# Internally coded TH–UWB–CDMA system and its performance evaluation

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**Abstract:** A new time-hopping ultrawideband (TH–UWB) CDMA scheme for indoor wireless communications is presented. In the proposed method, the duration of each bit is divided into  $Ns_1$  frames, each one containing  $Ns_2$  subframes. Two pseudorandom sequences are assigned to each user. During each bit interval, based on the output of a super-orthogonal encoder and the user's first dedicated pseudorandom sequence, the transmitter selects one of the  $Ns_1$  frames and then transmits  $Ns_2$  narrow pulses in that frame, one in each of the  $Ns_2$  subframes. The location of the pulse in each subframe is determined by the user's second dedicated PN sequence. Four different detection techniques are considered at the receiver front end, namely thresholded hard decision, strict hard decision, soft decision and chip-based decision. Their performances are analysed and the results are compared with those of the previously introduced coded and uncoded TH–UWB systems. The results indicate that the proposed scheme has the best performance without requiring any extra bandwidth. It is also shown that the chip-based decoding technique works better in moderate and high SNRs while the soft decision method has better performance in low SNRs.

#### 1 Introduction

In the conventional TH–UWB system which was first introduced in [1], extremely short pulses, about less than 1 ns, are used for transmitting data, so that the bandwidth is very wide from about DC to several GHz. The advantages of the TH–UWB system, such as capability to resolve multipath components with differential delays on the order of 1 nanosecond or less, penetrating materials and interference avoidance make it viable for high-quality mobile shortrange indoor radio communications [2].

In recent years, there has been intensive work on different aspects of UWB systems, such as coding, synchronisation, multiuser detection, narrow-band interference cancellation and multirate schemes [3-12]. In [3], a practical low-rate coding scheme is applied to the TH–UWB system, which does not require any extra bandwidth further than what is needed by TH-spread spectrum modulation. The system performance analysis in [3] indicates that the coded scheme outperforms the uncoded scheme significantly.

In this paper, we propose a new internal coding scheme for TH–UWB–CDMA that has much better performance than the conventional uncoded TH–UWB–CDMA [2]. In addition, it can have better performance or less complexity than the corresponding coded scheme described in [3]. In our new method, two pseudorandom sequences are dedicated to each user. The bit duration is divided into  $Ns_1$ 

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frames, where each frame consists of  $Ns_2$  subframes and each subframe is also segmented into  $N_h$  chips. The output symbol of a super-orthogonal encoder [13], along with the user's first pseudorandom sequence (PN1), select one of the  $Ns_1$  frames. Then,  $Ns_2$  short pulses are transmitted in that frame, one in each of the  $Ns_2$  subframes. The chip position of the transmitted pulse in each of the  $Ns_2$  subframes is determined by the user's second dedicated PN sequence (PN2).

Note that the basic difference between the proposed method and the scheme introduced in [3] is that in the new method the coded symbol does not directly modulate the pulses. Instead, it determines the position of the frame in which the pulses will be transmitted (compared to [3] in which the output of the encoder is considered as a sequence of bits and the pulses are transmitted using BPPM). As a result, a wider pulse can be used in the proposed method. In other words, the proposed method consumes less bandwidth and can be synchronised more easily. However, if we consider the same pulse duration or equivalently the same bandwidth for the two systems, the proposed method can possess a higher processing gain.

At the receiver front end of the proposed system, we consider four different detection techniques, namely thresholded hard decision, strict hard decision, soft decision and chip-based decision. Then, the Viterbi algorithm is applied to decode the underlying super-orthogonal code.

We evaluate the performance of the four mentioned decoding techniques and obtain the upper bound on the bit error rate (BER) using the Chernoff bound and the path generating function of the super-orthogonal code in a synchronous AWGN channel. For some cases, we also provide the analytical results using the Beaulieu series. Then, we discuss the results and compare them with those of the previously presented uncoded [2] and coded [3] TH–UWB systems. It must be noted that our analyses are verified by the simulation results. The performance analysis in fading channels is under consideration.

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#### 2 System description

Fig. 1 shows the block diagram of the proposed UWB system. The duration of each bit,  $T_b$ ; is divided into  $Ns_1$  frames each of duration  $T_f$ , that is  $T_b = Ns_1T_f$ . The duration of each frame is also partitioned into  $Ns_2$  subframes of duration  $T_{sf}$ , where each subframe consists of  $N_h$  chips with duration  $T_c$ , that is  $T_f = Ns_2T_{sf}$  and  $T_{sf} = N_hT_c$ . So, the processing gain is

$$PG = T_b/T_c = Ns_1 N s_2 N_h \tag{1}$$

Two PN codes, namely PN1 and PN2, are assigned to each user. The components of these two sequences are i.i.d integer-valued random variables with uniform distributions on  $\{0, 1, \ldots, N_{s_1} - 1\}$  and  $\{0, 1, \ldots, N_h - 1\}$ , respectively.

The data bit of each user is applied to a super-orthogonal encoder with the constraint length K, which generates  $2^{K-2}$  different symbols [13]. The encoder output is then added to the first pseudorandom sequence (PN1) in mod  $Ns_1$ , where  $Ns_1 = 2^{K-2}$ . The result specifies the position of one of the  $Ns_1$  frames. Then, at that frame,  $Ns_2$  pulses are transmitted in the  $Ns_2$  chips, one in each subframe like in a conventional TH–UWB system as described in [2, 3]. The transmitted signal of the user *m* can be written as

$$s^{(m)}(t) = \sum_{j} \sum_{i=0}^{N_{s_2}-1} w(t - c_{1,j}^{(m)}T_f - iT_{sf} - c_{2,i}^{(m)}T_c - jT_b)$$
(2)

where w(t) is a pulse with maximum duration  $T_c$  and energy  $m_p \triangleq \int_0^{T_c} w^2(t) dt, c_{1,j}^{(m)}$  is the coded symbol corresponding to the *j*th bit of user *m*, which is determined by the sum of the super-orthogonal encoder output and the PN1 sequence in mod  $Ns_1$ . It takes on an integer value between 0 and  $Ns_1-1$  uniformly and specifies the location of the frame, in which the  $Ns_2$  pulses are transmitted. In (2),  $c_{2,i}^{(m)}$  is the PN2 sequence which determines the position of the chips in the selected frame, in which the pulses are transmitted. Also, note that in Fig. 1,  $b_j^{(m)} \in \{0,1\}$  is the data bit of user *m* at time  $jT_b$ .

The received signal in a synchronous AWGN channel can be written as

$$r(t) = \sum_{m=1}^{N_u} s^{(m)}(t) + n(t)$$
(3)

where  $N_u$  is the number of active users and n(t) is the zeromean additive white Gaussian noise with two-sided power spectral density  $N_0/2$ . From this signal, the receiver must detect the frame containing pulses  $(\hat{c}_{1,j}^{(m)})$  and decode the transmitted data using the sequence of symbols  $\hat{c}_{1,j}^{(m)}$ .

Assuming that the first user is the desired user, in the thresholded hard decision, strict hard decision and soft decision techniques, the receiver uses a frame-based sliding correlator with the base signal as

$$v_j^{(1)}(t) = \sum_{i=0}^{Ns_2 - 1} w(t - iT_{sf} - c_{2,i}^{(1)}T_c - jT_b)$$
(4)

to find the frame in which the data is sent (note that  $v_j^{(1)}(t)$  depends only on PN2). In other words, during the *j*th bit interval, the receiver computes the following correlation values for each of the  $Ns_1$  frames

$$R_{j,h} = \int_{jT_b + hT_f}^{jT_b + (h+1)T_f} r(t)v_j^{(1)}(t - hT_f) dt$$
  
$$h = 0, 1, \dots, Ns_1 - 1$$
(5)

Then, based on the output of the corresponding correlator, a value is assigned to each frame.

In the thresholded hard decision technique, the value '1' is assigned to the frame *h* if  $R_{j,h}$  is greater than the threshold value *Thr*; otherwise the value '0' is assigned to that frame. In other words, we assign the value  $M_{j,h}$  to the frame *h* during the *j*th bit interval as follows

$$M_{j,h} = \begin{cases} 1 & \text{if } R_{j,h} > Thr\\ 0 & \text{if } R_{j,h} < Thr \end{cases}$$
(6)

In the strict hard decision technique, only the frame with the greatest correlation is assigned the value '1', that is

$$M_{j,h} = \begin{cases} 1 & \text{if } h = \hat{h} \\ 0 & \text{if } h \neq \hat{h} \end{cases}$$
  
where  $R_{j,\hat{h}} = \max\{R_{j,h}; h = 0, 1, \dots, Ns_1 - 1\}$  (7)

In the soft decision technique, the correlator output is directly assigned to the corresponding frame, that is  $M_{j,h} = R_{j,h}$ .

In the chip-based hard decision technique, for each frame h the receiver first calculates the  $Ns_2$  chip-based correlator outputs at the  $Ns_2$  mark chips as follows

$$\dot{r}_{j,h,i} = \int_{jT_b + hT_f + iT_c}^{jT_b + hT_f + (i+1)T_c} r(t)w(t - jT_b - hT_f - iT_{sf} - c_{2,i}^{(1)}T_c) dt; \quad i = 0, 1, \dots, Ns_2 - 1$$
(8)

Note that the mark chips are the chip positions in the  $Ns_2$  subframes in which the  $Ns_2$  pulses are transmitted. These positions are determined by the PN2 sequence.

These values are then compared to the threshold  $Thr_{chip}$  to make a decision on the existence of pulses on the mark chips of each frame *h*. Finally, the value '1' is assigned to the frame, if the pulses are detected in all of the  $Ns_2$  mark chips of that frame. Otherwise, the value '0' is assigned



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Fig. 1 Block diagram and time axis division of the proposed system

to the frame, that is we have

$$M_{j,h} = \begin{cases} 1 & \text{if } r_{j,h,i} > Thr_{chip}; \quad i = 0, 1, 2, \dots, Ns_2 - 1 \\ 0 & \text{otherwise} \end{cases}$$
(9)

In all of the above detection techniques, the value  $M_{j,h}$  is used as the metric of the branches in the trellis diagram of the underlying convolutional code. In other words, if the output symbol of a branch at the *j*th bit duration in the trellis diagram is *h*, then the value  $M_{j,h}$  is used as its metric.

In the following, we obtain the frame-based correlator output due to the desired signal, interference and noise, respectively. Without loss of generality, we consider the signalling period j = 0 and for simplicity, we drop all of the *j* indexes in the rest of the paper.

#### 2.1 Output due to signal of the desired user

The correlator output due to the first (desired) user is obtained by replacing  $s^{(1)}(t)$  instead of r(t) in (5). It can be easily observed that the output of the frame-based correlator is

$$S_{h}^{(1)} = \int_{hT_{f}}^{(h+1)T_{f}} s^{(1)}(t) v^{(1)}(t - hT_{f}) dt$$
$$= \begin{cases} Ns_{2} \int_{0}^{T_{c}} w^{2}(t) dt = Ns_{2}m_{p} & h = c_{1}^{(1)} \\ 0 & h \neq c_{1}^{(1)} \end{cases}$$
(10)

where  $m_p$  is the energy of the transmitted pulse.

# 2.2 Effect of multiple access interference (MAI)

Assuming that the interfering user *m* transmits its data in the frame *h*, the corresponding correlator output in that frame equals the total correlation of the overlapped pulses of the interfering user with those of the desired user (user 1). In other words, if we assume that the PN2 sequences of the user *m* and user 1 overlap in *n* chips (among the  $Ns_2$  mark chips of the frame), then the correlator output will be  $nm_p$ . For an i.i.d. PN sequence, the probability of such an event is equal to

$$\binom{Ns_2}{n} \alpha^n (1-\alpha)^{Ns_2-n}$$

where  $\alpha \triangleq 1/N_h$  is the probability that the users *m* and 1 send their pulses in the same chip. So, the probability density function (pdf) of the correlator output due to the interfering user *m* in the frame *h* is

$$f_{R_h}^{(m)}(R) = \sum_{n=0}^{N_{S_2}} {N_{S_2} \choose n} \alpha^n (1-\alpha)^{N_{S_2}-n} \delta(R-nm_p)$$
(11)

where  $\delta(\cdot)$  is the Dirac delta function. For simplicity, we assume that the desired user sends its data in the frame  $c_1^{(1)} = 0$ . If we consider that  $U_h$  independent users transmit their data in the frame  $h \neq 0$ , in which the desired user does not send its data, then the pdf of the related correlator output due to the total interference will be

$$f_{R_{h}}^{tot(U_{h})}(R) = f_{R_{h}}^{(m)\{U_{h}\}}(R) \triangleq \underbrace{f_{R_{h}}^{(m)}(R) * \cdots * f_{R_{h}}^{(m)}(R)}_{U_{h} times}; h \neq 0$$
(12)

where  $f^{\{k\}}(x)$  denotes the k times convolution of f(x) with itself. In the frame  $h = c_1^{(1)} = 0$ , in which the desired user

sends its data, there is also a correlation value  $Ns_2m_p$ due to the desired user's signal (see (10)). Hence, the pdf of the correlator output at frame h = 0 due to the  $U_0$  interfering users and the desired user is like (12) shifted by  $Ns_2m_p$ 

$$f_{R_0}^{tot(U_0)}(R) = f_{R_h}^{(m)\{U_0\}}(R - Ns_2m_p) \quad h = 0$$
(13)

Since  $f_{R_h}^{(m)}(R)$  in (11) is a sequence of impulses, we can rewrite the pdfs of (12) and (13) as

$$f_{R_h}^{tot(U_h)}(R) = \sum_{l=0}^{L_h - 1} a_l \delta(R - lm_p) \quad h \neq 0$$
(14)

$$f_{R_0}^{tot(U_0)}(R) = \sum_{l=0}^{L_0-1} b_l \delta(R - (l + Ns_2)m_p) \quad (h = 0)$$
(15)

where the coefficients  $a_1$  and  $b_1$  and the values  $L_h$  and  $L_0$  can be found by inserting (11) in (12) and (13), respectively.

# 2.3 Output noise

The noise component at the output of the frame-based correlator is computed as

$$n_h = \int_{hT_f}^{(h+1)T_f} n(t) v^{(1)} (t - hT_f) dt$$
  
$$h = 0, 1, \dots, Ns_1 - 1$$
(16)

These output noise components are all zero-mean with the following properties:

$$E\{n_h \ n_{h'}\} = 0 \ h \neq h' \ ; \ \sigma_n^2 = E\{n_h^2\} = \frac{N_0}{2}Ns_2m_p$$
(17)

We define the signal-to-noise ratio (SNR) as the energy of the desired signal to the variance of noise at the correlator output, which noting (10) and (17) yields

$$SNR = \frac{(Ns_2m_p)^2}{2\sigma_n^2} = \frac{Ns_2m_p}{N_0}$$
(18)

Noting the independence of noise and interference, the pdf of the correlator output due to the interfering users and noise in the frame  $h \neq (c_1^{(1)} = 0)$  is obtained as (see (14))

$$f_{R_h}^{tot(U_h),n}(R) = f_{R_h}^{tot(U_h)}(R) * f_n(R)$$
$$= \sum_{l=0}^{L_h-1} a_l f_n(R - lm_p) \quad h \neq 0$$
(19)

where  $f_n(\cdot)$  is the pdf of the noise component which is a zero-mean Gaussian distribution with variance  $\sigma_n^2$  given in (17). Similarly for the frame h = 0, we have (see (15))

$$f_{R_0}^{tot(U_0),n}(R) = f_{R_0}^{tot(U_0)}(R) * f_n(R)$$
$$= \sum_{l=0}^{L_0-1} b_l f_n(R - (l + Ns_2)m_p) \quad h = 0$$
(20)

# 2.4 Chernoff bound

According to the Chernoff bound [13], for random variable Z we have

$$P(Z > a) \le \min_{s \ge 0} \{e^{-sa}\varphi_z(s)\}$$
(21)

where  $\varphi_Z(s) \triangleq E\{e^{sZ}\}$  is the characteristic function of Z. To compute the BER of the underlying convolutional code, as usual, we must first compute an error event with Hamming distance d [13–15]. We denote the probability of such an event as  $P_d$ . Note that since the outputs of the superorthogonal encoder in our application are considered as symbols, the distance considered here is the symbol distance of the two paths. Without loss of generality, we assume that the all zero sequence is transmitted by the desired user. Thus,  $P_d$  is the probability that the metric of a nonzero path with symbol weight d is larger than that of the all zero path, i.e.

$$P_{d} = P\left\{\sum_{k=1}^{d} M_{y'_{k}\neq 0} > \sum_{k=1}^{d} M_{y_{k}=0}\right\} = P\left\{\sum_{k=1}^{d} Z_{k} > 0\right\}$$
$$= P\{Z > 0\}$$
(22)

where  $Z \triangleq \sum_{k=1}^{d} Z_k$  and  $Z_k \triangleq M_{y_k \neq 0} - M_{y_k = 0}$  indicates the difference in the metrics of the branch corresponding to the nonzero path and the branch corresponding to the all-zero path at the instant of the *k*th different branches of the two paths ( $y_k$  and  $y'_k$  denote the branch outputs of the two paths). Note that the two paths may have a length larger than *d* but they differ in only *d* branches.

In a memoryless channel, the variables  $Z_k$ , k = 1, 2, ..., d, are independent and have the same pdf; therefore, it suffices to find the characteristic function of one of them  $(\varphi_{Z_k}(s))$  which results in

$$\varphi_Z(s) = [\varphi_{Z_k}(s)]^d \tag{23}$$

Then from (21)–(23), we obtain an upper bound for  $P_d$  as

$$P_d = \Pr\{Z > 0\} \le \min_{s > 0} \{\varphi_Z(s)\} = [\min_{s > 0} \{\varphi_{Z_k}(s)\}]^d$$
(24)

# 2.5 Upper and lower bounds on the BER of the super-orthogonal code

For a convolutional code, only the lower and upper bounds on the BER are available analytically. As stated before, in the proposed method we use the output of the superorthogonal encoder as a symbol not as a sequence of bits. (This symbol determines the location of one of the  $Ns_1$ frames, in which  $Ns_2$  pulses are transmitted.) Consequently, the path-generating function in our method differs from that of [13], in which the encoder output is considered as a sequence of bits. The path-generating function in our application is computed in [14] as

$$T(D,N) = \frac{(1-D)ND^{K}}{1-D(1+N(1+D^{K-3}-2D^{K-2}))}$$
(25)

where, in the series expansion of the above equation, the power of D denotes the (symbol) Hamming weight of the encoder output and the power of N represents the bit weight of the input. The number of bit errors due to an error event with weight d, that is  $c_d$ , can be calculated from the path-generating function as follows [13–15]

$$\left. \frac{\partial T(D,N)}{\partial N} \right|_{N=1} = \sum_{d=K}^{\infty} c_d D^d$$
(26)

where  $K = \log_2 Ns_1 + 2$  is the free distance of the code [13–15]. Therefore, using the union bound [13–15] and the Chernoff bound (24), the upper bound on the BER is

obtained as

$$P_{b} < \sum_{d=K}^{\infty} c_{d} P_{d} < \sum_{d=K}^{\infty} c_{d} \left[ \min_{s>0} \{\varphi_{Z_{k}}(s)\} \right]^{d}$$
$$\Rightarrow P_{b} < \frac{\partial T(D, N)}{\partial N} \Big|_{N=1, D=\min_{s>0} \{\varphi_{Z_{k}}(s)\}}$$
(27)

Similarly, a lower bound can be calculated in some special cases. However, numerical evaluations show that the upper and lower bounds are very close to each other. Therefore, for the rest of the paper we consider only the upper bound of the BER.

In the following sections, we evaluate the performance of the proposed decoding techniques by calculating  $\min_{s>0}{\varphi_{Z_t}(s)}$  and using the upper bound (27).

#### 3 Performance analysis of hard decoding

#### 3.1 Thresholded hard decision

In this technique, as stated previously, the correlator output in each frame  $(R_h)$  is compared with a threshold and then the value  $M_h \in \{0,1\}$  is assigned to the frame. We choose the threshold value as a fraction of the desired user output (10), that is,  $Thr = \lambda Ns_2 m_p$  where  $\lambda \leq 1$ . So, we have

$$M_{h} = \begin{cases} 0 & R_{h} < \lambda N s_{2} m_{p} \\ 1 & R_{h} > \lambda N s_{2} m_{p} \end{cases} \quad \lambda \le 1$$
(28)

Note that it is possible that more than one frame is assigned the value '1' due to the multiple-access interference and noise. Considering (28),  $Z_k = M_{h\neq 0} - M_{h=0} \triangleq M_h - M_0$ takes on one of the three values -1, 0 or +1. Defining  $P_i \triangleq \Pr\{Z_k = i\}; i \in \{-1, 0, +1\}$ , the characteristic function of  $Z_k$  can be written as

$$\varphi_{Z_{k}}(s) = P_{-1}e^{-s} + P_{0} + P_{1}e^{s}$$
<sup>(29)</sup>

The optimum value of *s* that minimises the function  $\varphi_{Z_k}(s)$  in (29) (required in (27)) is easily computed to be  $s_{opt} = 1/2 \ln(P_{-1}/P_1)$ . Replacing  $s_{opt}$  in (29) yields

$$\min_{s>0} \{\varphi_{Z_k}(s)\} = \varphi_{Z_k}(s_{opt}) = P_0 + 2\sqrt{P_1 P_{-1}}$$
(30)

We define  $P(M_h, M_0|0, 1)$  as the joint probabilities of the metrics of the zero  $(M_0)$  and nonzero  $(M_h)$  branches, conditioned that the data is sent in the zero path (as a result, no data is sent in the nonzero path). Note that 1 and 0 in  $P(M_h, M_0|0, 1)$  stand for sending data in frame  $c_1^{(1)} = 0$  and, as a result, not sending data in frame  $h \neq 0$ , respectively. Then, we can compute  $P_i = P(Z_k = i) = P(M_h - M_0 = i)$  in (30) as

$$P_{-1} = P(0, 1|0, 1) \tag{31}$$

$$P_0 = P(0, 0|0, 1) + P(1, 1|0, 1)$$
(32)

$$P_1 = P(1, 0|0, 1) \tag{33}$$

Note that  $P(M_h, M_0|0, 1) \neq P(M_h|0)P(M_0|1)$ , i.e. the random variables  $M_0$  and  $M_h$  (the metrics of the two branches) are not independent. The reason is that if there exist  $U_0$  and  $U_h$  interfering users in the frames indicated by the zero and nonzero branches, respectively, then  $U_0$  and  $U_h$  must satisfy  $U_0 + U_h \leq N_U - 1$ , which makes the above two variables be dependent. However, conditioned on  $U_0$  and  $U_h$ , they will be independent, that is

$$P(M_h, M_0|0, 1, U_h, U_0) = P(M_h|0, U_h)P(M_0|1, U_0)$$
(34)

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So we can use the conditional probability to find  $P(M_h, M_0|0, 1)$ 

$$P(M_h, M_0|0, 1) = \sum_{U_h=0}^{N_u-1} \sum_{U_0=0}^{N_u-1-U_h} P(M_h|0, U_h) P(M_0|1, U_0)$$
$$\times P(U_h, U_0) \quad M_h, M_0 = \{0, 1\}$$
(35)

where  $P(U_h, U_0)$  is the probability of existing  $U_0$  and  $U_h$  interfering users in the zero and nonzero frames (branches), respectively. Its value can be easily computed as

$$P(U_{h}, U_{0}) = {\binom{N_{u} - 1}{U_{h}}} {\binom{N_{u} - 1 - U_{h}}{U_{0}}} \times \beta^{U_{h}} \beta^{U_{0}} (1 - 2\beta)^{N_{u} - 1 - U_{h} - U_{0}}$$
(36)

where  $\beta \triangleq 1/Ns_1$  is the probability that two users send their data in the same frame.

The probabilities  $P(M_h|0, U_h)$  and  $P(M_0|1, U_0)$  can be easily evaluated from (19), (20) and (28) as

$$P(1|0, U_h) = \int_{\lambda N s_2 m_p}^{\infty} f_{R_h}^{tot(U_h), n}(R) dR$$
$$= \sum_{l=0}^{L_h - 1} a_l Q\left(\frac{\lambda N s_2 m_p - l m_p}{\sigma_n}\right)$$
$$= \sum_{l=0}^{L_h - 1} a_l Q\left(\left(\lambda - \frac{l}{N s_2}\right)\sqrt{2SNR}\right) \qquad (37)$$

$$P(0|0, U_h) = 1 - P(1|0, U_h)$$
(38)

$$P(1|1, U_0) = \int_{\lambda N s_2 m_p}^{\infty} f_{R_0}^{tot(U_0), n}(R) dR$$
$$= \sum_{l=0}^{L_0 - 1} b_l Q \left( (\lambda - \frac{l}{N s_2} - 1) \sqrt{2SNR} \right) \quad (39)$$

$$P(0|1, U_0) = 1 - P(1|1, U_0)$$
(40)

where  $Q(x) \triangleq 1/\sqrt{2\pi} \int_x^{\infty} \exp(-u^2/2) du$  and SNR is defined in (18). By replacing (36)–(40) in (35) and using the results in (33)–(35), the values  $P_{-1}$ ,  $P_0$ , and  $P_1$  are computed. Then, by inserting these values in (30) and (27), the upper bound on the BER of this technique is evaluated.

# 3.2 Strict hard decoding

In the strict hard decoding technique, the value '1' is assigned to the frame  $\hat{h}$  with the greatest correlator output and the other frames are all assigned the value zero (see (7)). Note that in contrast with the thresholded hard decoding technique, in this case only one frame is assigned the value '1'. The performance analysis of the strict hard decoding technique is similar to that of the thresholded hard decision technique except that in the calculation of (31)-(33), the outputs of the correlators due to all frames must be considered simultaneously.

Again we assume that the all-zero sequence is transmitted. In other words, the pulses of the desired user are located in the frame h = 0 in all bit intervals. In order to evaluate  $P(M_h, M_0|0, 1)$  in (31)–(33), we first compute the probability of detection in the correct frame (h = 0),

in which data is sent

$$P_{c} = P(0, 1|0, 1) = \Pr\{\hat{h} = 0\}$$

$$= \Pr\{R_{0} > \max\{R_{h}, h = 1, ..., Ns_{1} - 1\}\}$$

$$= \sum_{U_{0}, U_{1}, ..., U_{Ns_{1}-1}} \times \left[ P\{R_{0} > \max\{R_{h}\} | U_{0}, U_{1}, ..., U_{Ns_{1}-1}\} \right]$$

$$\times P(U_{0}, U_{1}, ..., U_{Ns_{1}-1})$$

$$= \sum_{U_{0}, U_{1}, ..., U_{Ns_{1}-1}} \binom{N_{u} - 1}{U_{0}, U_{1}, ..., U_{Ns_{1}-1}} \beta^{N_{u}-1}$$

$$\times \int_{-\infty}^{\infty} \left\{ \prod_{h=1}^{N_{u}-1} \left[ \int_{R_{h} < R_{0}} f_{R_{h}}^{tot(U_{h}), n}(\mathbf{x}) \, d\mathbf{x} \right] f_{R_{0}}^{tot(U_{0}), n}(R_{0}) \right\} dR_{0}$$
(41)

where  $R_h$  is the correlator output of the frame *h* defined in (5) and the functions  $f_{R_h}^{tot(U_h),n}(x)$  and  $f_{R_0}^{tot(U_0),n}(x)$  are defined in (19) and (20), respectively. The probability of error in detection of the correct frame is  $P_f = 1 - P_c$ . There are totally  $Ns_1 - 1$  incorrect frames, which any of them may be detected instead of the correct frame (h = 0)with the same probability  $P_f/(Ns_1 - 1)$ . We remind that P(1, 0|0, 1) denotes the probability of detection in the frame indicated by the branch of the nonzero path (i.e. the frame  $h \neq 0$ ). Thus, we have

$$P(1,0|0,1) = \Pr\{\hat{h} = h \neq 0\} = \frac{P_f}{Ns_1 - 1}$$
(42)

Similarly, the probability of detection in a frame other than the correct frame and the frame corresponding to the nonzero path (i.e.  $P(0, 0|0, 1) = \Pr{\{\hat{h} \neq 0 \& \hat{h} \neq h\}}$ ) is

$$\frac{Ns_1 - 2}{Ns_1 - 1}P_f$$

Note that in this technique, the probability of assigning the value '1' to two frames is zero. Briefly, we have

$$P(0, 1|0, 1) = 1 - P_f$$

$$P(1, 0|0, 1) = \frac{P_f}{Ns_1 - 1} = \frac{\beta}{1 - \beta} P_f$$

$$P(0, 0|0, 1) = \frac{Ns_1 - 2}{Ns_1 - 1} P_f = \frac{1 - 2\beta}{1 - \beta} P_f$$

$$P(1, 1|0, 1) = 0$$
(43)

Using the above values, we obtain  $P_i$  in (31)–(33). Then, by replacing (30) in (27), the upper bound on the BER of this technique is computed.

#### 4 Performance analysis of soft decoding

In the soft decoding technique, the correlator output is used directly as the metric of each branch in the Viterbi algorithm. So, we have

$$Z_k = M_{h \neq 0} - M_{h=0} = R_{h \neq 0} - R_{h=0}$$
(44)

To compute  $\varphi_{Z_k}(s)$ , we remind that the variables  $R_{h\neq 0}$  and  $R_0 \triangleq R_{h=0}$  conditioned on  $U_h$  and  $U_0$  are independent.

Therefore, we can write the characteristic function of  $Z_k$  as

$$\varphi_{Z_{k}}(s) = \sum_{U_{h}=0}^{Nu-1} \sum_{U_{0}=0}^{Nu-1-U_{h}} \varphi_{R_{h\neq0}}(s|0, U_{h})\varphi_{R_{0}}(-s|1, U_{0})P(U_{h}, U_{0})$$
(45)

We can compute  $\varphi_{R_{h\neq 0}}(s|0, U_h)$  and  $\varphi_{R_0}(-s|1, U_0)$  using (12), (13) and noting that the characteristic function of the noise components  $n_{h\neq 0}$  and  $n_0$  (respectively included in  $R_{h\neq 0}$  and  $R_0$ ) is  $\varphi_{n_h}(s) = \exp(\sigma_n^2 s^2/2)$ . In this way, we have

$$\varphi_{R_{h\neq 0}}(s|0, U_{h}) = \left(\varphi_{R_{h}^{(m)}}(s)\right)^{U_{h}} e^{\sigma_{n}^{2}s^{2}/2}$$
$$\varphi_{R_{0}}(s|1, U_{0}) = e^{Ns_{2}m_{p}s} \left(\varphi_{R_{h}^{(m)}}(s)\right)^{U_{0}} e^{\sigma_{n}^{2}s^{2}/2}$$
(46)

where  $\varphi_{R_h}^{(m)}(s)$  is the characteristic function of one interfering user which is calculated from (11) as

$$\varphi_{R_h^{(m)}}(s) = \sum_{n=0}^{Ns_2} {Ns_2 \choose n} \alpha^n (1-\alpha)^{Ns_2-n} \exp(nm_p s)$$
(47)

Then, from  $(45)-(47) \varphi_{Z_k}(s)$  is computed. However, for this case it is difficult to find the optimum value of *s* (required in (27)) analytically. So, we will compute it numerically. Eventually, the upper bound on the BER is obtained using (27).

# 5 Performance of chip-based hard decoding

In this technique a chip-based correlator is used to calculate the correlation at the mark chips of each frame (determined by PN2). Then, the value '1' is assigned to the frame, in which the pulses are detected in all its  $Ns_2$  mark chips. Otherwise, the value '0' is assigned to that frame.

Assuming  $U_h$  users transmitting data in the frame  $h \neq 0$ , then the pdf of the chip-based correlator output in a specific chip of that frame due to the interfering users is easily computed as

$$f_{chip}^{tot(U_h)}(R) = \sum_{n=0}^{U_h} {U_h \choose n} \alpha^n (1-\alpha)^{U_h - n} \delta(R - nm_p) \quad (48)$$

where  $\alpha = 1/N_h$  is the probability that the two users send their pulses in the same chip.

It is easily shown that the output noise has Gaussian pdf  $f_{n_{chip}}(\cdot)$  with mean zero and variance  $\sigma_{n,chip}^2 = m_p N_0/2$ . So, the pdf of the output of the chip-based correlator in the frame  $h \neq 0$  will be

$$f_{chip}^{tot(U_h),n}(R) = \sum_{n=0}^{U_h} {U_h \choose n} \alpha^n (1-\alpha)^{U_h - n} f_{n_{chip}}(R - n m_p) \quad (49)$$

The output of the chip-based correlator due to a single pulse of the desired user in frame h = 0 is  $m_p$ . Thus, in order to obtain the pdf of the correlator output in the frame h = 0, we must replace  $U_h$  and  $R - nm_p$  in (49) by  $U_0$  and  $R - nm_p - m_p$ , respectively.

Considering the threshold level  $Thr_{chip} = \mu m_p$  where  $\mu \le 1$ , the probability of detecting a pulse in a single chip

at frame  $h \neq 0$  is

$$P_{chip}(1|0, U_h) = \int_{\mu m_p}^{\infty} f_{chip}^{tot(U_h), n}(R) dR$$
$$= \sum_{n=0}^{U_h} {U_h \choose n} \alpha^n (1-\alpha)^{U_h - n}$$
$$\times Q\left((\mu - n)\sqrt{\frac{2SNR}{Ns_2}}\right)$$
(50)

Then, the probability of detecting  $Ns_2$  pulses at the mark chips determined by the PN2 sequence equals  $P(1|0, U_h) = [P_{chip}(1|0, U_h)]^{Ns_2}$ . It is also obvious that  $P(0|0, U_h) = 1 - P(1|0, U_h)$ . Similarly, we can compute  $P(1|1, U_0)$  and  $P(0|1, U_0)$ . Inserting these values in (35), (31)–(33) and (27) yields the upper bound on the BER.

# 6 Numerical results

In this section, we present some numerical results based on the analytical evaluations obtained in the previous sections and the results achieved from the simulation. At first, we



**Fig. 2** Performance of the proposed method for different decoding techniques against the number of users Noiseless channel, i.e.  $N_0 = 0$ ,  $N_{S_1} = 4$ ,  $N_{S_2} = 8$ , and  $N_h = 8$ 



**Fig. 3** Performance comparison of the proposed method and the method introduced in [3] for the same processing gain  $N_0 = 0$ ,  $N_{s_1} = 8$ ,  $N_{s_2} = 2$  and  $N_h = 16$ 



**Fig. 4** Optimum value of the normalised threshold  $(\lambda_{opt})$  in the thresholded hard decoding against the number of users and SNR  $Ns_1 = 2, Ns_2 = 8, N_h = 8$ 

compare the performance of the four decoding techniques in the absence of noise to consider the effect of the MAI. It must be noted that Figs. 2-5 demonstrate the performance obtained only from the analytical evaluations and Figs. 6 and 7 show both the analytical and simulation results. Fig. 2 shows the BER of the proposed detection techniques against the number of users. Note that in this figure we have used the threshold values  $Thr = Ns_2m_p$  and  $Thr_{chip} = m_p$ , that is  $\lambda = \mu = 1$  (see (28), (37)–(40) and (50)). It is observed that in this case the chip-based hard decoding has the best performance. The reason is that the MAI always increases the correlation value. In other words, in the absence of noise, the existence of the pulses are always detected in the correct frame and the only possible source of error is that the MAI builds the expected pattern in a frame that does not contain the desired user's data. In addition, in the frame-based hard detection techniques, even though some mark chips of the frame may not indicate the existence of the pulse, but due to the large interference in the other mark chips, it is possible that the correlator output is greater than the threshold level or the correlator output of other frames, which may lead to detection error. Thus, by applying the hard decision technique on the chipbased correlator output, the effect of MAI can be substantially decreased.



**Fig. 5** *Performance of the three decoding techniques for the proposed system* 

 $N_u = 15, Ns_1 = 4, Ns_2 = 8 \text{ and } N_h = 16$ 



**Fig. 6** *Performance of the thresholded hard decoding and chipbased decoding techniques for both simulation and analytical evaluations* 

 $N_0 = 0$ ,  $Ns_1 = 2$ ,  $Ns_2 = 4$  and  $N_h = 8$ 

It is also observed from Fig. 2 that in the absence of noise, the frame-based hard decision technique performs better than the soft detection technique. The reason is that MAI always increases the correlation. Thus, the output of the frame-based correlator in the frame containing data is always equal to or greater than  $Ns_2m_p$ . As a result, a correlation value smaller than  $Thr = Ns_2m_p$  implies no data at the corresponding frame. Thus, assigning the value '0' to the frames with correlation less than the threshold mitigates the effect of MAI. However, as we will see, the above result does not hold true when the Gaussian noise dominates the interference.

Fig. 3 compares the performance of our method using soft and thresholded hard decoding techniques with that of the uncoded scheme in [2] and the coded scheme in [3] using soft decoding. It is observed from this figure that the proposed method significantly outperforms the uncoded system. It also shows better performance than the coded scheme in [3], without requiring any extra bandwidth or considerable complexity. Note that in this comparison the parameters of the new method and those of the uncoded and coded schemes of [2] and [3] are chosen so that all schemes have the same bandwidth, bit rate, energy per bit and coding gain. We also note that with these identical parameters the processing gain



**Fig. 7** BER of the soft decoding technique against the number of users for both analytical and simulation evaluations  $N_0 = 0$ ,  $N_{S_1} = 2$ ,  $N_{S_2} = 8$  and  $N_h = 8$ 

of the proposed method is slightly higher. (As stated in Section 1, since the proposed method does not use BPPM, for the same bandwidth and bit rate, its processing gain can be selected higher than that of [3].)

Fig. 4 indicates that, in the presence of noise, the performances of the threshold-based technique are very sensitive to the threshold value. It can be shown that the optimum values of the normalised thresholds  $\lambda_{opt}$  and  $\mu_{opt}$  (see (28), (37)–(40) and (50)) are a function of the number of users and SNR. The results indicates that, as expected, the optimum normalised threshold is about 0.5 for a small number of users and low SNRs, while it is 1.0 for a large number of users or high SNRs.

In Fig. 5, we have demonstrated the performance of different decoding techniques against SNR (the optimum threshold is used for the threshold-based techniques). It can be observed that the soft decoding technique performs better in low SNRs, where the noise is dominant, while the chip-based hard decoding technique has a better performance for rather high SNRs, where the MAI is dominant.

We have also simulated the proposed techniques to verify the analytical results. Fig. 6 demonstrates the results of simulation for the thresholded hard decoding and chip-based hard decoding techniques in a multiple access environment. It is observed that the simulation results confirm the analyses derived in the previous sections. However, Fig. 7 shows that the Chernoff bound is not tight enough to match the results of simulation in the soft decoding technique. To obtain a tighter bound for the BER in this case, we used the Beaulieu series [16]. The other analytical results were also verified by the simulation results.

# 7 Conclusions

We proposed a new scheme for a TH-UWB system which outperforms the previous scheme significantly. We considered four decoding techniques at the receiver front end. Then, we used the Viterbi algorithm to decode the underlying convolutional code. We obtained the upper bound on the BER using the Chernoff bound, and the pathgenerating function of the super-orthogonal code. For the soft decoding technique, we also provided analytical results using the Beaulieu series. It was observed that for high SNRs, where the MAI is dominant, the chip-based hard decoding has a better performance than the other decoding techniques. While for low SNRs, in which the noise is dominant, the soft decoding works better. We also showed that the proposed method also outperforms the previously introduced coded scheme. The analytical results were verified by the simulations.

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