ELEC 344

Applied Electronics and Electromechanics

Fall 2016

Instructor: Dr. Hamid Atighechi
My Webpage: www.ece.ubc.ca/~hamida

Class Webpage:
TBD

NOTE: Class notes are originally prepared by Dr. Juri Jatskevich

Major Topics Covered

- Principles of electromagnetics, inductance and reluctance
- Magnetic circuits & magnetically coupled systems
- Linear and rotating electromechanical devices
- Electromechanical energy conversion, developed forces and torques
- AC power, three-phase system, connections and applications
- Rotating magnetic field, poly-phase systems
- Induction motor, operation, equivalent circuit
- Synchronous motor, operation, steady-state equivalent circuit
- Brushless dc motors, operation, steady-state characteristics
- Stepper motors, principle of operation, full-step, micro-stepping, driver circuits
- Single phase AC motors

Module 1, Part 1

Introduction & Magnetic Circuits
(Read Chapter 1)

Most Important Topics

- Applications of Electromechanics
- Fundamentals of Electromagnetics, Maxwell's Equations
- Sign & direction conventions
- Basic magnetic circuits, concepts, analogies, calculations
- Flux, flux linkage, inductance
- Magnetic materials, saturation, hysteresis loop
- Coil under ac excitation, type of core losses

Applications of Electromechanics

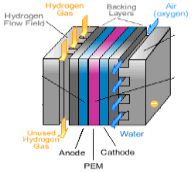
Production of Electric Energy & Modern Electric Grid

Hydro

PV Solar

Wind Energy

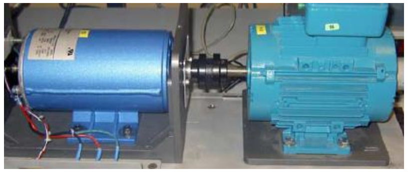
Battery Storage

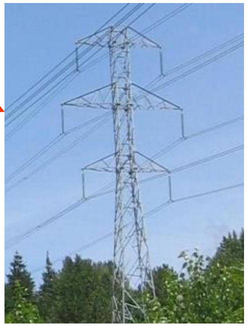


Fuel Cells

HVDC

Industrial Loads
Motor Drives





Transmission System

Applications of Electromechanics

Mining Industry

Electric Cable Shovel

Electric Dragline

AC and DC
Electric Drives

Applications of Electromechanics

Heating and Melting

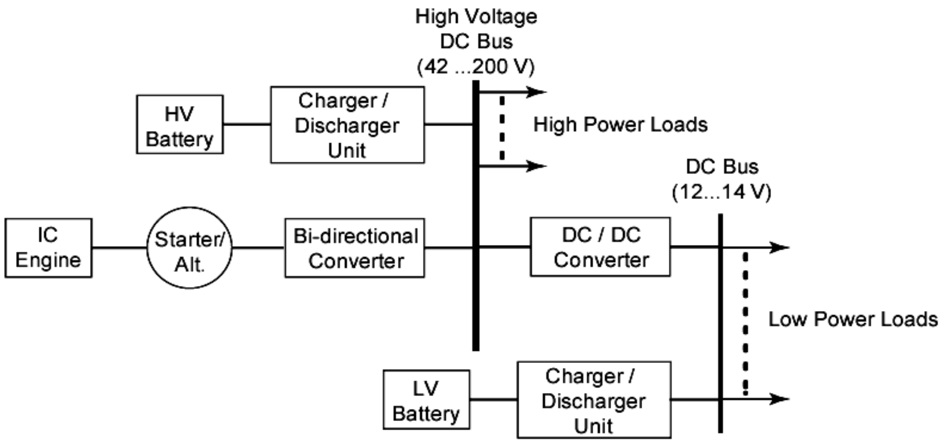
Induction Furnace

Induction Heater

No Flame Heater for
Dental Instrument

Applications of Electromechanics

Modern Transportation



Diesel-Electric

Liebherr T282B earth-hauling truck 2.7MW AC Propulsion

Hybrid

Toyota Hybrid, operates at 288V, reaches 30kW

All-Electric => Zero Emission Transportation

Canada Line (Richmond-Airport-Vancouver Line) SNC-Lavalin & Rotem Company

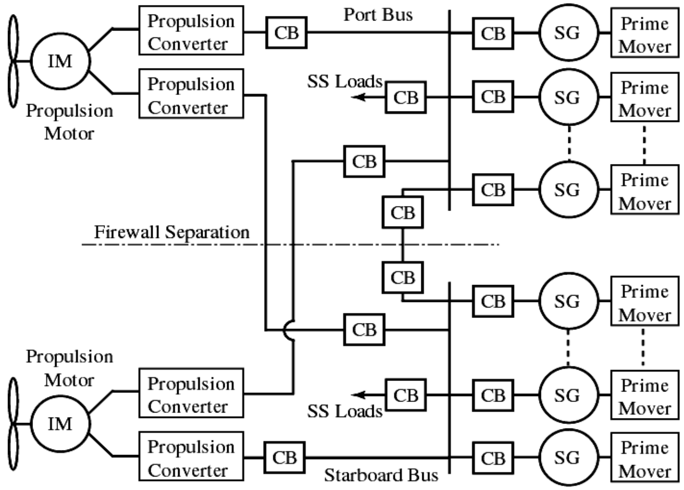
Vancouver TransLink Trolley Bus New Flyer Industries

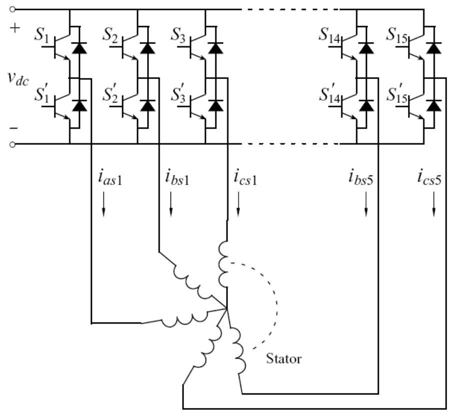
All-Electric

Tesla Roadster, Induction Motor, reaches 200 kW

Applications of Electromechanics

Modern Ships





High-Phase Count Motor Drives

20MW, 15-Phase Induction Motor

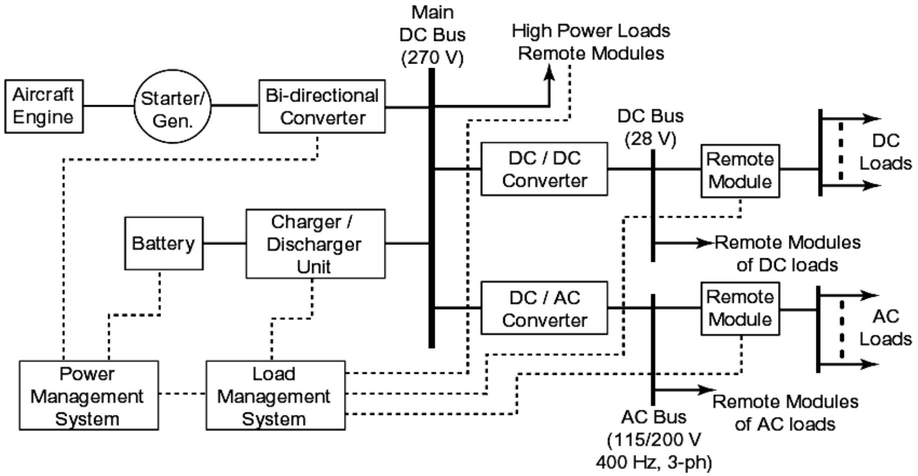
Future Canadian Ship: Joint Support Ship (JSS) 30.4MW Electric Propulsion

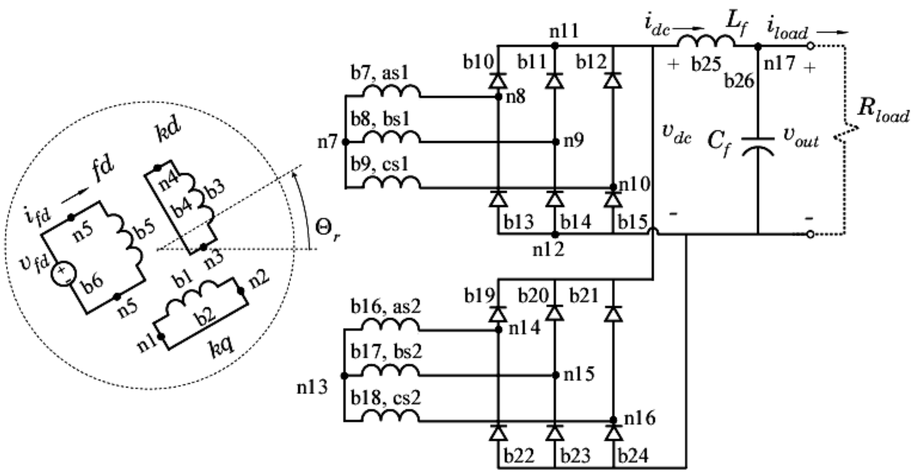
HMCS Windsor Diesel-Electric, 2x5MW motor-generators

Carnival Liberty Cruise Ship 2 x 20MW Electric Propulsion

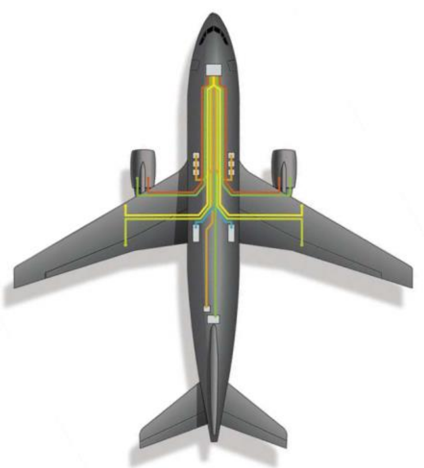
Applications of Electromechanics

Modern Aircraft





High-Speed, Low-Weight, High-Phase-Count Motors, Generators and Converters



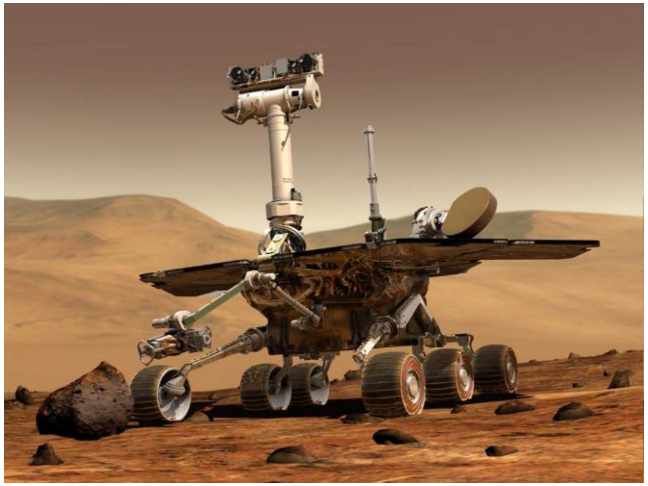
Airbus A380

Antares 20E
42kW BLDC Propulsion

Electric Toys
BLDC Propulsion

Applications of Electromechanics

High & Low



BC hydro
POWER SMART

Electromechanical Devices

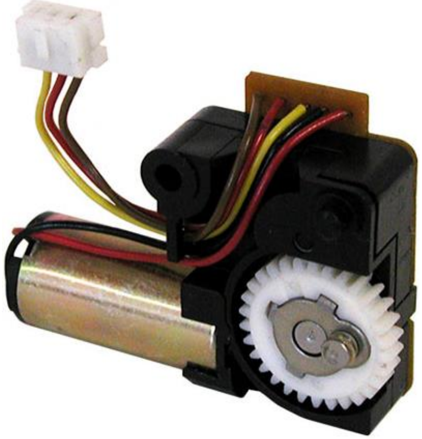
Industrial
Manufacturing
Automotive
Aircraft
Ships
Computers
Office
Household
…



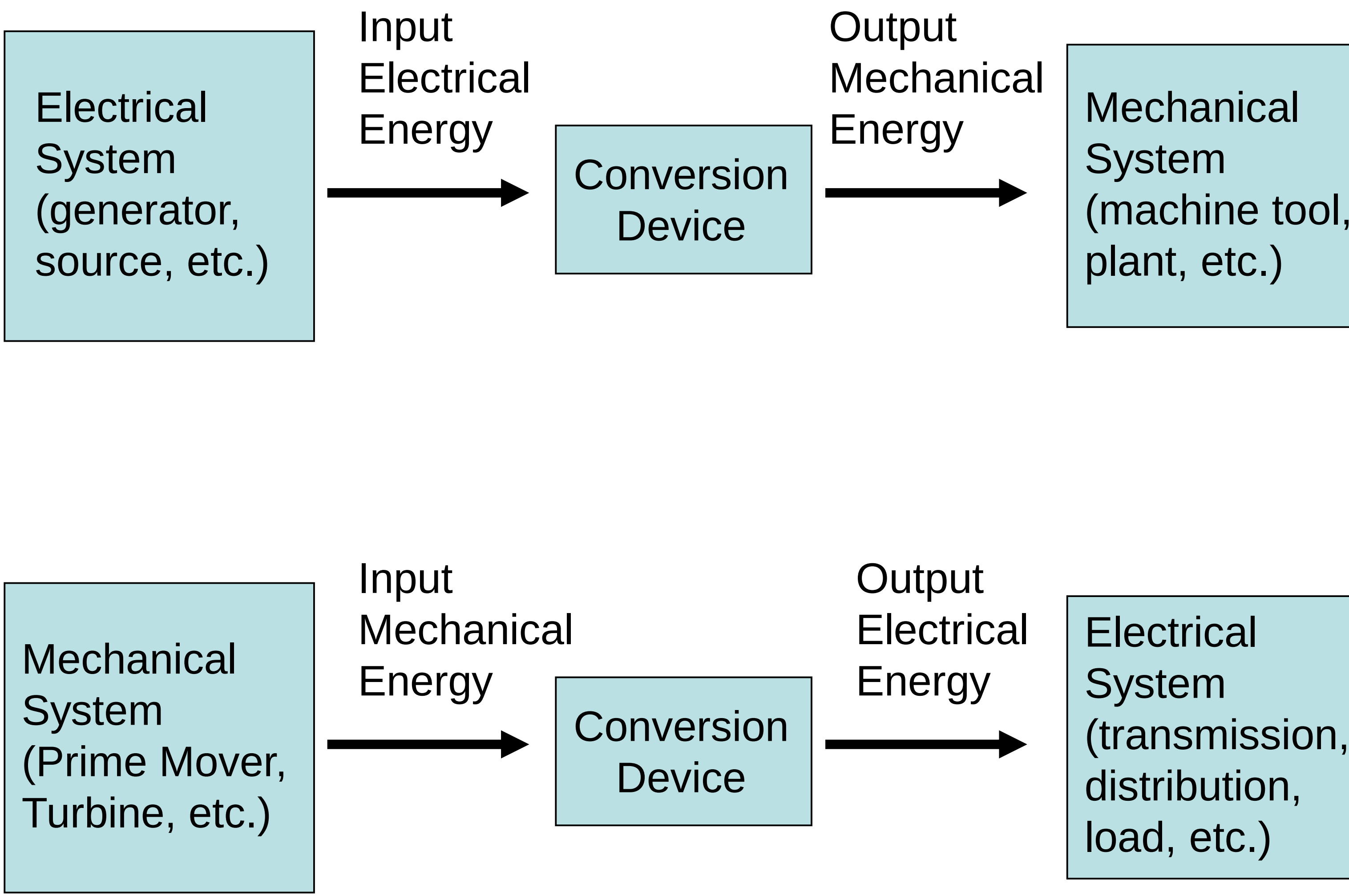






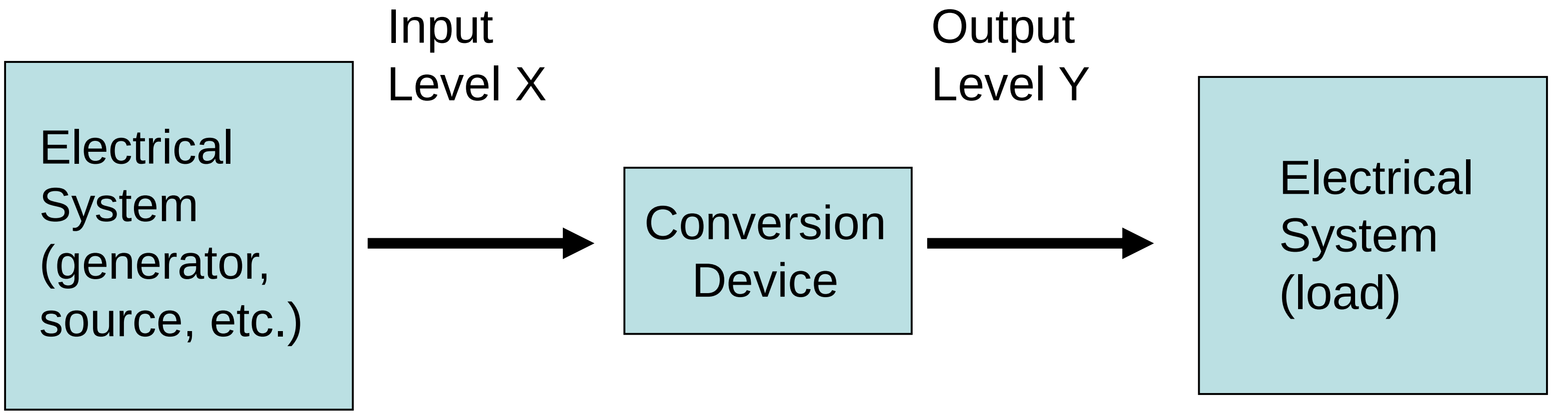


Electromechanical Energy Conversion



Electromechanical Energy Conversion

- Transformation of Electrical Energy



Electromechanical Energy Conversion

- Electrical Machines
  - Stationary
    - Transformers
  - Rotating
    - Motors, generators
  - Linear Devices
    - Solenoids, linear motors, other actuators

Conversion Device

- Power Electronics (Switched Mode PSs, Motor & Actuator Drivers, …)
  - Rectifiers
    - AC to DC
  - Converters
    - DC to DC
  - Inverters
    - DC to AC

Very broad & interesting area, requires its own course!

Magnetic Circuits: Basic Units

E – electric field intensity $\left[\frac{V}{m}\right]$

B – magnetic flux density $\left[Tesla = \frac{Weber}{meter^2}\right]$ $\left[T = \frac{Wb}{m^2}\right]$

H – magnetic field intensity $\left[\frac{A}{m}\right]$

Φ – magnetic flux $\left[Wb = T \cdot m^2\right]$

B-H Relation

- Current produces the H field (see Ampere's law)
- H is related to B

$$B = \mu H = \mu_0 \mu_r H$$

μ – permeability (characteristic of the medium) $\left[\frac{T \cdot m}{A} = \frac{Henry}{meter} = \frac{H}{m}\right]$

μ_0 – permeability of vacuum $= 4 \cdot \pi \cdot 10^{-7} [H/m]$

μ_r – relative permeability of material

magnetic materials $\mu_r = 100 \cdots 100{,}000$

Fundamentals

- **Maxwell's equations** are a set of partial differential equations that, together with the Lorentz force law, form the foundation of classical electrodynamics, classical optics, and electric circuits [Wikipedia].

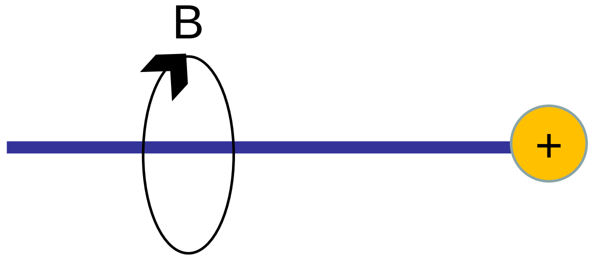
Fundamentals

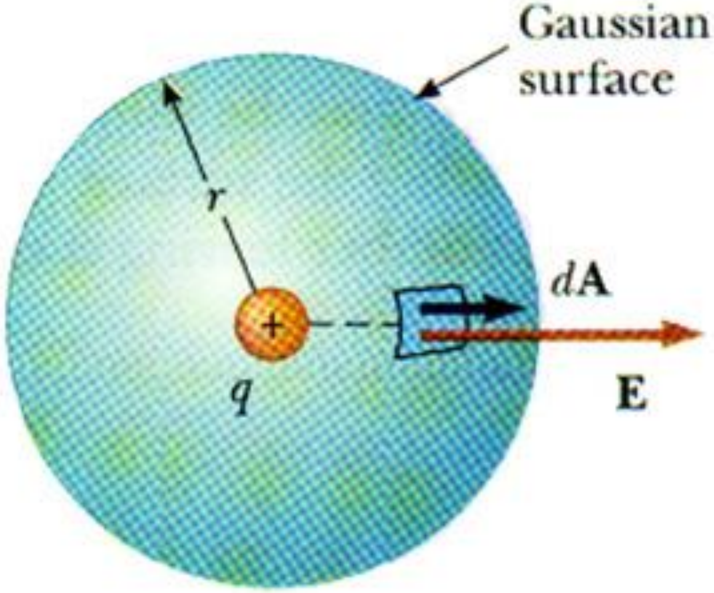
Summarized in Maxwell's Equations (1870s)

1) Gauss's Law for Electric Field

$$\oint_s \mathbf{E}\cdot d\mathbf{a} = \frac{q}{\varepsilon_0} = \Phi_e = \int E\cos\theta da$$

Electric flux out of any closed surface is proportional to the total charge enclosed





Fundamentals

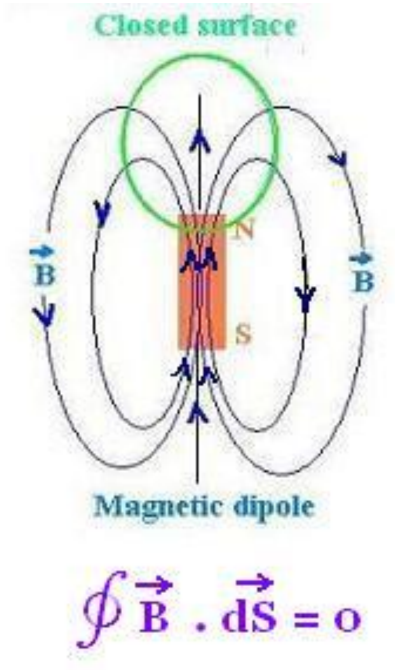
Summarized in Maxwell’s Equations (1870s)

2) Gauss’s Law for Magnetic Field

$$\oint_s \mathbf{B} \cdot d\mathbf{a} = \Phi_m = 0$$

Magnetic flux out of any closed surface is zero

There are no magnetic charges



Fundamentals

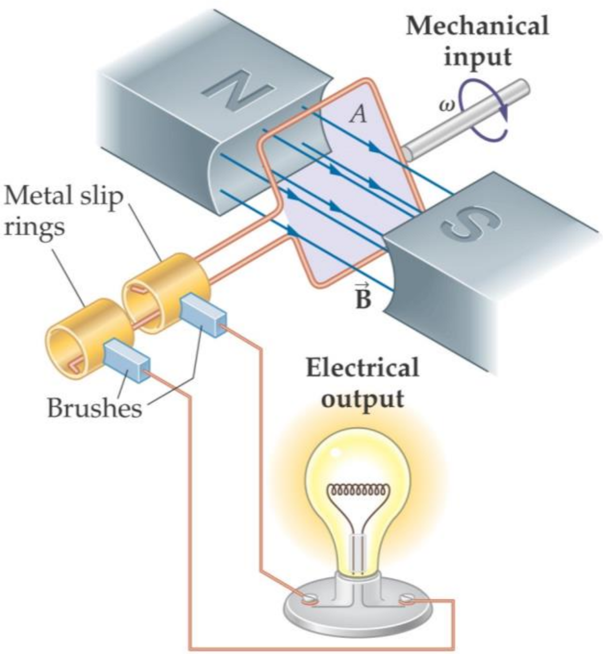
Summarized in Maxwell's Equations (1870s)

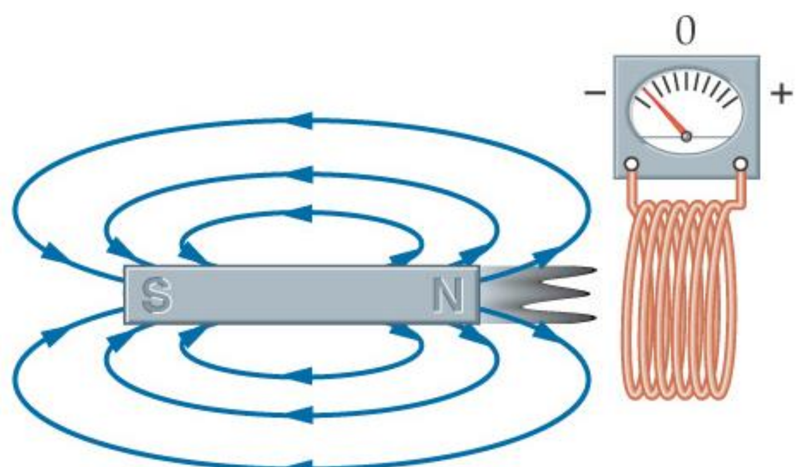
3) Faraday's Law

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{a} = -\frac{d\Phi}{dt} = emf$$

Electro**M**otive **F**orce (emf)

The line integral of the electric field around a closed loop/contour C is equal to the negative of the rate of change of the magnetic flux through that loop/contour



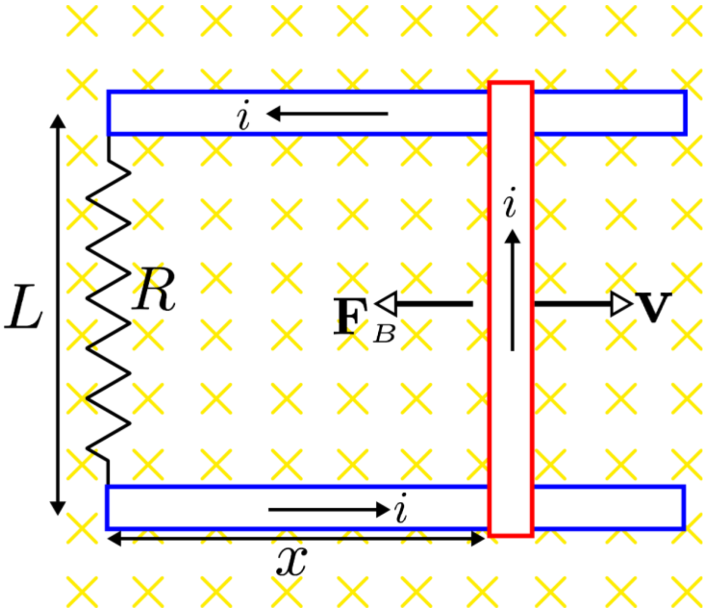


Fundamentals

Summarized in Maxwell's Equations (1870s)

$$\int_s \mathbf{B} da = \Phi_m \quad [Wb]$$

- Induced emf:



$$emf = -\frac{d\Phi_m}{dt} \quad [V]$$

- Lenz's Law:
  The direction of the voltage induced will produce a current that opposes the original magnetic field. This gives the negative sign, which we do not always include

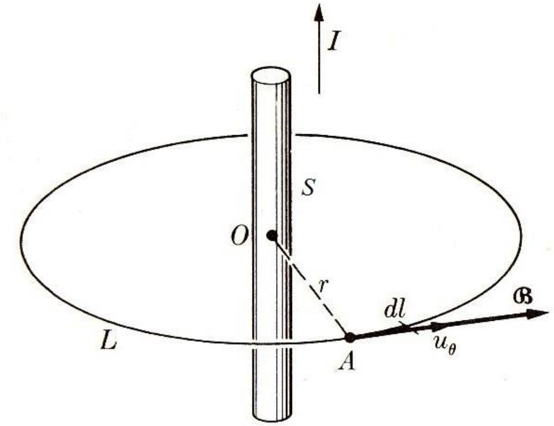
Fundamentals

Summarized in Maxwell's Equations (1870s)

4) Ampere's Law (for static electric field)

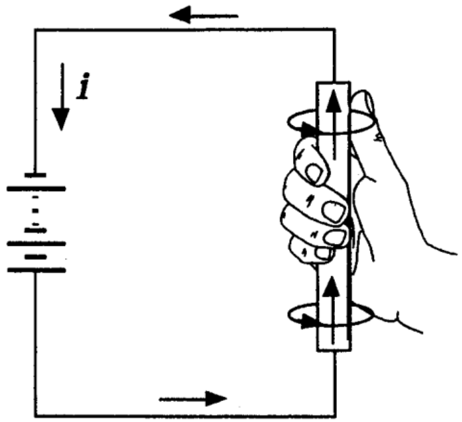
$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 \int_S J \cdot da = \mu_0 I_{net}$$

The line integral of the magnetic field B around a closed loop C is proportional to the net electric current flowing through that loop/contour C

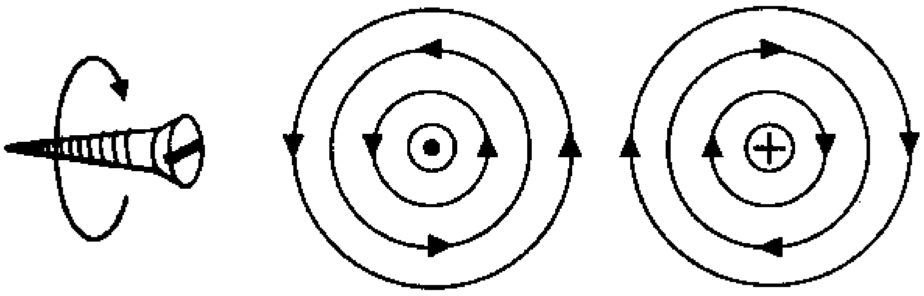


Conventions

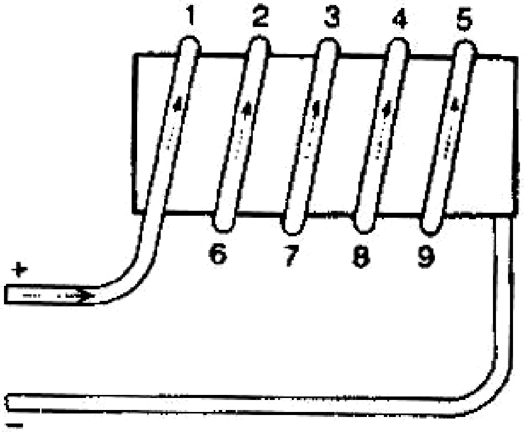
Right hand rule

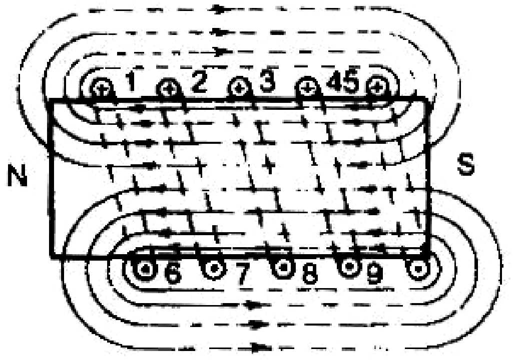


Right-screw rule
Dot and cross notations



Magnetic field produced by coil (solenoid)





Flux Lines:

- form a closed loop/path
- Lines do not cut across or merge
- Go from North to South magnetic poles

Some Definitions

Magnetic Flux

$$\Phi = \int_S \mathbf{B} \cdot \mathbf{da} = B_c A_c$$

Flux is always continuous

Recall Faraday's Law - **E**lectro**m**otive **F**orce (emf)

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a} = -\frac{d\Phi}{dt}$$

- voltage induced in one turn due to the changing magnetic flux

For coil with *N* turns: $e = N \cdot \frac{d\Phi}{dt}$

Flux Linkage $\lambda = N \cdot \Phi \quad [Wb \cdot t]$

flux scaled by the number of turns

Total induced emf $e = \frac{d\lambda}{dt} \quad [V]$

Some Definitions

Inductance

Need a function that relates Flux Linkage to the Current

Consider $\lambda = f(i) = L(\cdot) \cdot i$ $\quad L = \dfrac{\lambda}{i} \quad \left[\dfrac{Wb \cdot t}{A} = H \right]$

Then

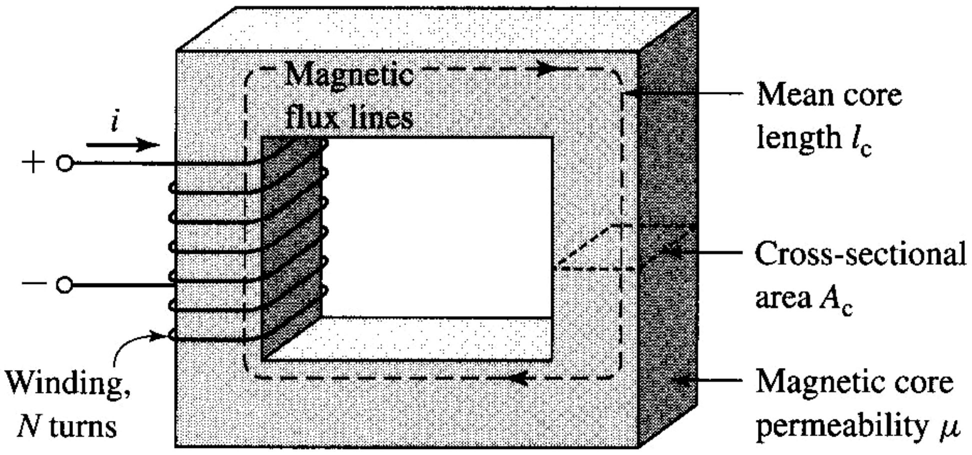
$$L = \frac{\lambda}{i} = \frac{N \cdot \Phi}{i} = \frac{N \cdot B \cdot A}{i} = \frac{N \cdot \mu \cdot H \cdot A}{i}$$

$$i = \frac{Hl}{N}$$

$$L = \frac{N^2}{l / \mu A}$$

Magnetic Circuits

- Basic magnetic circuit



Assume all magnetic field is continuous and confined inside the core

$$\Phi = \int_S \mathbf{B} \cdot \mathbf{da} = B_c A_c \qquad [Wb]$$

What does it remind you of?

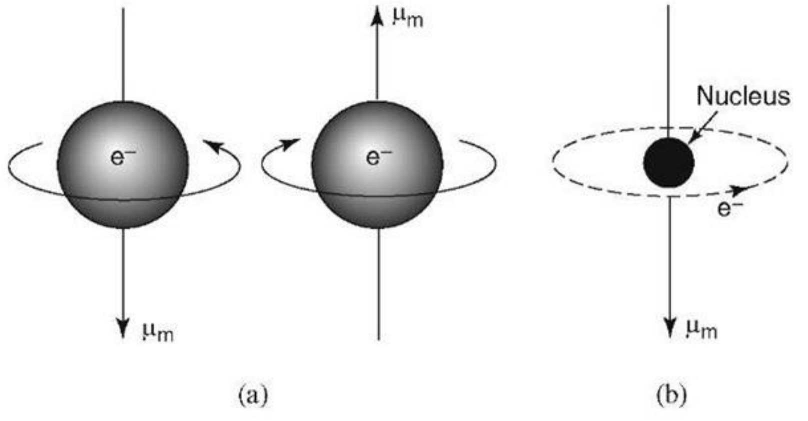
Consider mmf $F = Ni = H_c l_c = \frac{B_c l_c}{\mu} = \Phi \frac{l_c}{\mu A_c} = \Phi \Re_c$

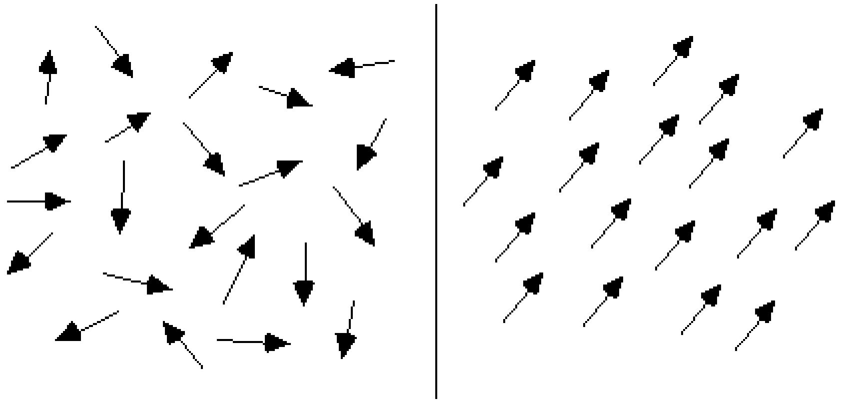
Define Reluctance (of the given magnetic path) $\Re_c = \frac{l_c}{\mu A_c} \qquad \left[\frac{A}{Wb}\right]$

$$L = \frac{N^2}{l/\mu A} = \frac{N^2}{R_C}$$

Ferromagnetism

- Magnetic moment:
  - ✓ Orbital motion of electrons
  - ✓ Spin of an electron
- In most of the materials, the net magnetic moment of one atom <u>if exists</u> is cancelled out by the other atom
- The five ferromagnetic elements are:
  - ✓ Iron, Nicle, Cobalt, Dysprosum, and Gadolinium





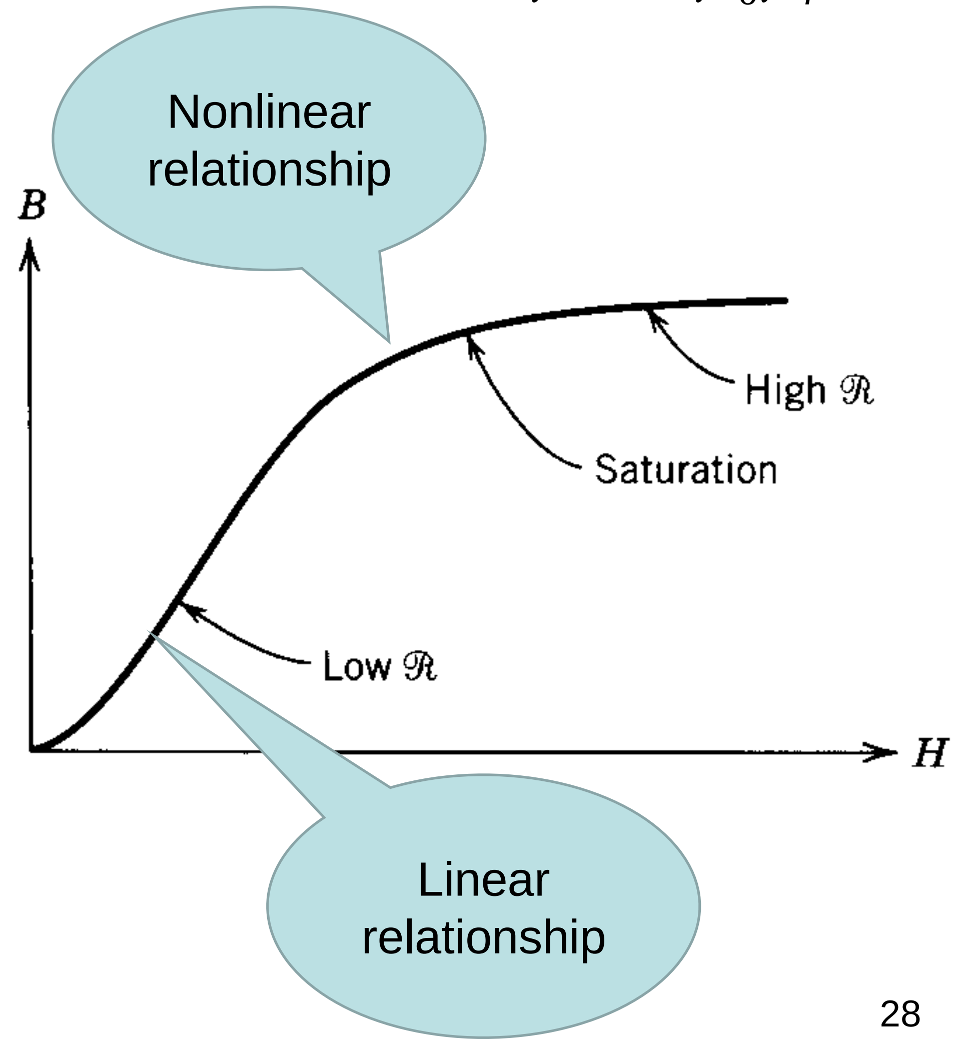
Classes of Magnetic Materials

- Diamagnetism
- Paramagnetism
- Ferromagnetism
- Ferrimagnetism
- Antiferromagnetism

Magnetization Curve

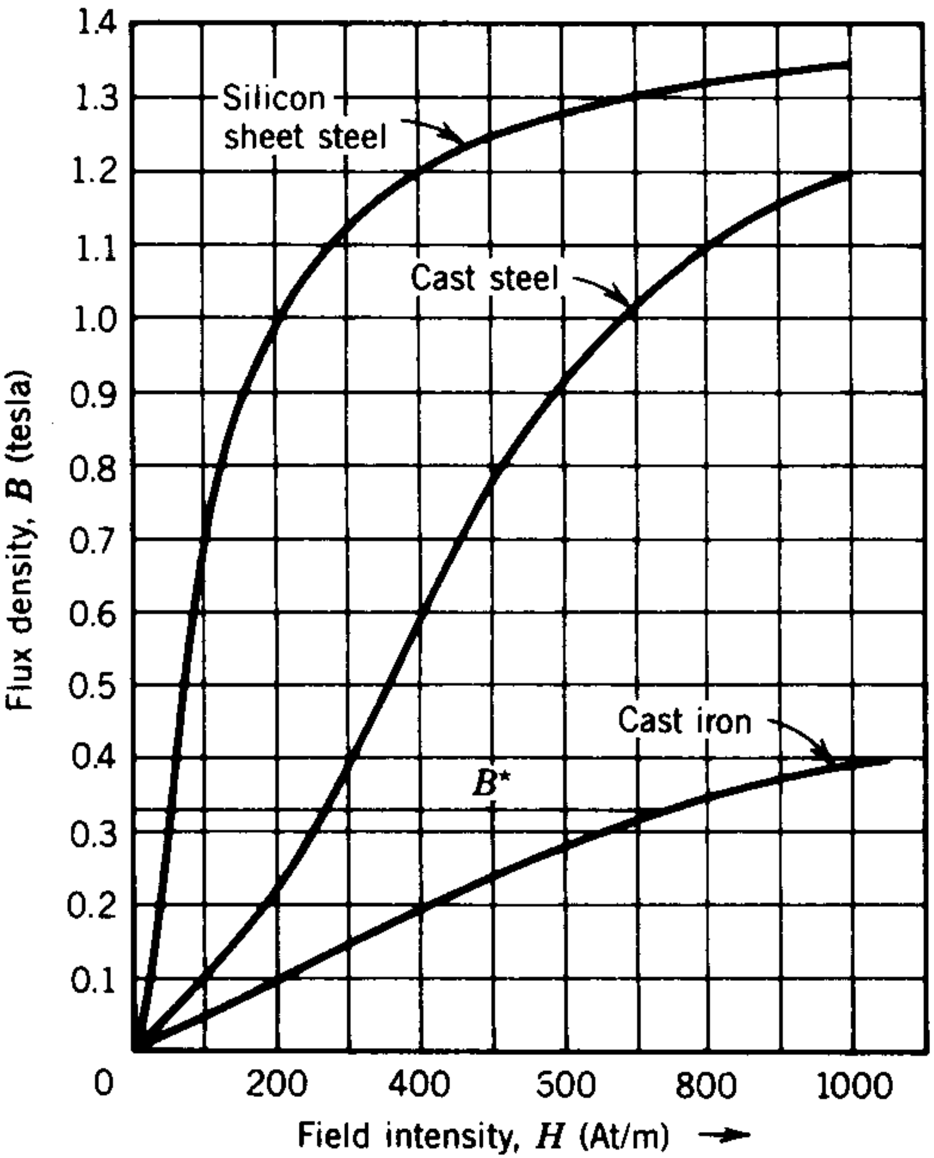
$$B = \mu H = \mu_0 \mu_r H$$

- The magnetic material shows the effect of saturation
- The reluctance is dependent on the flux density



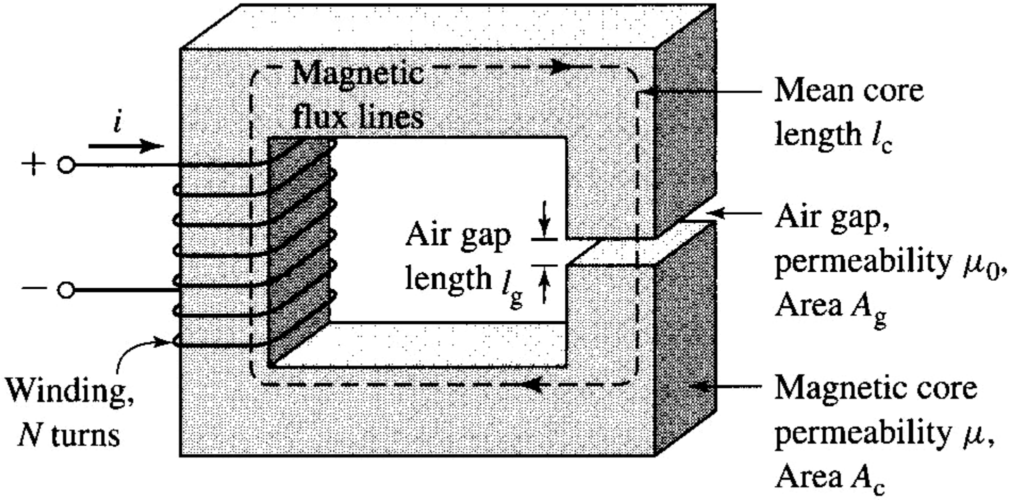
Magnetization Curve

- Depending on the applications, material with specific magnetization curve is selected



Magnetic Circuits

- Magnetic circuit with air gap



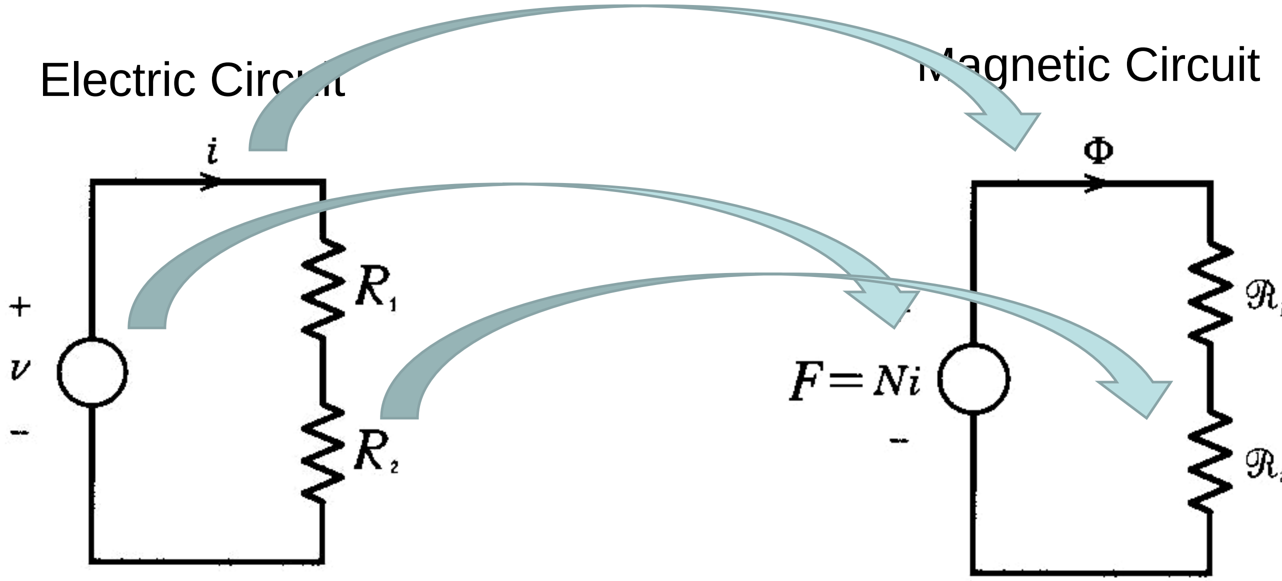
Consider mmf

$$F = Ni = \oint_C \mathbf{H} \cdot \mathbf{dl}$$
$$= H_c l_c + H_g l_g$$
$$= \frac{B_c l_c}{\mu} + \frac{B_g l_g}{\mu_0}$$

Assuming all magnetic flux is confined inside the core $B_c = \frac{\Phi}{A_c}$ and $B_g = \frac{\Phi}{A_g}$

$$F = \Phi\left(\frac{l_c}{\mu A_c} + \frac{l_g}{\mu_0 A_g}\right) = \Phi(\Re_c + \Re_g) = \Phi \sum \Re_i = \Phi \Re_{total}$$

Magnetic and Electric Circuits Analogy



$$i = \frac{v}{R_1 + R_2}$$

$$\Phi = \frac{F}{\Re_1 + \Re_2}$$

Magnetic and Electric Circuits Analogy

Electric Circuit

- Voltage (emf), $V, [Volt]$
- Current, $I, [Amps]$
- Resistance, $R = \frac{l}{\sigma A}, [\Omega]$
- Conductance, $G = \frac{1}{R}, [Siemens]$
- Conductivity, $\sigma, \left[\frac{Siemens}{m}\right]$

For loop $v = \sum R_n i_n$

For node $\sum_N i_n = 0$

Magnetic Circuit

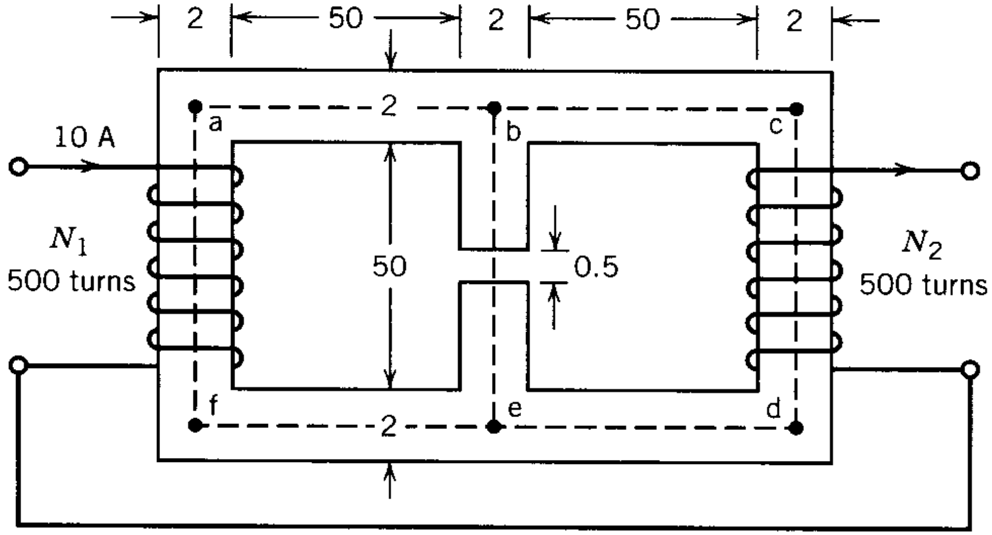
- mmf, $F, [A \cdot t]$
- Flux $\Phi, [Wb]$
- Reluctance, $\Re = \frac{l}{\mu A}, \left[\frac{A}{Wb}\right]$
- Permeance, $\rho = \frac{1}{\Re}, \left[\frac{Wb}{A}\right]$
- Permeability, $\mu, \left[\frac{H}{m}\right]$

For loop $F = \sum H_n l_n$

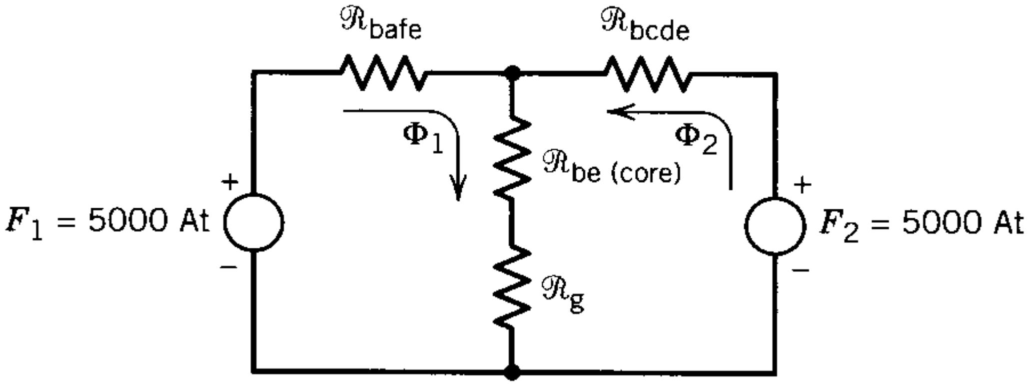
For node $\sum_N \Phi_n = 0$

Inductance: Example

Consider the following electromagnetic system (device)



Equivalent Electric Circuit



$$\mathcal{R}_{bcde} = \mathcal{R}_{bafe}$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g}$$

$$= \frac{5 \times 10^{-3}}{4\pi 10^{-7} \times 2 \times 2 \times 10^{-4}}$$

$$= 9.94 \times 10^6 \text{ At/Wb}$$

$$\mathcal{R}_{be(core)} = \frac{l_{be(core)}}{\mu_c A_c}$$

$$= \frac{51.5 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}}$$

$$= 0.82 \times 10^6 \text{ At/Wb}$$

$$\Phi_1(\mathcal{R}_{bafe} + \mathcal{R}_{be} + \mathcal{R}_g) + \Phi_2(\mathcal{R}_{be} + \mathcal{R}_g) = F_1$$

$$\Phi_1(\mathcal{R}_{be} + \mathcal{R}_g) + \Phi_2(\mathcal{R}_{bcde} + \mathcal{R}_{be} + \mathcal{R}_g) = F_2$$

$$\Phi_1 = \Phi_2 = 2.067 \times 10^{-4} \text{ Wb}$$

The air gap flux is

$$\Phi_g = \Phi_1 + \Phi_2 = 4.134 \times 10^{-4} \text{ Wb}$$

Inductance: Example

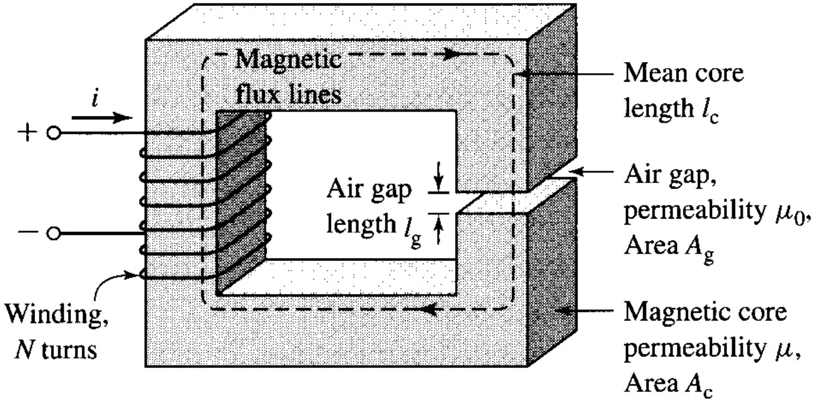
Consider the following electromagnetic system

$i = 1\,A,\ N = 400$

$l_c = 50\,cm,\ l_g = 1\,mm$

$A_c = A_g = 15\,cm^2$

$\mu_r = 3000$



Find inductance $L = \dfrac{N^2}{\mathcal{R}_c + \mathcal{R}_g}$

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{50e-2}{3000 \cdot 4\pi \cdot 1e-7 \cdot 15e-4} \approx 88.42e+3\ At/Wb$$

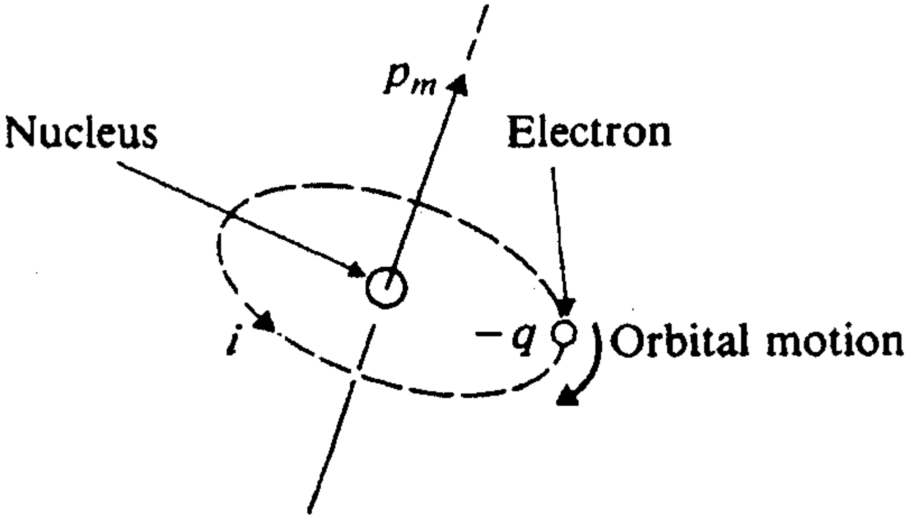
$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{1e-3}{4\pi \cdot 1e-7 \cdot 15e-4} \approx 530.515e+3\ At/Wb$$

$$L = \frac{400^2}{(88.42 + 530.515) \cdot e+3} = 258.52e-3\ H$$

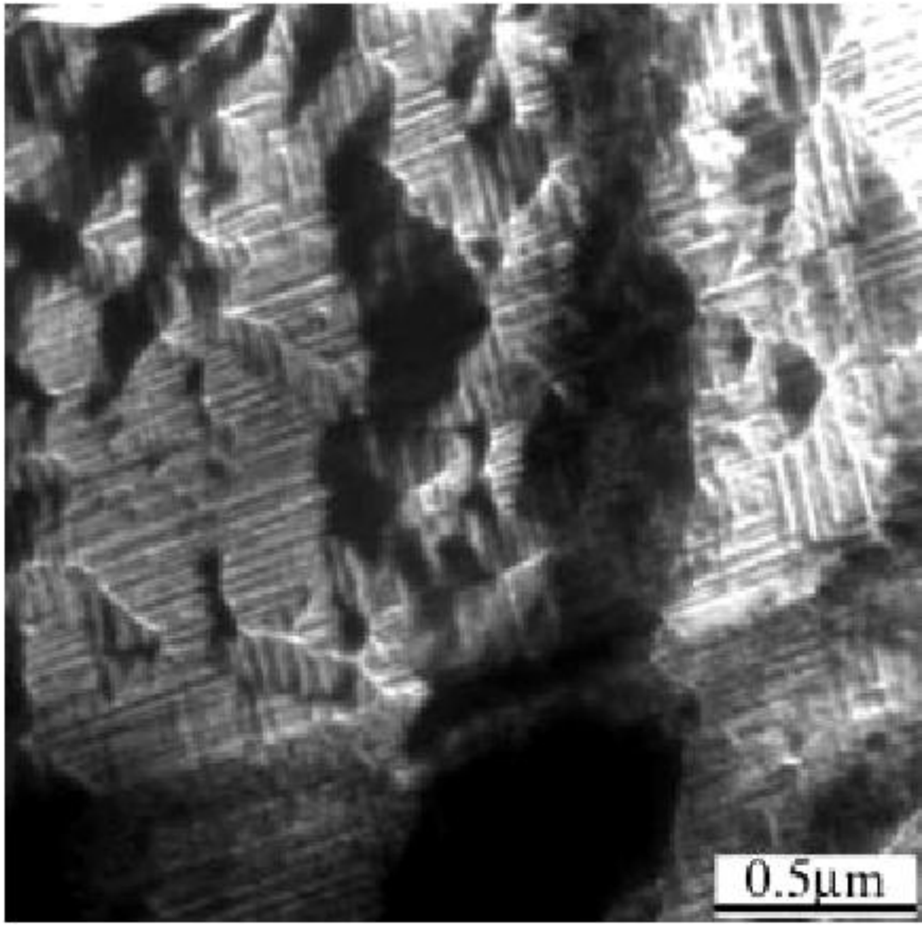
$$= 258.52\ mH$$

Magnetic Materials

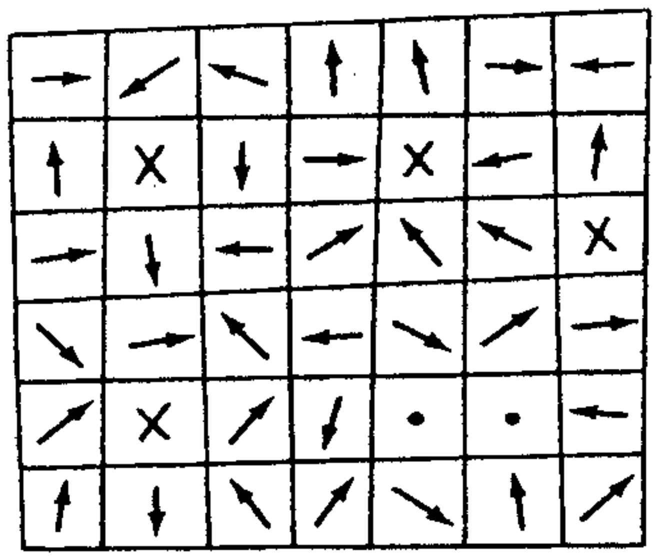
Magnetic moment
of an atom



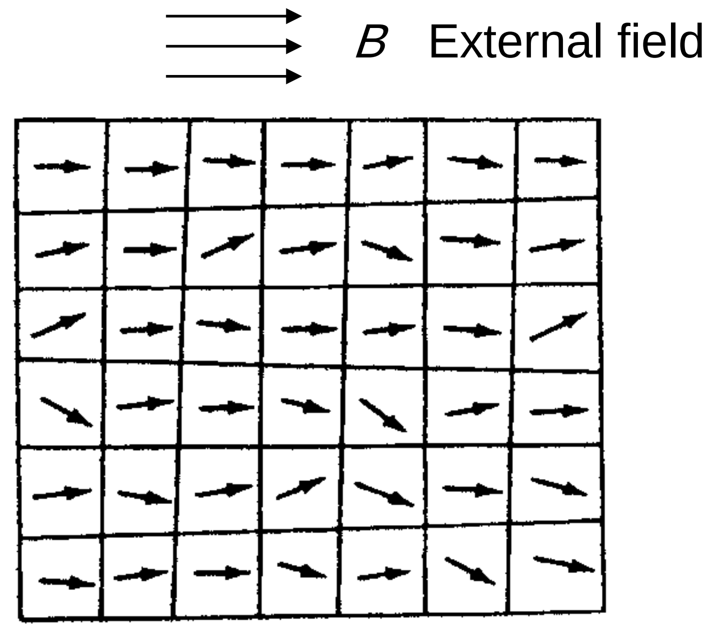
Magnetic Domain Structure



Magnetic Material Domain Model

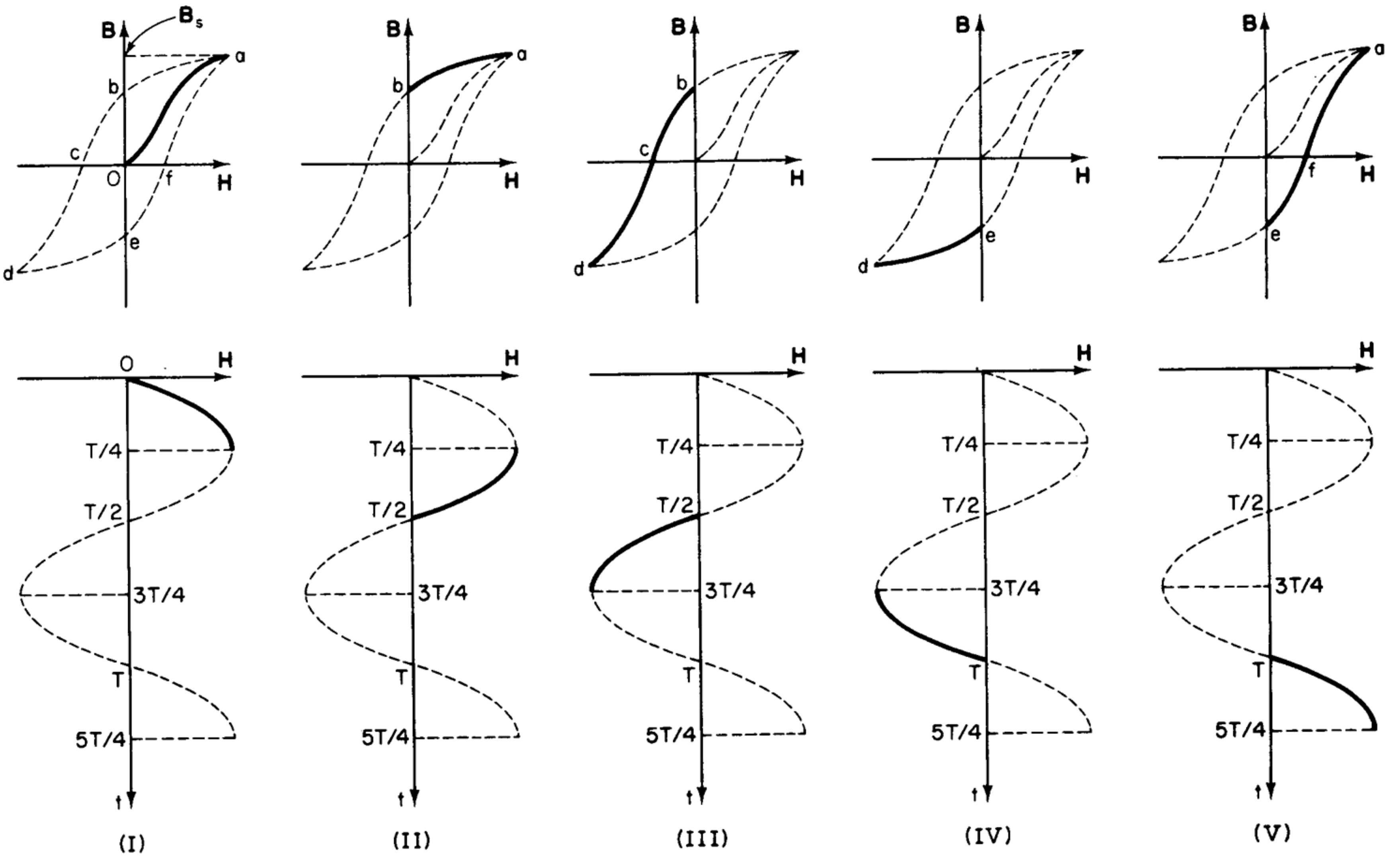


demagnetized

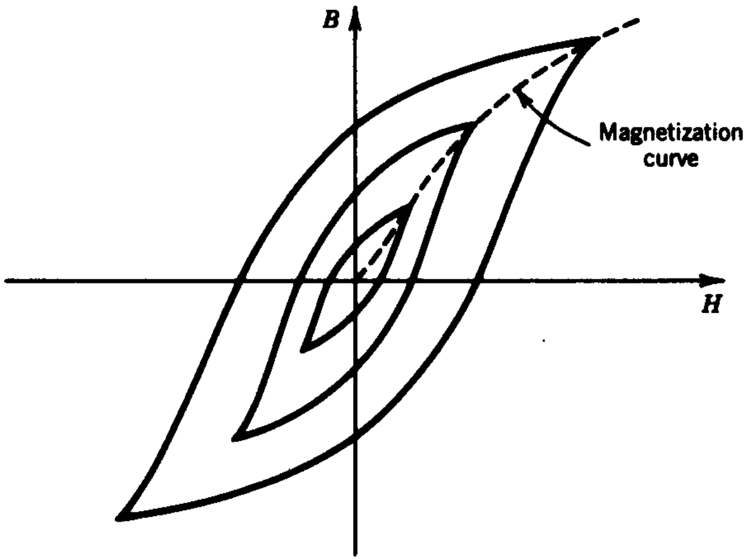


magnetized

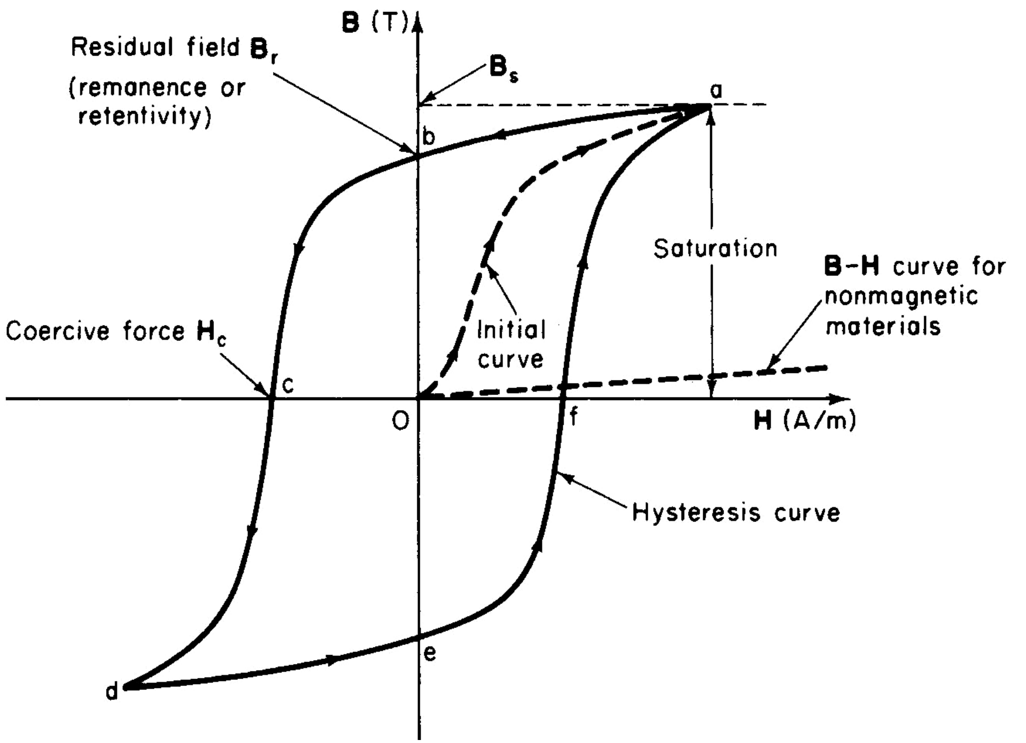
Hysteresis Loop



Hysteresis Loop



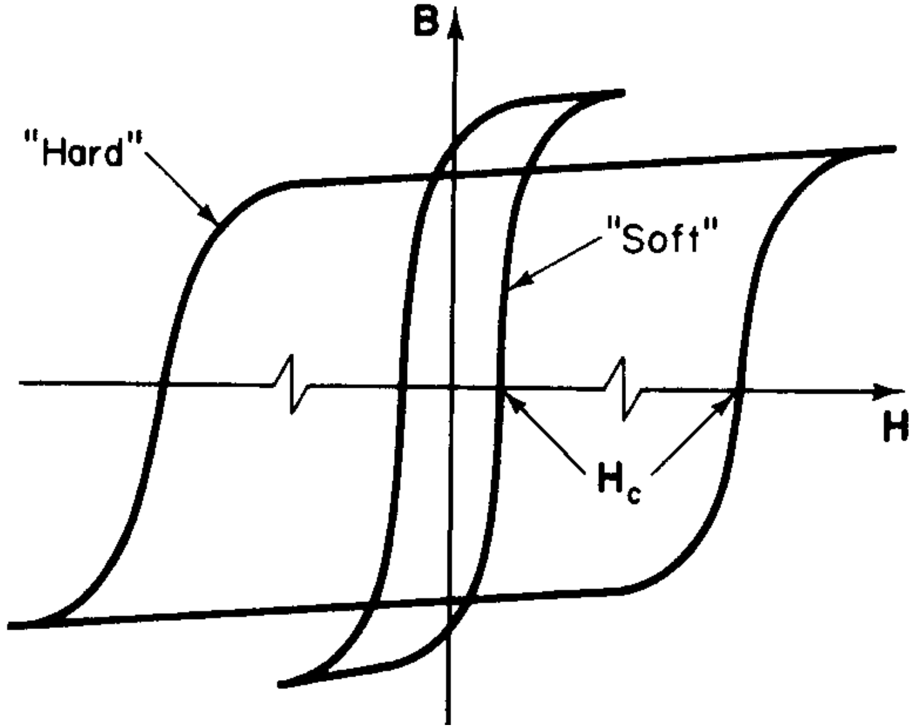
Hysteresis loops for different excitation levels



B_r – residual magnetism
H_c – coercivity force, external field required to demagnetize the material

Magnetic Materials

Classes of Magnetic Materials



Soft mag. materials

$$H_c \sim 0.1 \cdots 100 \, [A/m]$$

Hard mag. materials

$$H_c > 100 \, [A/m]$$

Permanent magnets (PM)

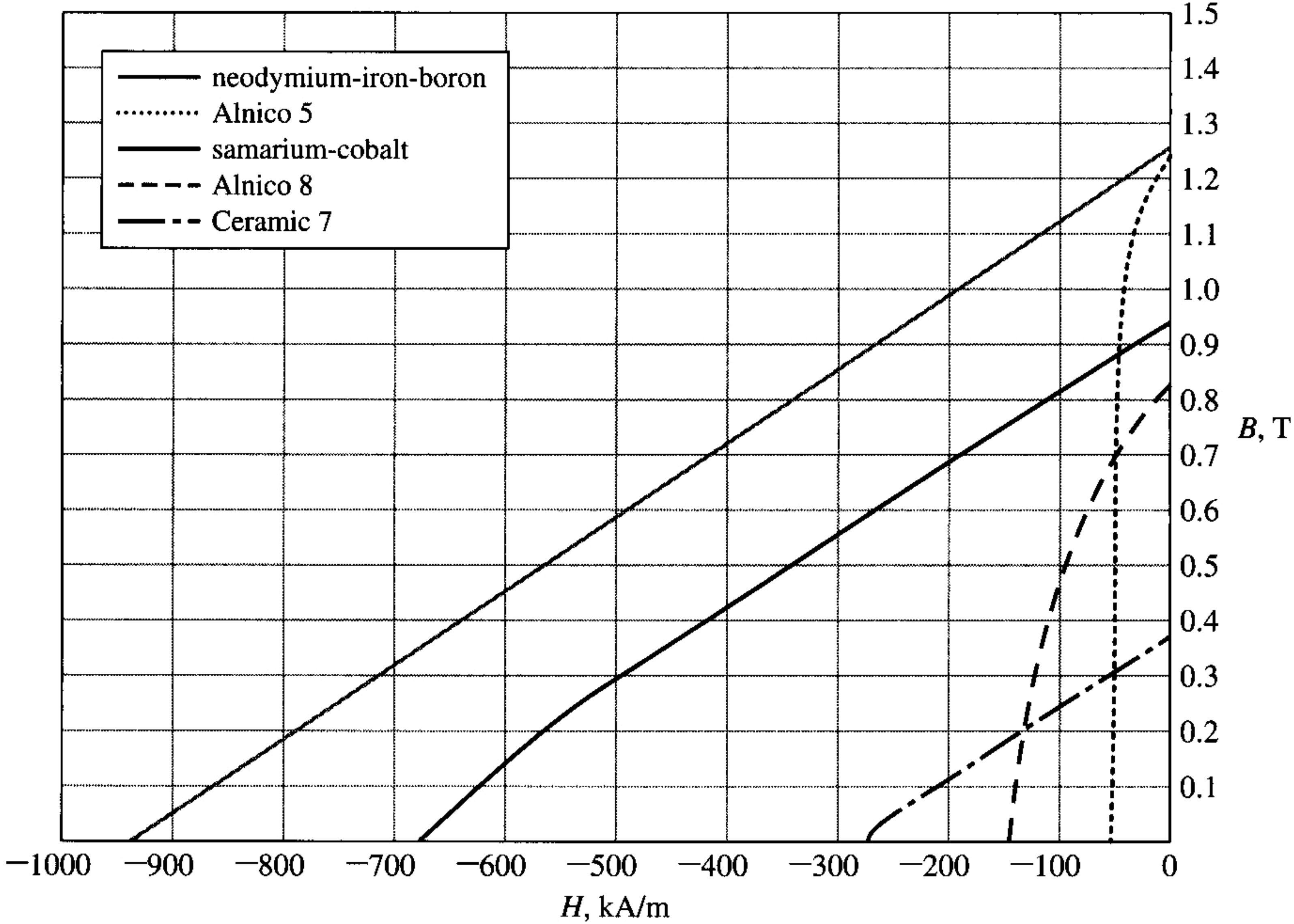
$$H_c \sim 10^4 \cdots 10^6 \, [A/m]$$

Types of PMs

- Neodymium Iron Boron (NdFeB or NIB)
- Samarium Cobalt (SmCo)
- Aluminum Nickel Cobalt (Alnico)
- Ceramic or Ferrite, very popular
  Iron-oxide, barium, etc. compressed powder

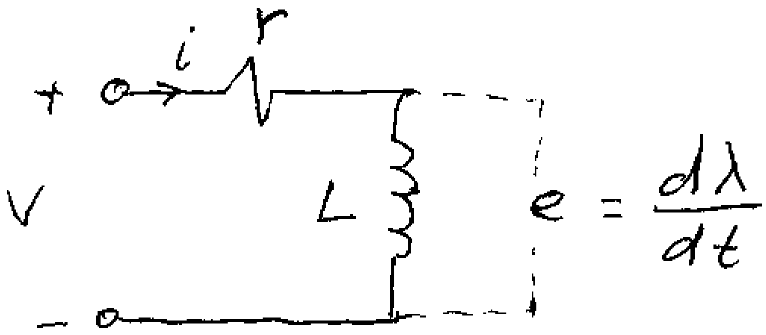
Magnetic Materials

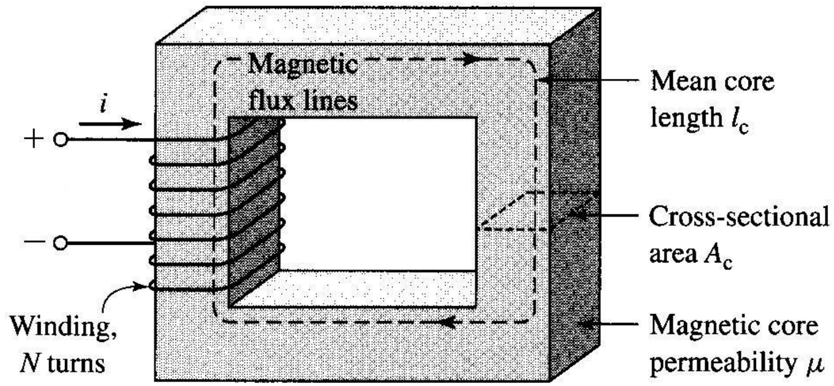
Second quadrant hysteresis curve for some common PM materials



Energy Stored in Inductor

Consider the following electromagnetic system





Instantaneous Power – rate of doing work

$$P_{in}(t) = i(t)\cdot v(t) = i^2\cdot r + i\cdot e = P_{loss} + P_f$$

Recall mmf $\quad F = Ni = H_c l_c$

Power going into the field $\quad P_f = i\cdot e \quad$ where $\quad e = \dfrac{d\lambda}{dt} \quad$ and $\quad \lambda = N\Phi = NA_cB_c$

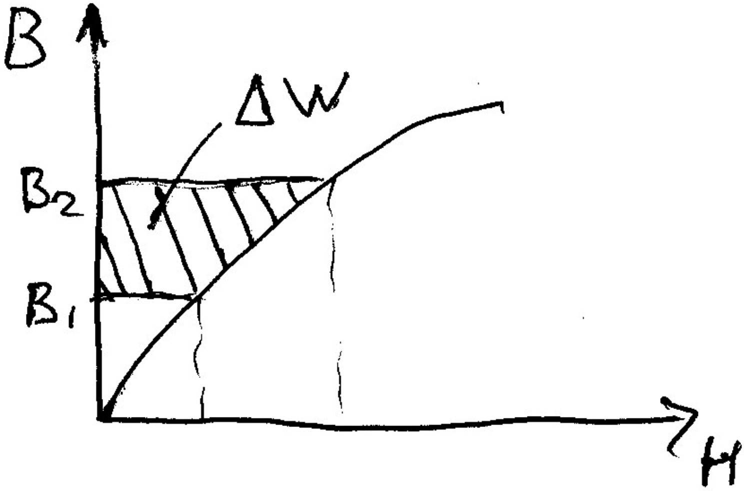
$$P_f(t) = i\cdot e = \frac{H_c l_c}{N}\cdot\frac{d}{dt}(NA_cB_c) = l_c A_c H_c \frac{dB_c}{dt}$$

Energy $\quad W_f = \int P_f\, dt$

Change of Energy $\quad \Delta W_f = \int_{t_1}^{t_2} P_f\, dt = l_c A_c \int H_c \dfrac{dB_c}{dt} dt = l_c A_c \int_{B_1}^{B_2} H_c\, dB_c$

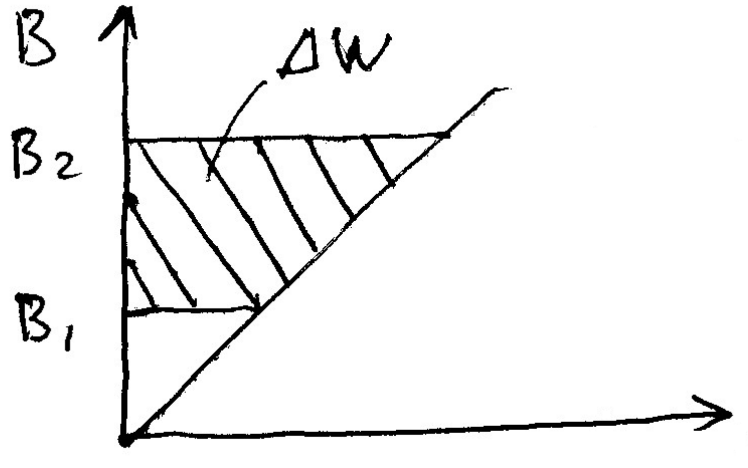
Energy Stored in Inductor

Energy per-unit-volume $\Delta w_f = \int_{B_1}^{B_2} H_c dB_c$



Magnetically Nonlinear System

$$\Delta W_f = l_c A_c \int_{B_1}^{B_2} H_c dB_c$$



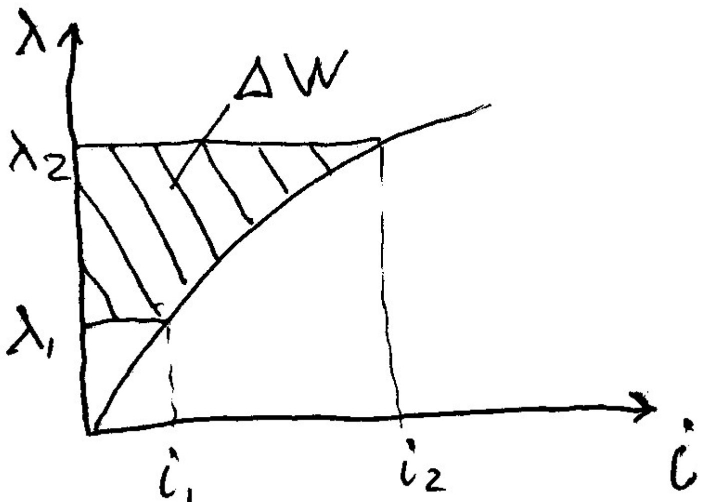
Magnetically Linear (Approximate) System

$$B = \mu H; \mu = const$$

$$\Delta W_f = \frac{l_c A_c}{\mu} \int_{B_1}^{B_2} B_c dB_c = \frac{l_c A_c}{2\mu}\left(B_2^2 - B_1^2\right)$$

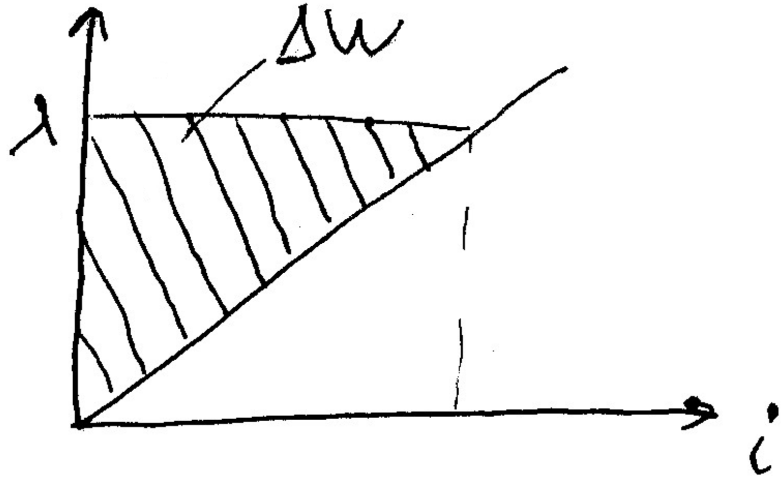
Energy Stored in Inductor

Energy in terms of flux linkage λ



Magnetically Nonlinear System

$$\Delta W_f = \int_{t_1}^{t_2} P_f \, dt = \int_{\lambda_1}^{\lambda_2} i \, d\lambda$$



Magnetically Linear (Approximate) System

$$i = \frac{\lambda}{L}; L = const$$

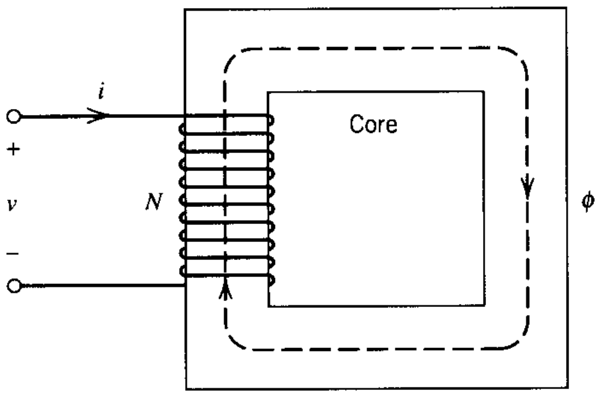
$$\Delta W_f = \frac{1}{L} \int_{\lambda_1}^{\lambda_2} \lambda \, d\lambda = \frac{1}{2L}\left(\lambda_2^2 - \lambda_1^2\right)$$

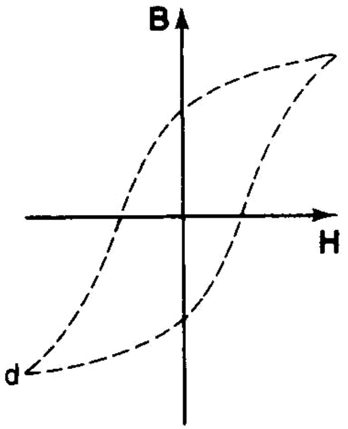
If $\lambda_1 = 0$ => $W = \frac{1}{2L}\lambda^2 = \frac{Li^2}{2}$

Core Losses

Hysteresis Losses

Consider AC excitation





$$\Delta W_{h,cycle} = \oint i d\lambda = \oint \left(\frac{H_c l_c}{N} \right) \left(N A_c dB_c \right) = l_c A_c \oint H_c dB_c$$

Power loss can be approximated as

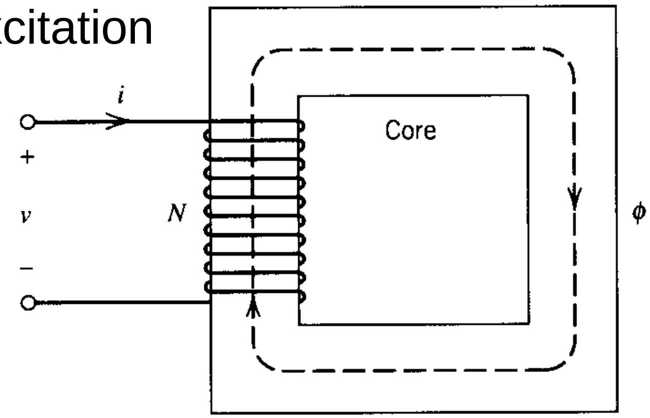
$$P_h = K_h \cdot f \cdot \left(B_{c,\max} \right)^n \qquad n \sim 1.5 \cdots 2.5$$

Where the constants K_h and n determined experimentally

Core Losses

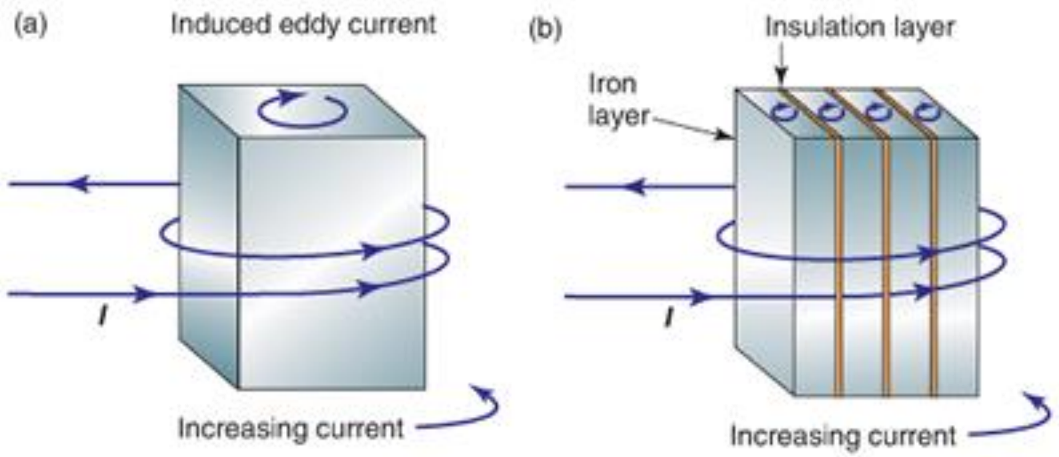
Eddy Current Losses

Consider AC excitation



Faraday's law

$$\oint_C \overline{E} \cdot dl = -\frac{d}{dt}\int_S B \cdot da$$



Power loss can be approximated as

$$P_e = K_e \cdot f^2 \cdot \left(B_{c,\max}\right)^2$$

Where the constant K_e depends on lamination thickness and is determined experimentally

EECE 365: Module 1, Part 2

Basic Electromechanical devices and Energy Conversion

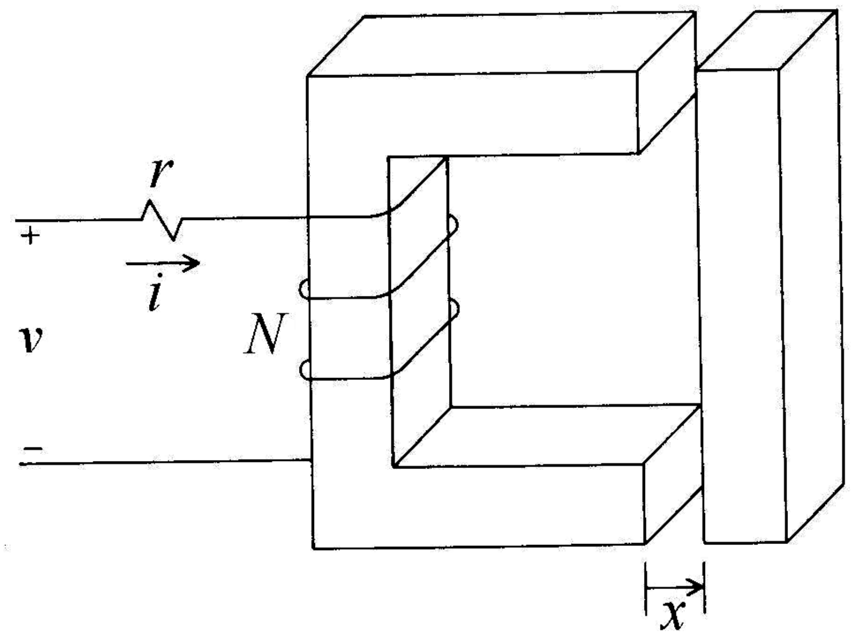
Most Important Topics and Concepts (Read Chap. 3)

- Basic linear devices with position-dependent reluctance & inductance
- Basic rotating devices with position-dependent reluctance & inductance
- Concept of coupling field
- Energy & Co-Energy
- Graphical interpretation of energy conversion
- Electromechanical force and torque

Basic Electromagnet

Voltage equation
(Faraday's law + KVL)

$$v = ri + \frac{d\lambda}{dt}$$



Flux linkage

$$\lambda = N\Phi = N\left(\Phi_m + \Phi_l\right)$$

Magnetizing flux $\Phi_m = Ni/\Re_m$

Flux leakage $\Phi_l = Ni/\Re_l$

Flux linkage & inductances

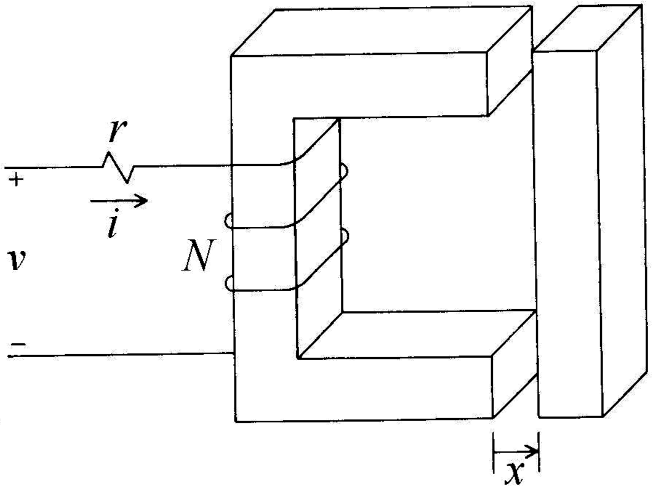
$$\lambda = \left(\frac{N^2}{\Re_l} + \frac{N^2}{\Re_m}\right) i = \left(L_l + L_m\right) i$$

L_l - Leakage inductance (assume constant)

L_m - Magnetizing inductance (depends of position x)

Consider Magnetizing Path $\Re_m = \Re_c + 2\Re_g$

Basic Electromagnet



Consider Magnetizing Path $\Re_m = \Re_c + 2\Re_g$

$$\Re_c = \frac{l_c}{\mu_r \mu_0 A_c}$$ - Reluctance of the stationary + movable core

$$\Re_g(x) = \frac{x}{\mu_0 A_g}$$ - Reluctance of the air-gap

Assume $A_c = A_g = A$ we get $\Re_m(x) = \frac{1}{\mu_0 A}\left(\frac{l_c}{\mu_c} + 2x\right)$

Magnetizing inductance $L_m = \frac{N^2}{\Re_m} = N^2 \mu_0 A \frac{1}{(l_c/\mu_c + 2x)} = \frac{k_1}{k_2 + x}$

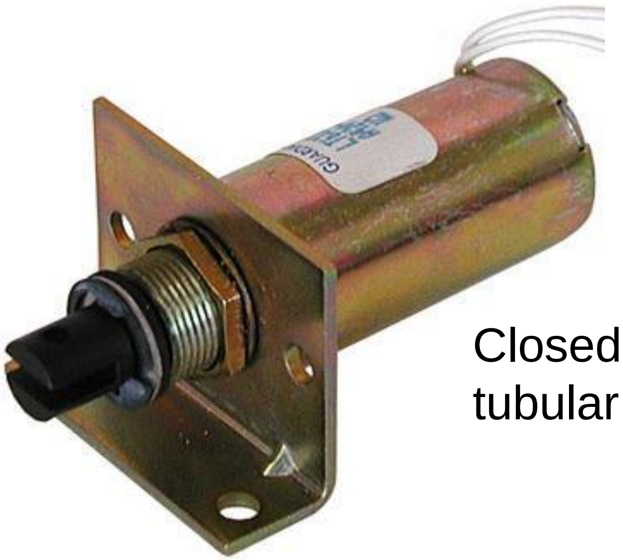
Total inductance

$$L = L_m + L_l = \frac{k_1}{k_2 + x} + L_l$$

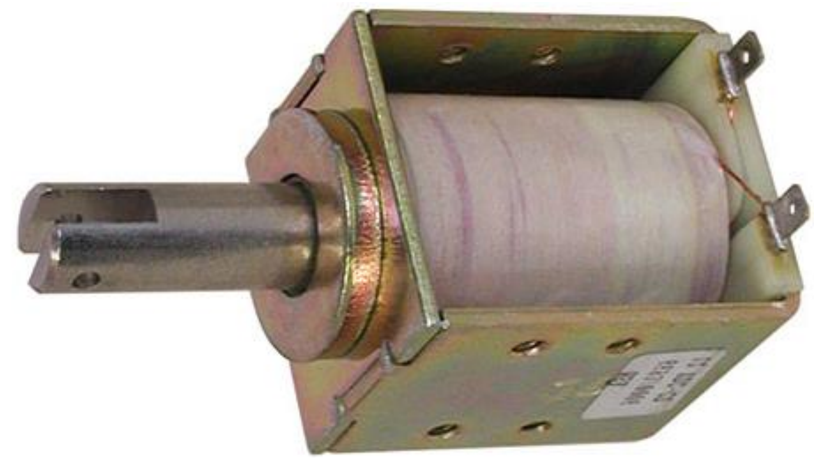
where $k_1 = \frac{N^2 \mu_0 A}{2}$ and $k_2 = \frac{l_c}{2\mu_c}$

Practical Reluctance Devices

Plunger solenoid (Lab-1)

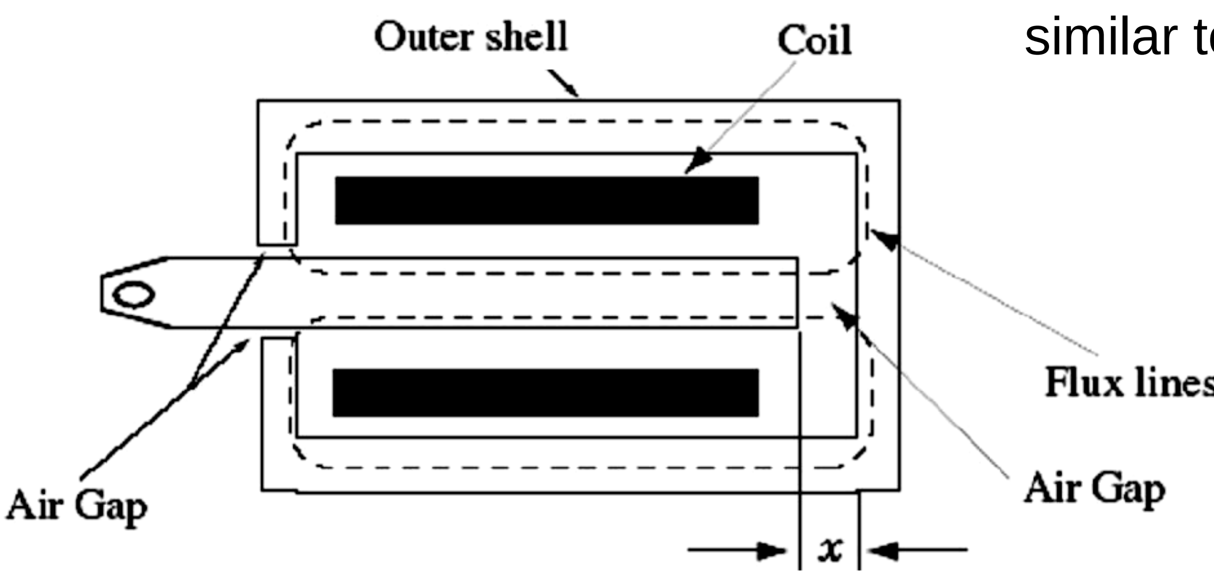


Closed-frame tubular solenoid



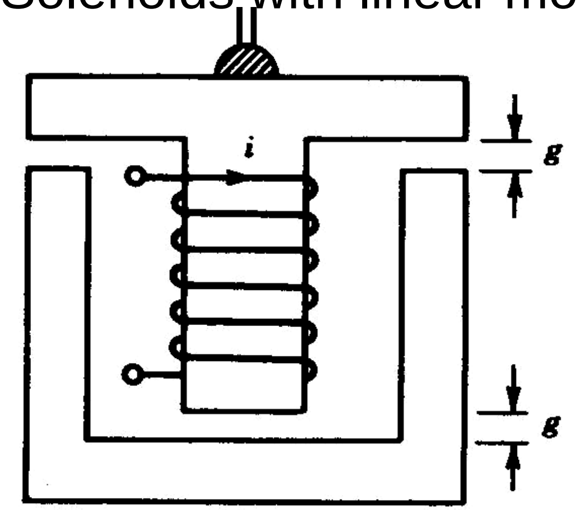
Open-frame solenoid

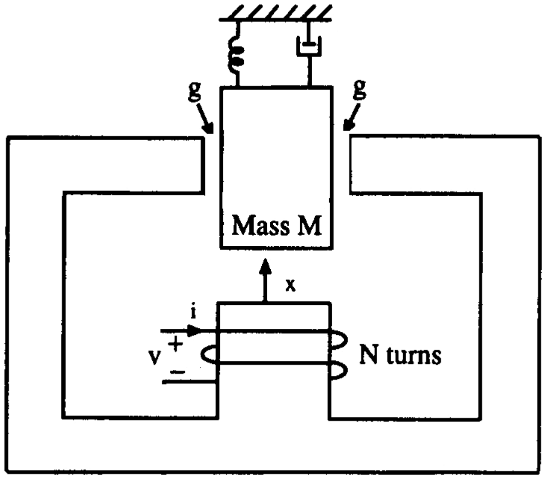
The basic electromagnet is very similar to a plunger solenoid (Lab-1)

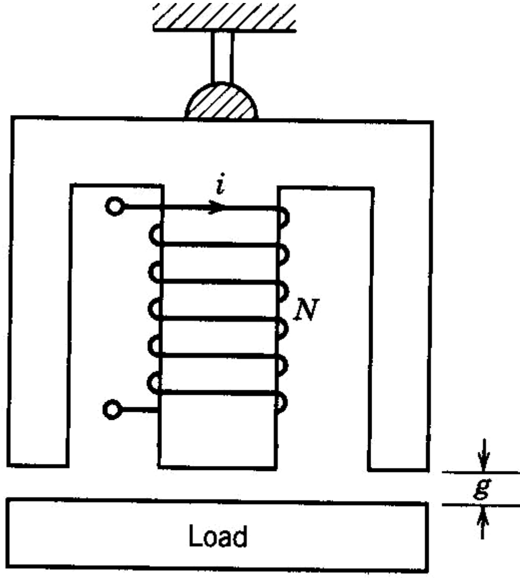


Other Reluctance Devices

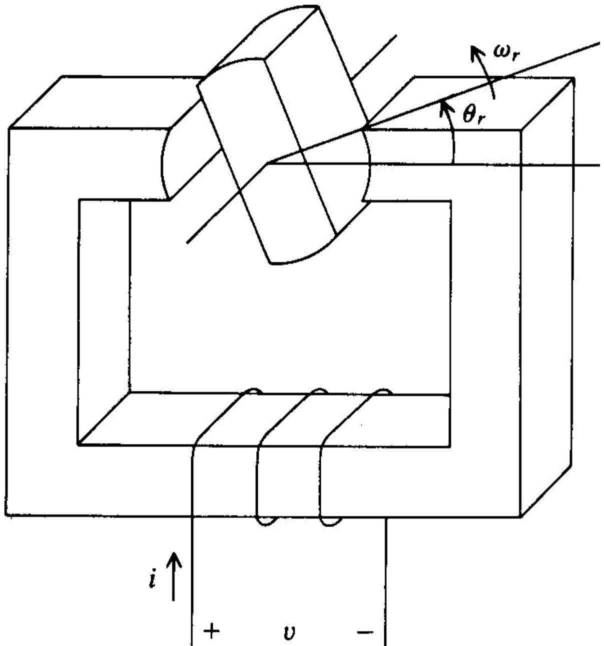
Solenoids with linear motion

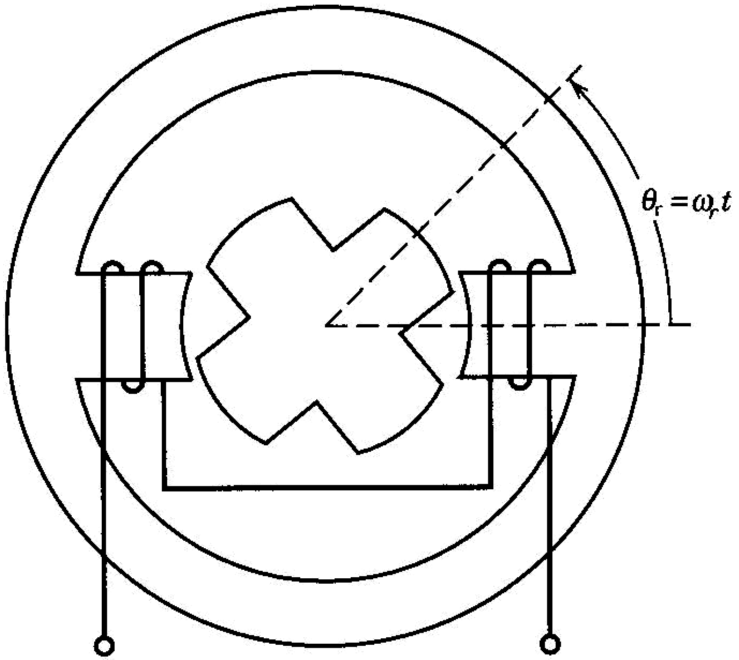




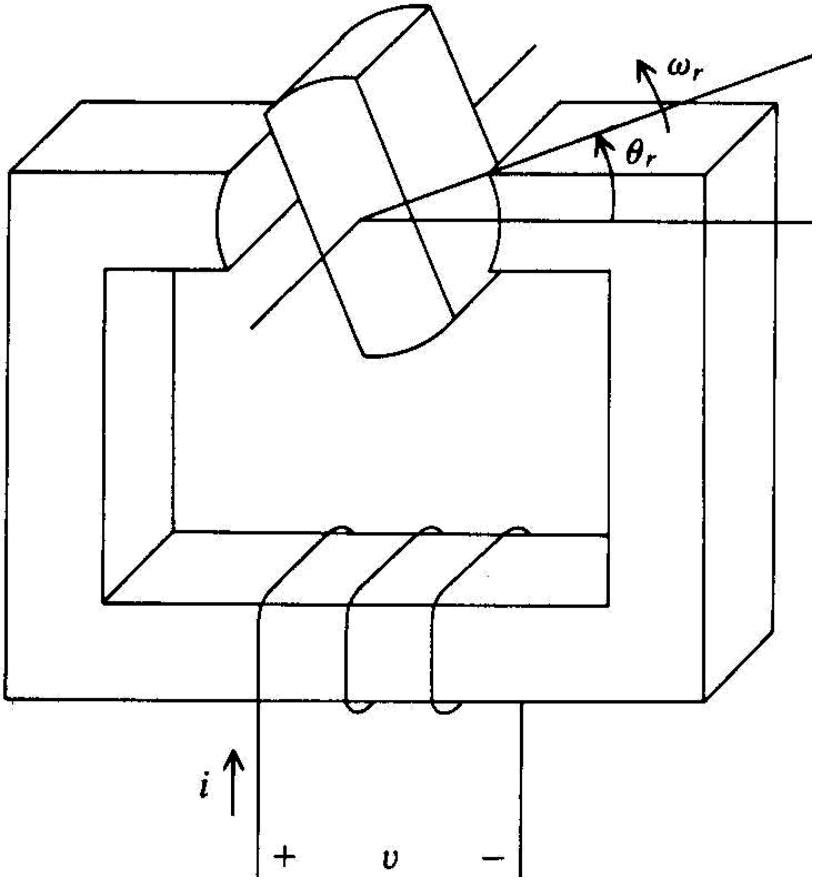


Rotating reluctance devices





Rotating Reluctance Devices



Flux linkage & inductances

$$\lambda = \left(\frac{N^2}{\Re_l} + \frac{N^2}{\Re_m} \right) i = \left(L_l + L_m \right) i$$

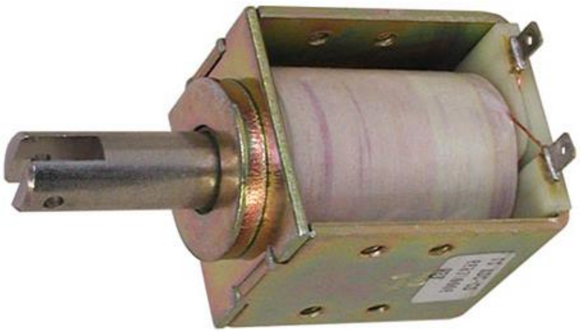
Magnetizing inductance

$$L_m = L_m(\theta_r) = \frac{N^2}{\Re_m(\theta_r)}$$

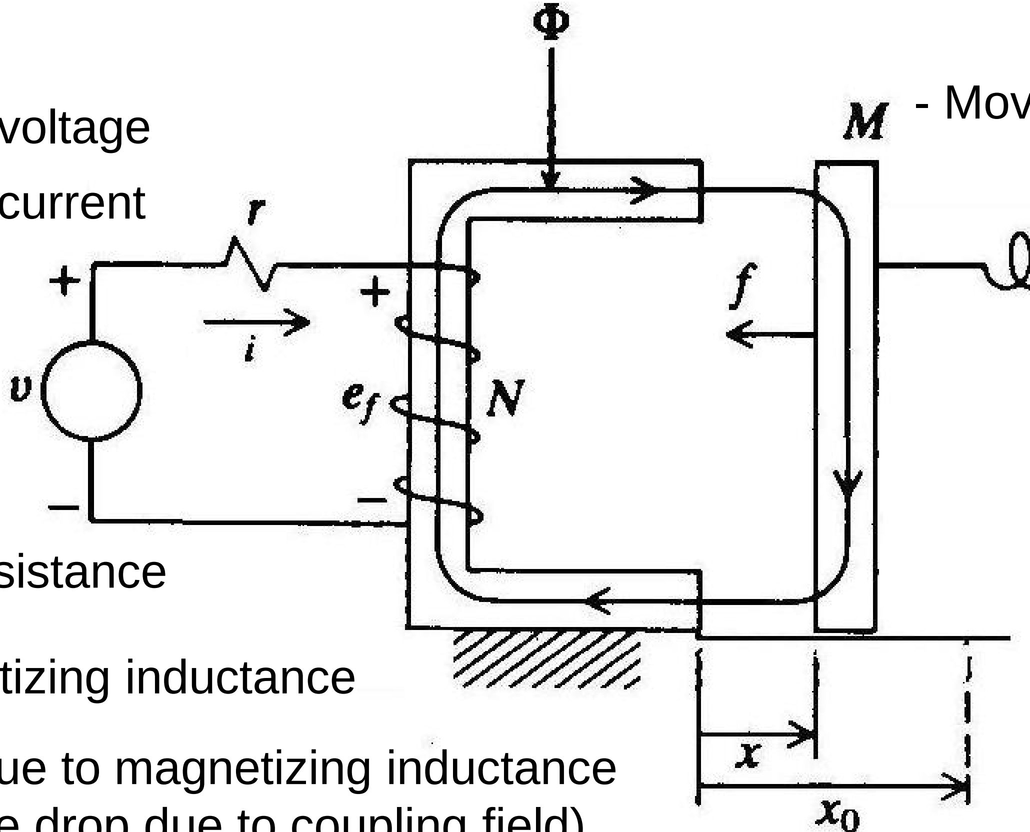
$\Re_m(0)$ - Maximum reluctance

$\Re_m(\pi/2)$ - Minimum reluctance

Electromechanical Energy Conversion



Basic Electromechanical System



v - Source voltage

i - Source current

M - Movable mass

K - Spring constant

f_m - Electromagnetic force

r - Coil resistance

L_m - Magnetizing inductance

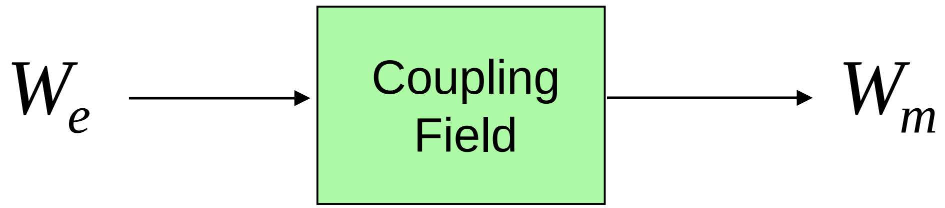
e_f - EMF due to magnetizing inductance (voltage drop due to coupling field)

W_e - Energy supplied from electrical system (going into coupling field)

W_f - Energy in coupling field

W_m - Energy going into mechanical system (from coupling field)

Electromechanical Energy Conversion



Energy Balance
$$W_e = W_f + W_m = \int e_f i dt = W_f + \int f_m dx$$

First, lets consider fixed position, and assume $dx = 0$

$$W_f = \int e_f i dt = \int \frac{d\lambda}{dt} i dt = \int i d\lambda$$

Energy in Coupling Field

Consider a state of the system

$$i = i_a \qquad \lambda = \lambda_a$$

Energy going in coupling field

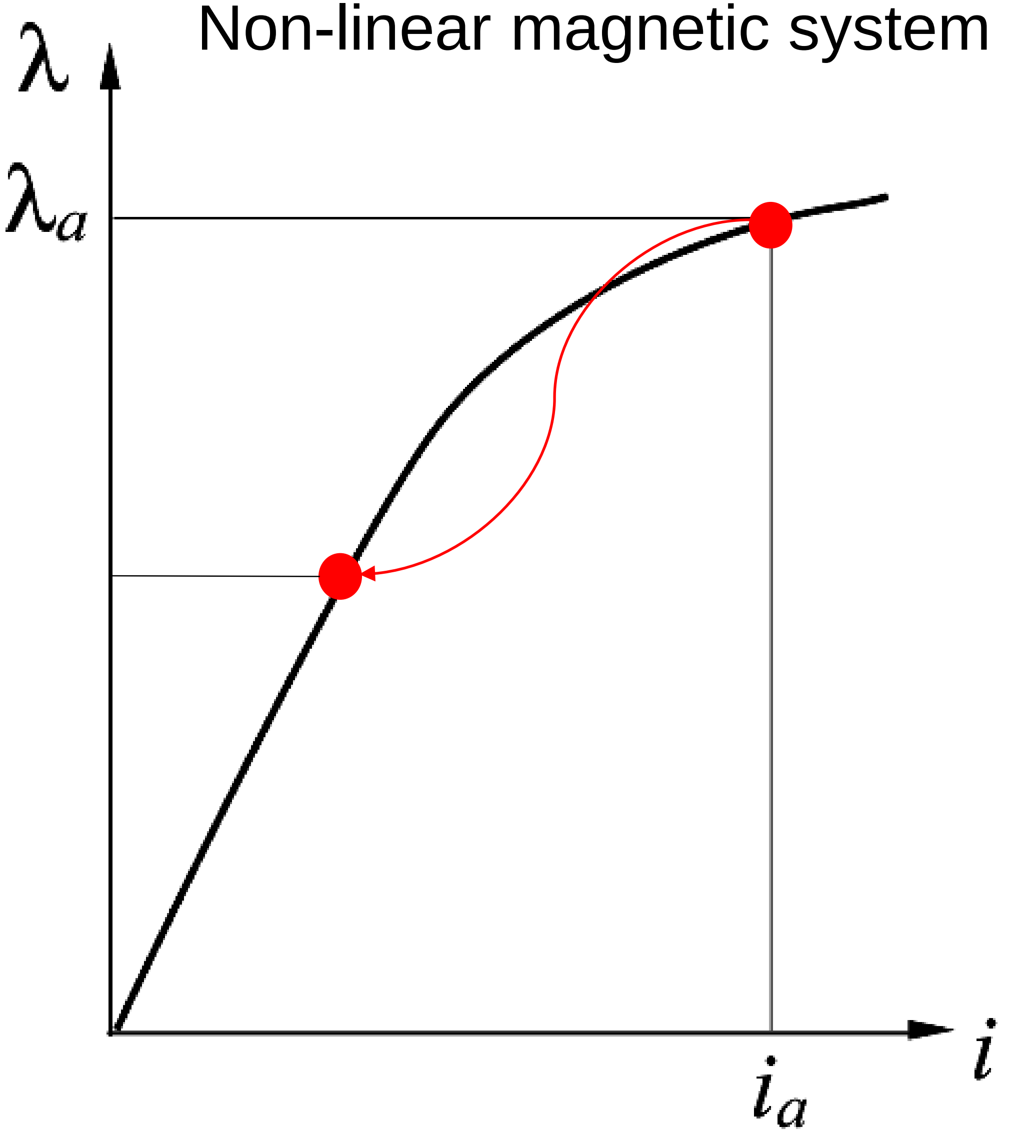
$$W_f = \int i d\lambda$$

Co-Energy associated with this state

$$W_c = \int \lambda di$$, assuming $dx = 0$

Energy and Co-Energy Balance

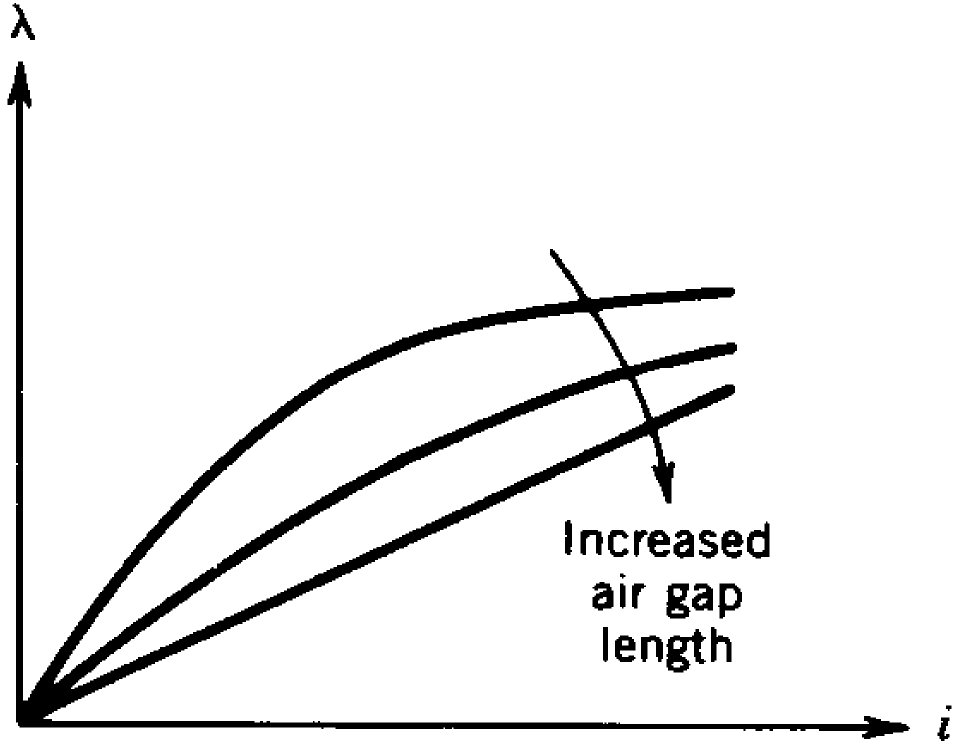
$$\lambda i = W_f + W_c$$



Coupling Field is Conservative – The stored energy does not depend on the history of electromechanical variables, it depends only on their final state/values

Energy in Coupling Field

The characteristic becomes linear by increasing the air gap



Energy in Coupling Field

Consider a state of the system

$$i = i_a \qquad \lambda = \lambda_a$$

Energy going in coupling field

$$W_f = \int i d\lambda$$

Co-Energy associated with this state

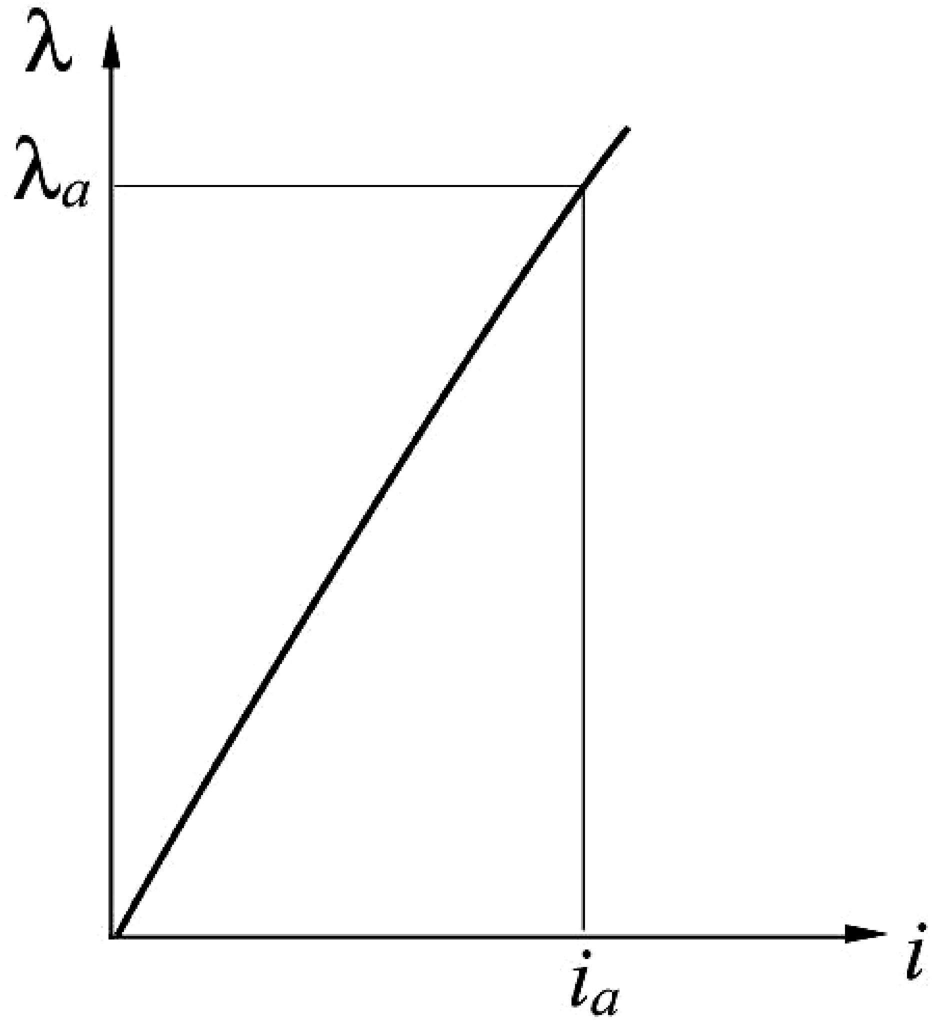
$$W_c = \int \lambda di$$, assuming $dx = 0$

For magnetically linear systems
Energy and Co-Energy Balance

$$W_f = W_c = \frac{1}{2}\lambda i$$

Coupling Field is Conservative – The stored energy does not depend on the history of electromechanical variables, it depends only on their final state/values

Linear (Approximate) magnetic system

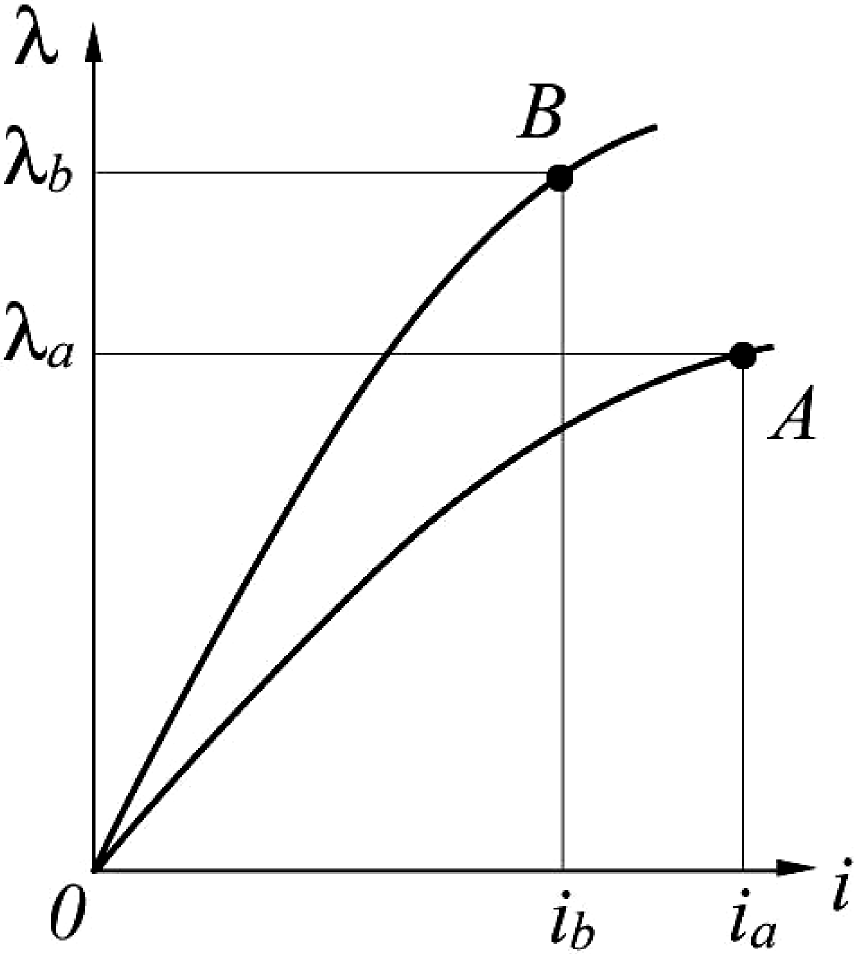


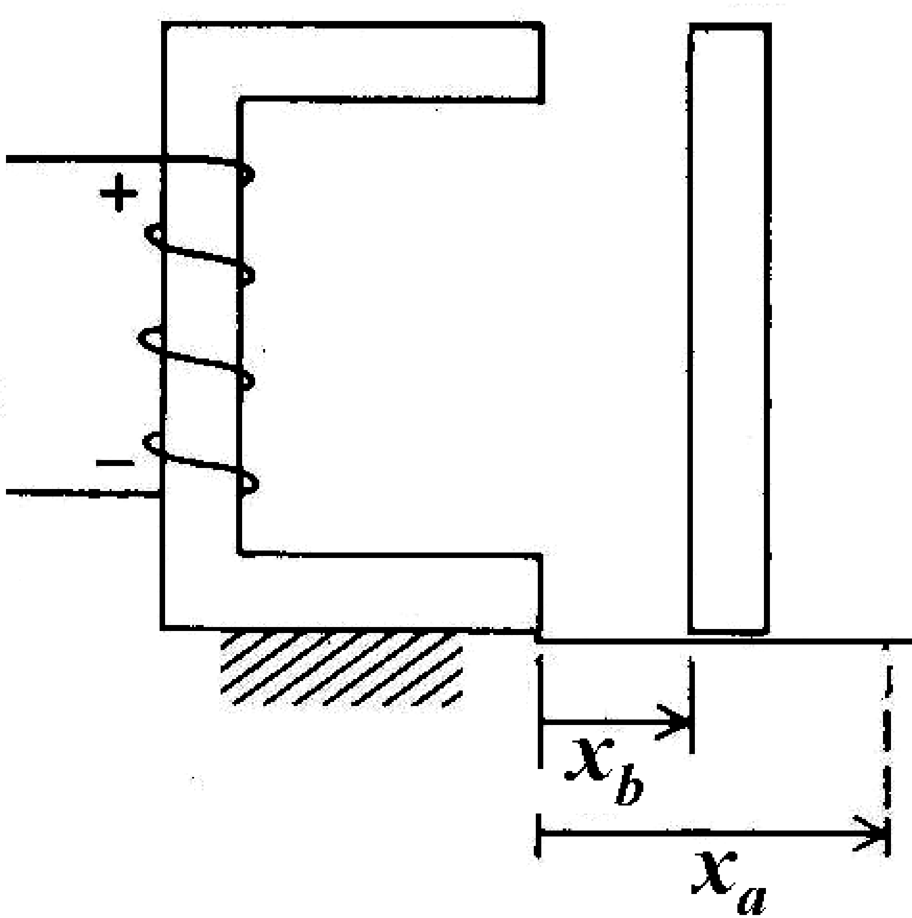
Electromechanical Energy Conversion

Graphical interpretation

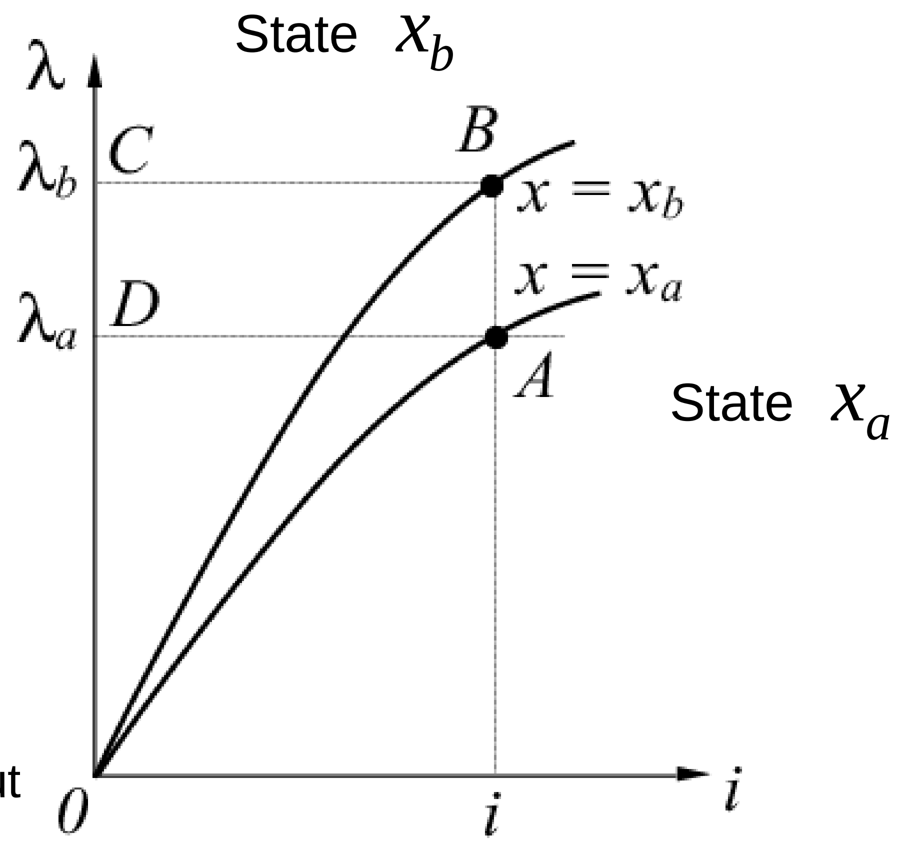
Assume move from x_a to x_b

Consider $\lambda - i$ relationship





Change in Energy



Change in electrical input

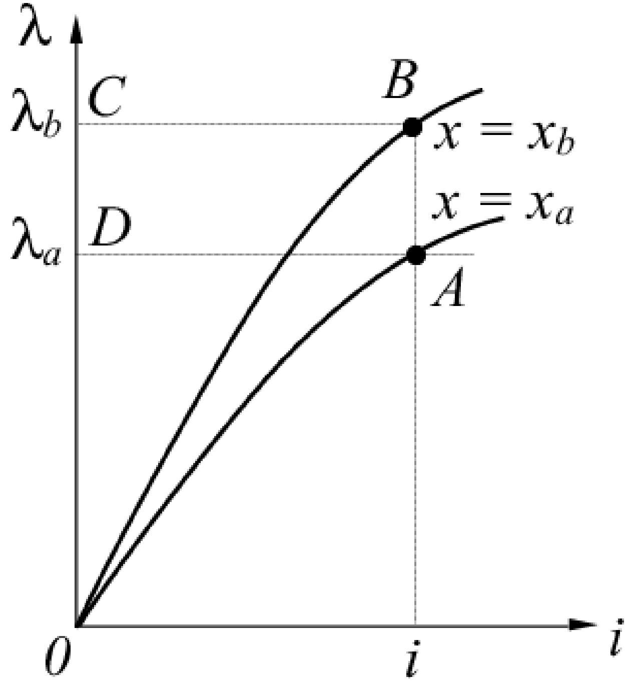
$$\Delta W_e = \int i e_f \, dt = \int_{\lambda_a}^{\lambda_b} i \, d\lambda = ABCD$$

Coupling Field Energy

$$\Delta W_f = OBC - OAD$$

Change in Energy

Change in Mechanical Energy $\Delta W_m = \Delta W_e - \Delta W_f = ABCD - (OBC - OAD)$



$$\Delta W_m =$$

Remember Co-Energy

$$W_c = \int \lambda di$$

Change in mechanical work under constant current

$$\Delta W_m = \Delta W_c = f_m \Delta x$$

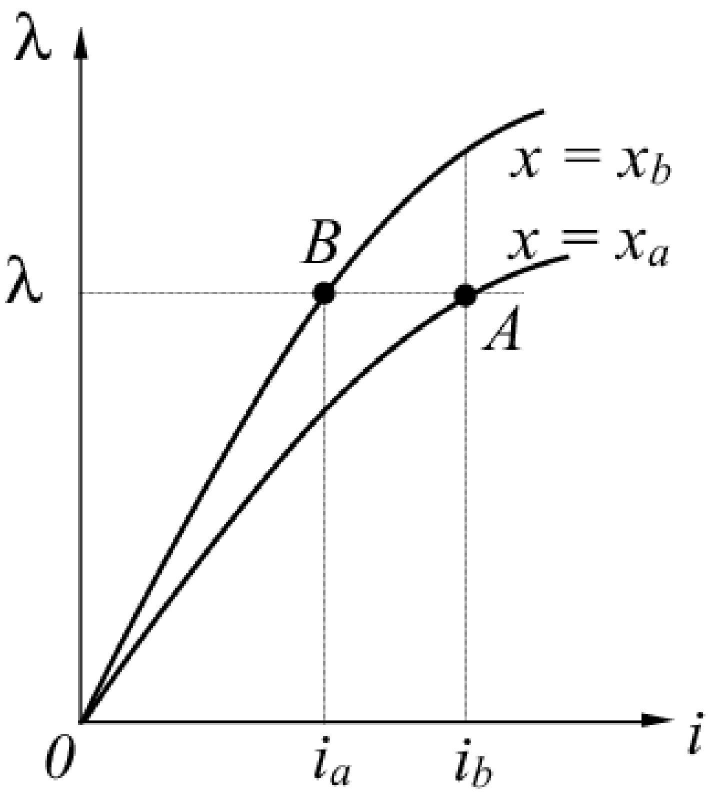
Developed Electromagnetic Force

$$f_m(i,x) = \left. \frac{\partial W_c(i,x)}{\partial x} \right|_{i=const}$$

Change in Energy

Change in Mechanical Energy at constant flux

$$\Delta W_m = \Delta W_e - \Delta W_f = OAB$$



Change in mechanical work under constant current

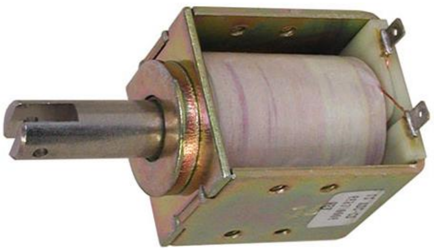
$$\Delta W_m = f_m \Delta x = -\Delta W_f$$

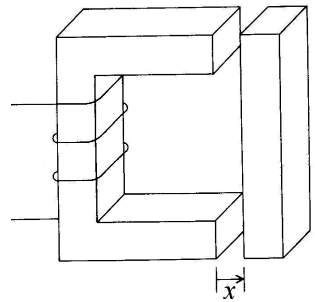
Developed Electromagnetic Force

$$f_m(i,\lambda) = -\left.\frac{\partial W_f(i,\lambda)}{\partial x}\right|_{\lambda=const}$$

Electromagnetic Forces & Torques

Linear Devices





Mechanical Energy/Work

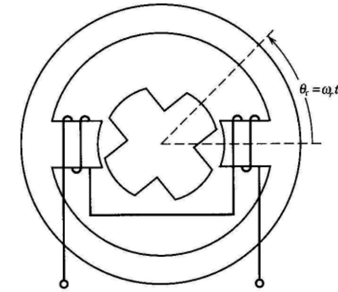
$$W_m = \int f_m dx$$

Electromagnetic Force f_m

$$dW_m = f_m dx$$

$$f_e(i, x) = \frac{\partial W_c}{\partial x} \qquad f_e(\lambda, x) = -\frac{\partial W_f}{\partial x}$$

Rotating Devices



Mechanical Energy/Work

$$W_m = \int T_m d\theta$$

Electromagnetic Torque T_m

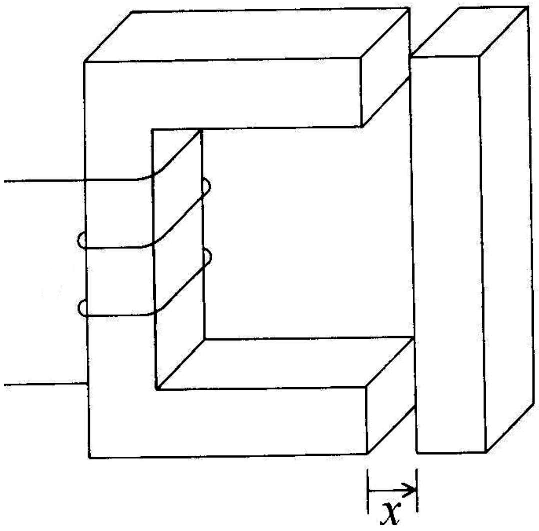
$$dW_m = T_m d\theta$$

$$T_e(i, \theta) = \frac{\partial W_c}{\partial \theta} \qquad T_e(\lambda, \theta) = -\frac{\partial W_f}{\partial \theta}$$

For magnetically linear systems
Energy and Co-Energy are the same

$$W_f = W_c = \frac{1}{2}\lambda i = \frac{1}{2}L(x)i^2$$

Linear Devices



Example

Given that

$$\lambda(i,x) = Li = [L_l + L_m(x)]i = \left(L_l + \frac{k}{x} \right) i$$

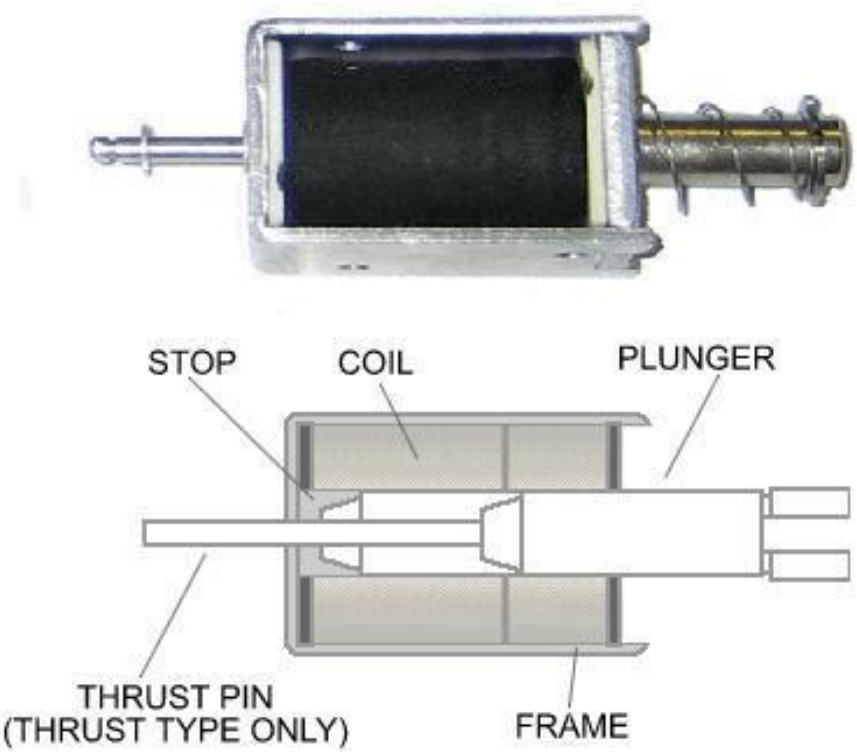
Calculate $f_m(i,x)$ at given current $i = i_a$

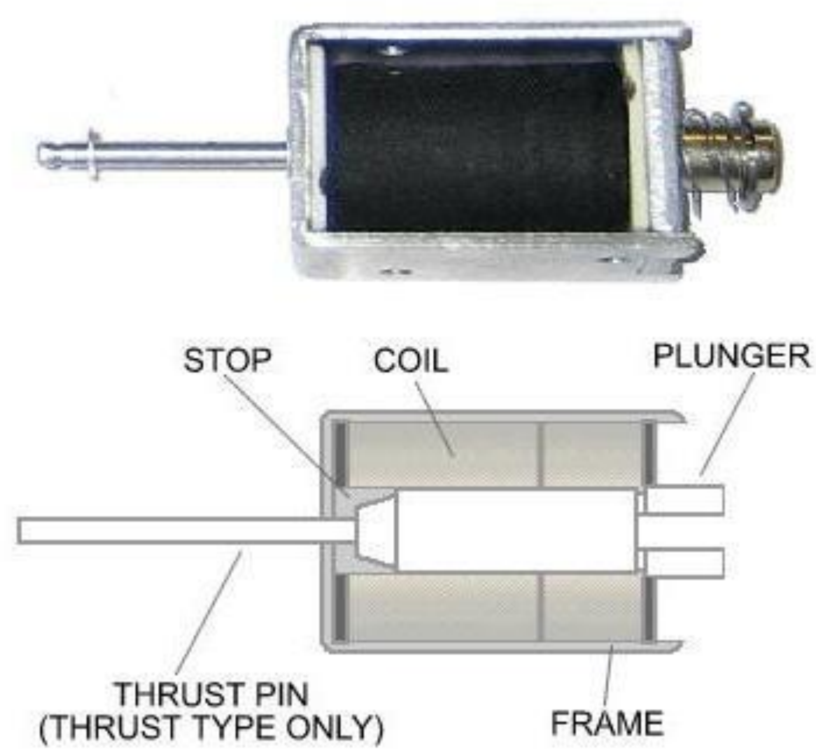
$$W_f(i,x) = W_c(i,x) = \left(L_l + \frac{k}{x} \right) i_a^2$$

$$f_m(i,x) = \frac{\partial W_c}{\partial x} = \frac{1}{2} i_a^2 \frac{\partial L}{\partial x} = -\frac{1}{2} i_a^2 \frac{k}{x^2}$$

Typical Push-Pull Solenoids

Naturally, solenoid coil pulls in the plunger. To get a push action, a thrust pin is added.





Typical Industrial Push-Pull Solenoids