmission system by relying on online measurements of synchronous-generator frequencies, which are commonly collected to compute the system-wide frequency [15].

The proposed approach begins with a classical power-system dynamical model perturbed around an initial operating point [16]. Using DC power-flow assumptions, we embed the impact of load changes on synchronous-generator dynamics by algebraically manipulating and incorporating pertinent admittance-like matrices in the dynamical model [17]. Then, leveraging this model along with generator frequency measurements collected at a particular sampling instant, we solve a convex optimization problem to simultaneously estimate the magnitude and location of load changes in a computationally efficient manner. The main advantage of the proposed method is that it uses online measurements of synchronous-generator frequencies, which can be obtained from sensors, such as phasor measurement units (PMUs), equipped at buses connected to generators. Furthermore, unlike most previous work in power-system change detection and identification, we explicitly incorporate the underlying system dynamics that give rise to measured quantities into the modelling framework, thereby accomplishing the intended estimation task using measurements of dynamic states prior to reaching the post-disturbance steady state. We demonstrate the effectiveness of the proposed method using synthetic measurements periodically sampled from a time-domain simulation of a detailed, lossy, and nonlinear power-system DAE model.

The remainder of this paper is organized as follows. Section II outlines the network power-flow formulation and pertinent synchronous-generator dynamics, which are combined to establish the system dynamical model. In Section III, we describe the proposed method to estimate load changes using generator frequency measurements. The efficacy of the proposed load-change estimation method is verified via numerical case studies in Section IV. Finally, Section V offers concluding remarks and directions for future work.

II. Preliminaries

In this section, we describe the power network and dynamics attributable to the synchronous generators. We then combine these to formulate the system dynamical model that will be used for load-change detection and identification.

A. Network Description

Consider a power transmission network with N buses collected in the set $\mathcal{N} = \{1, \ldots, N\}$ and transmission lines in the set $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$. Let $V_k(t) = |V_k(t)|\angle\theta_k(t)$ represent the voltage phasor at bus k and time t. Denote, by $\mathcal{G} = \{1, \ldots, G\} \subset \mathcal{N}$, the set of G buses that are connected to conventional turbine-based synchronous generators. Each generator $g \in \mathcal{G}$ is modelled as a voltage source $E_g(t) = |E_g|\angle\delta_g(t)$ behind constant reactance $\mathrm{j}X'_{d,g}$. Furthermore, denote by $\mathcal{L} = \mathcal{N} \setminus \mathcal{G} = \{G+1, \ldots, N\}$, the set of L load buses, which are connected to non-frequency-sensitive loads, e.g., constant-power loads. Finally, $P_k(t)$ denotes the non-frequency-sensitive active-power injection at bus $k \in \mathcal{N}$, which is a negative quantity if it corresponds to a constant-power load.[1]

A transmission line with current flowing from bus k to ℓ is denoted by $(k, \ell) \in \mathcal{E}$. Transmission line (k, ℓ) is modelled using the lumped-element Π-model with series admittance $y_{k\ell} = y_{\ell k} = g_{k\ell} + \mathrm{j}b_{k\ell} \in \mathbb{C} \setminus \{0\}$ and shunt admittance $y^{\mathrm{sh}}_{k\ell} = g^{\mathrm{sh}}_{k\ell} + \mathrm{j}b^{\mathrm{sh}}_{k\ell} \in \mathbb{C} \setminus \{0\}$ on both ends of the line. The active-power flow in line (k, ℓ) is denoted by $P_{k\ell}(t)$, $\ell \in \mathcal{N}_k$ with $\mathcal{N}_k$ representing the set of buses electrically connected to bus k. Since we aim to develop a perturbative model around an initial operating point, we will find the definition of the following small-signal variables useful: $\Delta\delta_g(t) := \delta_g(t) - \delta_g(0)$, $\Delta\theta_k(t) := \theta_k(t) - \theta_k(0)$, $\Delta P_{k\ell}(t) = P_{k\ell}(t) - P_{k\ell}(0)$, and $\Delta P_k(t) = P_k(t) - P_k(0)$. Under so-called DC assumptions, active-power flow variations in line (k, ℓ) can be captured by

$$\Delta P_{k\ell}(t) = -b_{k\ell}(\Delta\theta_k(t) - \Delta\theta_\ell(t)). \tag{1}$$

Also, the active-power balance at generator-connected bus $g \in \mathcal{G}$ is given by

$$0 = \Delta P_g(t) + \frac{1}{X'_{d,g}}(\Delta\delta_g(t) - \Delta\theta_g(t)) - \sum_{k \in \mathcal{N}_g} \Delta P_{gk}(t), \tag{2}$$

and active-power balance at load bus $\ell \in \mathcal{L}$ yields

$$0 = \Delta P_\ell(t) - \sum_{k \in \mathcal{N}_\ell} \Delta P_{\ell k}(t). \tag{3}$$

[1]Notation: The M-dimensional vector with all 1s is denoted by $\mathbb{1}_M$. The diagonal matrix $\mathrm{diag}(x)$ is formed with entries of the vector x stacked on the main diagonal; $\mathrm{diag}(x/y)$ forms a diagonal matrix with the ith diagonal entry given by x_i/y_i, where x_i and y_i are the ith entries of vectors x and y, respectively. The $M \times N$ matrix of all 0s is denoted by $\mathbb{0}_{M\times N}$. The vector $x_{\mathcal{G}} = [x_1, \ldots, x_G]^{\mathrm{T}}$ collects x_g's for all $g \in \mathcal{G}$, and $y_{\mathcal{L}} = [y_{G+1}, \ldots, y_N]^{\mathrm{T}}$ collects y_ℓ's for all $\ell \in \mathcal{L}$.

B. Synchronous-generator Dynamical Model

Let $\delta_{\mathcal{G}} \in \mathbb{R}^G$ and $\omega_{\mathcal{G}} \in \mathbb{R}^G$ denote the vector of rotor electrical angular positions and frequencies, respectively, for all generators $g \in \mathcal{G}$. Further let $P^{\mathrm{m}}_{\mathcal{G}} \in \mathbb{R}^G$ represent the vector of turbine mechanical powers. Also collect the variations in non-frequency-sensitive injections at all generator-connected buses into $\Delta P_{\mathcal{G}} \in \mathbb{R}^G$. Assume each generator $g \in \mathcal{G}$ initially operates at synchronous steady state with $\omega_g(0) = \omega_{\mathrm{s}} = 2\pi 60\,\mathrm{rad/s}$, the synchronous frequency. Define small-signal variations $\Delta\delta_{\mathcal{G}}(t) := \delta_{\mathcal{G}}(t) - \delta_{\mathcal{G}}(0)$, $\Delta\omega_{\mathcal{G}}(t) := \omega_{\mathcal{G}}(t) - \omega_{\mathrm{s}}\mathbb{1}_G$, and $\Delta P^{\mathrm{m}}_{\mathcal{G}}(t) := P^{\mathrm{m}}_{\mathcal{G}}(t) - P^{\mathrm{m}}_{\mathcal{G}}(0)$. Then, dynamics of generators can be described by the following model obtained by combining the swing equations with a speed-governor model [16]:

$$\Delta\dot{\delta}_{\mathcal{G}}(t) = \Delta\omega_{\mathcal{G}}(t), \tag{4}$$

$$\begin{aligned}\mathrm{diag}(M_{\mathcal{G}})\Delta\dot{\omega}_{\mathcal{G}}(t) = {}& \Delta P^{\mathrm{m}}_{\mathcal{G}}(t) - \mathrm{diag}(D_{\mathcal{G}})\Delta\omega_{\mathcal{G}}(t) \\ &+ K\Delta\theta(t) + \Delta P_{\mathcal{G}}(t),\end{aligned} \tag{5}$$

$$\mathrm{diag}(\tau_{\mathcal{G}})\Delta\dot{P}^{\mathrm{m}}_{\mathcal{G}}(t) = \Delta P^{\mathrm{r}}_{\mathcal{G}} - \Delta P^{\mathrm{m}}_{\mathcal{G}}(t) - \mathrm{diag}(R_{\mathcal{G}})\Delta\omega_{\mathcal{G}}(t), \tag{6}$$

where $M_{\mathcal{G}} \in \mathbb{R}^G$ and $D_{\mathcal{G}} \in \mathbb{R}^G$ denote, respectively, vectors containing generator inertia and damping constants. Furthermore, $\tau_{\mathcal{G}} \in \mathbb{R}^G$, $\Delta P^{\mathrm{r}}_{\mathcal{G}} \in \mathbb{R}^G$, and $R_{\mathcal{G}} \in \mathbb{R}^G$ collect the governor time constants, changes in reference power inputs, and *inverse* droop constants, respectively.[2] Also, in (5), $\Delta\theta = [\Delta\theta_1, \ldots, \Delta\theta_N]^{\mathrm{T}}$ collects the variations in bus voltage angles. Furthermore, matrix $K \in \mathbb{R}^{G\times L}$ is composed by appropriately evaluating (1) for each $g \in \mathcal{G}$, as follows:

$$K = -\left[\mathrm{diag}\left(\frac{\mathbb{1}_G}{X'_{d,\mathcal{G}}}\right) + B_{\mathcal{G}\mathcal{G}} \quad B_{\mathcal{G}\mathcal{L}}\right], \tag{7}$$

where $X'_{d,\mathcal{G}} = [X'_{d,1}, \ldots, X'_{d,G}]^{\mathrm{T}}$, and matrices $B_{\mathcal{G}\mathcal{G}}$ and $B_{\mathcal{G}\mathcal{L}}$ are formed based on the network topology (see Appendix A for details).

C. Load-change Estimation and Identification Model

Collect the load variations at all buses in vector $\Delta P = [\Delta P_1, \ldots, \Delta P_N]^{\mathrm{T}} \in \mathbb{R}^N$. Next, we develop a model that helps identify nonzero entries in ΔP. Substituting suitable instances of (1) into (2) and (3), and collecting the resultant into matrix-vector form, we obtain [17]

$$\Delta\theta(t) = B^{-1}\left(D\Delta\delta_{\mathcal{G}}(t) + \Delta P(t)\right), \tag{8}$$

where

$$B = \begin{bmatrix} B_{\mathcal{G}\mathcal{G}} & B_{\mathcal{G}\mathcal{L}} \\ B_{\mathcal{L}\mathcal{G}} & B_{\mathcal{L}\mathcal{L}} \end{bmatrix}, \quad D = \begin{bmatrix} \mathrm{diag}\left(\frac{\mathbb{1}_G}{X'_{d,\mathcal{G}}}\right) \\ \mathbb{0}_{L\times G} \end{bmatrix}. \tag{9}$$

Entries in and structures of matrices $B_{\mathcal{G}\mathcal{G}}$, $B_{\mathcal{G}\mathcal{L}}$, $B_{\mathcal{L}\mathcal{G}}$, and $B_{\mathcal{L}\mathcal{L}}$ are detailed in Appendix A. Then, we substitute (8) into (5) to get the following:

$$\begin{aligned}\mathrm{diag}(M_{\mathcal{G}})\Delta\dot{\omega}_{\mathcal{G}}(t) = {}& \Delta P^{\mathrm{m}}_{\mathcal{G}}(t) - \mathrm{diag}(D_{\mathcal{G}})\Delta\omega_{\mathcal{G}}(t) \\ &+ H\Delta\delta_{\mathcal{G}}(t) + W\Delta P(t),\end{aligned} \tag{10}$$

[2]In most reference textbooks, R_g refers to the droop constant of generator g; in this paper, we deviate from the standard to contain notational burden.

where matrices $H \in \mathbb{R}^{G\times G}$ and $W \in \mathbb{R}^{G\times N}$ are given by

$$H = KB^{-1}D, \tag{11}$$
$$W = KB^{-1} + \begin{bmatrix}\mathrm{diag}(\mathbb{1}_G) & \mathbb{0}_{G\times L}\end{bmatrix}. \tag{12}$$

With a view that $\Delta P(t)$ in (10) represents a disturbance input to the power system, entries of W capture the effects of load-variation disturbances at different buses on generator frequency dynamics. Next, we outline a method to estimate load changes in the network using generator frequency measurements in conjunction with (10).

III. Load-change Estimation and Identification

In this section, we propose a method to estimate load changes and identify their locations using measurements of generator frequencies. We further leverage the sparsity structure of the load-variation vector to estimate load disturbances using a limited number of measurements.

A. Problem Formulation

Suppose we have measurements of variations in synchronous-generator frequencies sampled at $t = k\Delta t$, $k \in \mathbb{Z}$, where $\Delta t > 0$ is the time interval between consecutive samples. Denote the measurement at time $t = k\Delta t$ (or time step k) by $\Delta\omega_{\mathcal{G}}[k]$. Then, at time step k, we can rearrange terms in (10) to get

$$\Delta y[k] = W\Delta P[k], \tag{13}$$

where

$$\Delta y[k] = \mathrm{diag}(M_{\mathcal{G}})\Delta\dot{\omega}_{\mathcal{G}}[k] - \Delta P^{\mathrm{m}}_{\mathcal{G}}[k] + \mathrm{diag}(D_{\mathcal{G}})\Delta\omega_{\mathcal{G}}[k] - H\Delta\delta_{\mathcal{G}}[k]. \tag{14}$$

Our goal is to estimate the value of $\Delta P[k]$ that satisfies (13) by evaluating $\Delta y[k]$ using online measurements of $\Delta\omega_{\mathcal{G}}[k]$. In addition to measurements of $\Delta\omega_{\mathcal{G}}[k]$, (14) requires samples of $\Delta\dot{\omega}_{\mathcal{G}}[k]$, $\Delta\delta_{\mathcal{G}}[k]$, and $\Delta P^{\mathrm{m}}_{\mathcal{G}}[k]$, which are easily computed using measurements of $\Delta\omega_{\mathcal{G}}[k]$, as shown next.

1) Computing $\Delta\dot{\omega}_{\mathcal{G}}[k]$: We approximate $\Delta\dot{\omega}_{\mathcal{G}}[k]$ as the slope between two consecutive measurements: $\Delta\omega_{\mathcal{G}}[k-1]$ and $\Delta\omega_{\mathcal{G}}[k]$. Specifically,

$$\Delta\dot{\omega}_{\mathcal{G}}[k] \approx \Delta t^{-1}\left(\Delta\omega_{\mathcal{G}}[k] - \Delta\omega_{\mathcal{G}}[k-1]\right). \tag{15}$$

2) Computing $\Delta\delta_{\mathcal{G}}[k]$: Discretization of the differential equation in (4) results in the following expression:

$$\Delta\delta_{\mathcal{G}}[k+1] = \Delta\delta_{\mathcal{G}}[k] + \Delta t\Delta\omega_{\mathcal{G}}[k]. \tag{16}$$

Given the initial value $\Delta\delta_{\mathcal{G}}[0] = \mathbb{0}_G$, we can compute $\Delta\delta_{\mathcal{G}}[k]$ at all time steps $k > 0$.

3) Computing $\Delta P^{\mathrm{m}}_{\mathcal{G}}[k]$: We first note that variations in reference power inputs, denoted by $\Delta P^{\mathrm{r}}_{\mathcal{G}}(t)$, in (6) are the output of automatic generation control (AGC), which is used to restore system frequency back to synchronous value. In the timescales that we consider, we assume that the reference power inputs do not change significantly, i.e., $\Delta P^{\mathrm{r}}_{\mathcal{G}}(t) \approx \mathbb{0}_G$, $t \geq 0$, in (6). Then, discretization of (6) yields

$$\Delta P^{\mathrm{m}}_{\mathcal{G}}[k+1] = A\Delta P^{\mathrm{m}}_{\mathcal{G}}[k] + B\Delta\omega_{\mathcal{G}}[k], \tag{17}$$

where matrices $A, B \in \mathbb{R}^{G\times G}$ are given by

$$A = \exp(-\mathrm{diag}(\tau_{\mathcal{G}})^{-1}\Delta t),$$
$$B = \mathrm{diag}(R_{\mathcal{G}})A - \mathrm{diag}(R_{\mathcal{G}}).$$

Then, given the initial value $\Delta P^{\mathrm{m}}_{\mathcal{G}}[0] = \mathbb{0}_G$, we can compute $\Delta P^{\mathrm{m}}_{\mathcal{G}}[k]$, $k > 0$, using the recurrence relation in (17).

We close this discussion with a few remarks on the estimation of $\Delta P[k]$ from (13). By suitably shifting time indices in (15), (16), and (17), and substituting them into (14), we get

$$\begin{aligned}\Delta y[k] = &\left(\Delta t^{-1}\mathrm{diag}(M_{\mathcal{G}}) + \mathrm{diag}(D_{\mathcal{G}})\right)\Delta\omega_{\mathcal{G}}[k] \\ &- \left(\Delta t^{-1}\mathrm{diag}(M_{\mathcal{G}}) + B_d + \Delta tH\right)\Delta\omega_{\mathcal{G}}[k-1] \\ &- A_d\Delta P^{\mathrm{m}}_{\mathcal{G}}[k-1] - H\Delta\delta_{\mathcal{G}}[k-1].\end{aligned} \tag{18}$$

The observation in (18) indeed requires only measurements of synchronous-generator frequencies, and information at time step $k-1$ is known from previous frequency samples. However, the estimation problem in (13) cannot be solved by naïvely inverting W. This is because, in general, not all buses are connected to a generator, i.e., $G < N$, hence W is not a square matrix. In fact, the estimation problem in (13) is under-determined, and it cannot be solved via conventional least-squares estimation. One way to bypass this issue is to assume that the load variations remain constant and stack up observations over multiple time steps and formulate an over-determined estimation problem. However, in an effort to minimize the number of measurements needed, we next outline a method to solve the estimation problem by leveraging the expectedly sparse nature of load changes in the system.

B. Solution Approach

At any given point in time, we expect the vast majority of loads in the system to remain close to their steady-state operating point, i.e., for a given time step $k \in \mathbb{Z}$, $\Delta P[k]$ is a sparse vector. Thus, we cast the load-change detection and identification problem as an optimization program that minimizes the number of nonzero elements in $\Delta P[k]$:

$$\begin{aligned}\min_{\Delta P[k]} \quad & \|\Delta P[k]\|_0 \\ \text{s.t.} \quad & \Delta y[k] = W\Delta P[k].\end{aligned} \tag{19}$$

The problem in (19) is NP-hard due to the unavoidable combinatorial search [18]. A common solution approach is to relax the nonconvex and discontinuous ℓ_0-norm cost function with the convex and continuous ℓ_1-norm, yielding the following convex optimization problem:

$$\begin{aligned}\min_{\Delta P[k]} \quad & \|\Delta P[k]\|_1 \\ \text{s.t.} \quad & \|\Delta y[k] - W\Delta P[k]\|_2^2 \leq \epsilon,\end{aligned} \tag{20}$$

where $\epsilon \geq 0$ is a small tolerance introduced to facilitate computations. The optimization problem in (20) is known as the *basis pursuit denoising* (BPDN) problem and is widely used in compressed sensing for signal reconstruction, where noise is inherent in signal measurements [19]. There are several algorithms that can be utilized to solve the BPDN problem in (20), such as the alternating direction method of multipliers (ADMM) [20] and the spectral projected gradient for ℓ_1-norm minimization (SPGL1) algorithm [21], [22]. For $\Delta y[k] = W \Delta P[k]$ to have at least one solution, the matrix W must be full rank. We utilize the SPGL1 algorithm in [21] to solve the BPDN problem in (20), which relies only on matrix-vector operations and is computationally tractable for large-scale estimation problems [22].

IV. Case Studies

We demonstrate the proposed method via numerical case studies involving the Western Electricity Coordinating Council (WECC) 3-machine 9-bus test system; the one-line diagram for this system can be found in [16]. Synchronous generators are connected to buses collected in $\mathcal{G} = \{1, 2, 3\}$, loads are connected to buses in $\mathcal{L} = \{5, 6, 8\}$.

A. Simulation Setup

Although our analytical development is grounded in a simplified synchronous-generator model and leverages several approximations (e.g., lossless network, small voltage-angle differences, etc.), we verify the proposed method with time-domain simulations for a detailed, lossy, and nonlinear DAE model of the power network that includes generator dynamics from a two-axis synchronous-machine model, a turbine-governor model, and an exciter model. The simulations are performed using PSAT [23]. Synthetic measurements are sampled from the PSAT simulation at discrete intervals of $\Delta t = 0.0333\,\mathrm{s}$, which is within the capability of current measurement technology [24]. In our setup, at time $t = 0$, the load at bus 6 follows the Pennsylvania-New Jersey-Maryland Interconnection's (PJM) dynamic regulation signal (regD) trajectory [25]. The PJM regD signal is fed over a period of 40 seconds, where the disturbance setpoint changes every time step $\Delta t = 0.0333\,\mathrm{s}$. Other loads remain constant.

B. Load-change Estimation

We restrict measurements sampled from the time-domain simulation to only $\Delta\omega_{\mathcal{G}}[k]$ and compute $\Delta\dot{\omega}_{\mathcal{G}}[k]$, $\Delta\delta_{\mathcal{G}}[k]$, and $\Delta P^{\mathrm{m}}_{\mathcal{G}}[k]$ using (15), (16), and (17), respectively. As shown in Fig. 1a, the computed dynamic state trajectories match the actual ones reasonably well. With the measured $\Delta\omega_{\mathcal{G}}[k]$, $\Delta y[k]$ is evaluated using (18). Then, at each time step, we solve the optimization problem in (20) using the SPGL1 algorithm [21], [22]. The estimated load changes are plotted in Fig. 1b. The load change is detected and its location is identified at $k = 1$. The load changes estimated at bus 6 from measurements obtained during the transient period follow the regD signal closely, while other loads remain near zero, as desired. Indeed, the proposed load-change detection and identification method using only generator frequency measurements is very effective.

V. Concluding Remarks

In this paper, we proposed a method to estimate load changes using generator frequency measurements along with a reduced-order power system dynamical model. The model accounts for locational effects of load disturbances on generator frequency dynamics. The utility of the proposed method was demonstrated via numerical case studies involving the WECC test system. Results show that we can accurately detect and identify load disturbances in the network. Future work includes extending the proposed framework for generator and line outage detection as well as frequency-dependent loads.

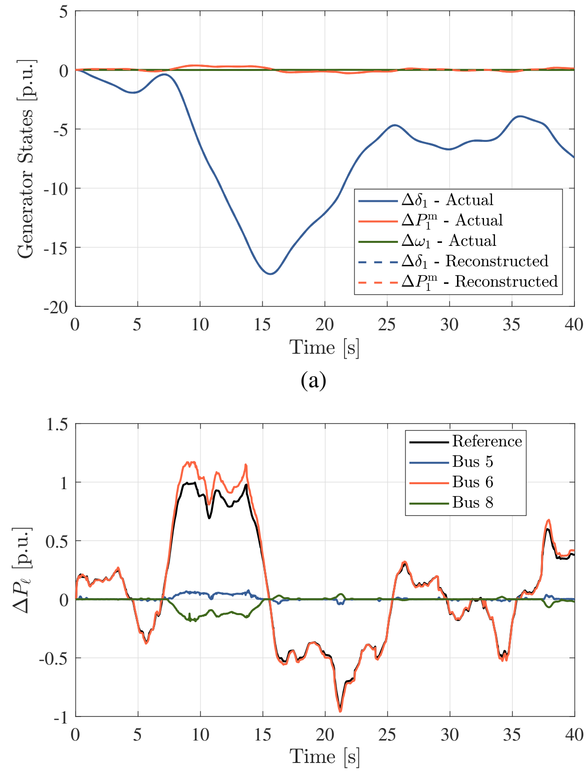

Fig. 1: WECC test system: load change detection, estimation, and identification algorithm performance evaluation. (a) Demonstrating that dynamic state trajectories reconstructed from $\Delta\omega_{\mathcal{G}}[k]$ measurements (dashed traces) are accurate as compared to actual trajectories (solid traces). (b) Estimated load changes due to PJM regD regulation signal disturbance in load at bus 6 using measurements $\Delta\omega_{\mathcal{G}}[k]$.

Appendix

A. Details of Model in (10)

Based on the structures of (2) and (3), matrices $B_{\mathcal{G}\mathcal{G}}$, $B_{\mathcal{G}\mathcal{L}}$, $B_{\mathcal{L}\mathcal{G}}$, and $B_{\mathcal{L}\mathcal{L}}$ can be expressed as

$$B_{\mathcal{G}\mathcal{G}} = B_{GG} - \mathrm{diag}\left(\begin{bmatrix} B_{GG} & B_{GL} \end{bmatrix} \mathbb{1}_N + \frac{\mathbb{1}_G}{X'_{d,\mathcal{G}}} \right), \quad (21)$$

$$B_{\mathcal{G}\mathcal{L}} = B_{GL}, \quad (22)$$

$$B_{\mathcal{L}\mathcal{G}} = B_{LG}, \quad (23)$$

$$B_{\mathcal{L}\mathcal{L}} = B_{LL} - \mathrm{diag}\left(\begin{bmatrix} B_{LG} & B_{LL} \end{bmatrix} \mathbb{1}_N \right), \quad (24)$$

where submatrices B_{GG}, B_{GL}, B_{LG}, and B_{LL} are extracted from the standard (and appropriately reordered) network admittance matrix Y for a lossless system, i.e.,

$$Y = \mathrm{j} \begin{bmatrix} B_{GG} & B_{GL} \\ B_{LG} & B_{LL} \end{bmatrix}. \quad (25)$$

REFERENCES

[1] Y. C. Chen, T. Banerjee, A. D. Domínguez-García, and V. V. Veeravalli, "Quickest line outage detection and identification," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 749–758, Jan 2016.

[2] R. Emami and A. Abur, "External system line outage identification using phasor measurement units," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1035–1040, May 2013.

[3] H. Zhu and G. B. Giannakis, "Sparse overcomplete representations for efficient identification of power line outages," *IEEE Transactions on Power Systems*, vol. 27, no. 4, pp. 2215–2224, Nov 2012.

[4] J. E. Tate and T. J. Overbye, "Line outage detection using phasor angle measurements," *IEEE Transactions on Power Systems*, vol. 23, no. 4, pp. 1644–1652, Nov 2008.

[5] Y. Shin, E. J. Powers, M. Grady, and A. Arapostathis, "Power quality indices for transient disturbances," *IEEE Transactions on Power Delivery*, vol. 21, no. 1, pp. 253–261, Jan 2006.

[6] W. G. Morsi and M. E. El-Hawary, "Fuzzy-wavelet-based electric power quality assessment of distribution systems under stationary and nonstationary disturbances," *IEEE Transactions on Power Delivery*, vol. 24, no. 4, pp. 2099–2106, Oct 2009.

[7] R. H. G. Tan and V. K. Ramachandaramurthy, "Voltage sag acceptability assessment using multiple magnitude-duration function," *IEEE Transactions on Power Delivery*, vol. 27, no. 4, pp. 1984–1990, Oct 2012.

[8] L. Xie, Y. Chen, and P. R. Kumar, "Dimensionality reduction of synchrophasor data for early event detection: linearized analysis," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 2784–2794, Nov 2014.

[9] I. Kamwa, A. K. Pradhan, and G. Joos, "Robust detection and analysis of power system oscillations using the Teager-Kaiser energy operator," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 323–333, Feb 2011.

[10] M. J. Smith and K. Wedeward, "Event detection and location in electric power systems using constrained optimization," in *2009 IEEE Power Energy Society General Meeting*, Jul 2009, pp. 1–6.

[11] G. Rigatos, P. Siano, and A. Piccolo, "Neural network-based approach for early detection of cascading events in electric power systems," *IET Generation, Transmission Distribution*, vol. 3, no. 7, pp. 650–665, Jul 2009.

[12] G. W. Hart, "Nonintrusive appliance load monitoring," *Proceedings of the IEEE*, vol. 80, no. 12, pp. 1870–1891, Dec 1992.

[13] Y. Su, K. Lian, and H. Chang, "Feature selection of non-intrusive load monitoring system using stft and wavelet transform," in *2011 IEEE 8th International Conference on e-Business Engineering*, Oct 2011, pp. 293–298.

[14] A. F. Ebrahim and O. Mohammed, "Household load forecasting based on a pre-processing non-intrusive load monitoring techniques," in *2018 IEEE Green Technologies Conference*, Apr 2018, pp. 107–114.

[15] A. G. Phadke and B. Kasztenny, "Synchronized phasor and frequency measurement under transient conditions," *IEEE Transactions on Power Delivery*, vol. 24, no. 1, pp. 89–95, Jan 2009.

[16] P. W. Sauer and M. A. Pai, *Power System Dynamics and Stability*. Upper Saddle River, NJ: Prentice-Hall, Inc., 1998.

[17] B. Chen, A. Al-Digs, and Y. C. Chen, "A network-cognizant aggregate-frequency reduced-order power system dynamical model," in *2018 North American Power Symposium (NAPS)*, Sep. 2018, pp. 1–6.

[18] M. Hyder and K. Mahata, "An approximate L0 norm minimization algorithm for compressed sensing," in *2009 IEEE International Conference on Acoustics, Speech and Signal Processing*, Apr 2009, pp. 3365–3368.

[19] J. F. C. Mota, J. M. F. Xavier, P. M. Q. Aguiar, and M. Puschel, "Distributed basis pursuit," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 1942–1956, Apr 2012.

[20] J. F. C. Mota, J. M. F. Xavier, P. M. Q. Aguiar, and M. Pschel, "D-ADMM: A communication-efficient distributed algorithm for separable optimization," *IEEE Transactions on Signal Processing*, vol. 61, no. 10, pp. 2718–2723, May 2013.

[21] E. van den Berg and M. P. Friedlander, "SPGL1: A solver for large-scale sparse reconstruction," Jun 2007, http://www.cs.ubc.ca/labs/scl/spgl1.

[22] ——, "Probing the Pareto frontier for basis pursuit solutions," *SIAM Journal on Scientific Computing*, vol. 31, no. 2, pp. 890–912, 2008.

[23] F. Milano, "An open source power system analysis toolbox," *IEEE Trans. on Power Systems*, vol. 20, no. 3, pp. 1199–1206, Aug 2005.

[24] US DOE & FERC. (2006, Feb) Steps to establish a real-time transmission monitoring system for transmission owners and operators within the eastern and western interconnections. [Online]. Available: http://energy.gov/sites/prod/files/oeprod/DocumentsandMedia/final_1839.pdf

[25] PJM. (2019, Feb) PJM manual 12: Balancing operations. [Online]. Available: http://www.pjm.com